A Leakage-Based Precoding Scheme for Downlink Multi-User MIMO Channels

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***Abstract*— In multiuser MIMO downlink communications, it is necessary to design precoding schemes that are able to suppress co-channel interference. This paper proposes designing precoders by maximizing the so-called *signal-to-leakage-and-noise ratio* (SLNR) for all users simultaneously. The presentation considers communications with both single- and multi-stream cases, as well as MIMO systems that employ Alamouti coding. The effect of channel estimation errors on system performance is also studied. Compared with zero-forcing solutions, the proposed method does not impose a condition on the relation between the number of transmit and receive antennas, and it also avoids noise enhancement. Simulations illustrate the performance of the scheme.**

***Index Terms*— MIMO communications, multiuser precoding, Alamouti coding, multiuser beamforming, generalized eigenvalue problem.**

I. INTRODUCTION

IN multiuser MIMO downlink communications, a base station communicates with several co-channel users in the same frequency and time slots. It is therefore necessary to rely on transmission schemes that are able to suppress co-channel interference (CCI) at the end users. The suppression of CCI can be pursued by using linear precoders and decoders at both the transmitter and receiver. Both joint and independent optimization schemes for precoders and decoders have already been studied in the literature, e.g., for the single user case in [1], [2], [3].

In the multiuser case, several works have proposed schemes for choosing the weights of the precoders and decoders. For instance, some schemes choose the precoders and decoders optimally in order to maximize the output signal-to-interference-plus-noise ratio (SINR) [4]. In these cases, the solution can only be obtained iteratively due to the coupled nature of the corresponding optimization problem and its complexity.

Other works have proposed schemes for perfectly canceling the CCI for each user (also referred to as zero-forcing solutions) [5], [6], [7], [8]. These schemes impose a restriction on the system configuration in terms of the number of antennas. Roughly, they require the number of transmit antennas at the base station to be larger than the sum of receive antennas of all users. This condition is necessary in order to provide enough degrees of freedom for the zero-forcing solution to force the CCI to zero at each user. One way to apply such zero-forcing solutions when the (dimension) condition is not met is to resort to time-scheduling [9]. In this case, a subset of the users communicates at each time slot such that the total number of receive antennas for active users at any time instant satisfies the required dimension condition [10].

In this paper, we pursue an alternative approach for designing transmit beamforming vectors based on the concept of signal *leakage*, as advanced in [11] and subsequently used in [12]. While CCI refers to the interference at a desired user that is caused by all other users, leakage refers to the interference caused by the signal intended for a desired user on the remaining users. That is, leakage is a measure of how much signal power leaks into the other users. The performance criterion for choosing the beamforming coefficients will be based on maximizing the *signal-to-leakage-and-noise ratio* (SLNR) for all users simultaneously. While the problem of maximizing the alternative so-called signal-to-interference-noise ratio (SINR) for all users has already been studied in the literature [13], no closed form solutions are available due to the complexity and the coupled nature of the resulting optimization problem. On the other hand, the leakage-based criterion leads to a decoupled optimization problem and admits an analytical closed form solution [11], [12]. Moreover, in contrast to the zero-forcing solution, the leakage scheme does not require any dimension condition on the number of transmit/receive antennas. It further takes into account the influence of noise when designing the beamforming vectors. By doing so, the leakage solution outperforms zero-forcing solutions even when the dimension requirement for zero-forcing solutions is satisfied.

The development and analysis in [11], [12] focused on the single stream case, which will be reviewed below in Sec. III. In this paper, we consider at least three extensions: (1) we investigate the incorporation of the leakage-based solution to MIMO systems that employ Alamouti coding, (2) we also apply the leakage-based solution to the case of multiple streams per user, and (3)finally, we examine the effect of channel estimation errors on system performance. In the article [12] we used the SLNR criterion of the single-stream, and a detailed random matrix spectral analysis, to motivate an antenna selection mechanism for multi-user MIMO systems.

The paper is organized as follows. The next section de-

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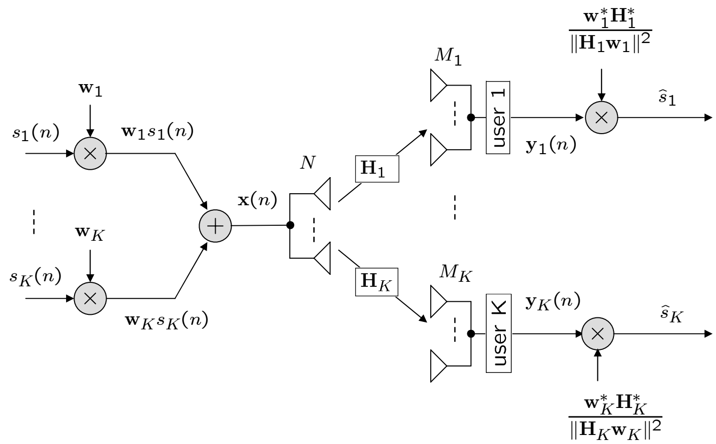


Fig. 1. Block diagram of a multi-user beamforming wireless communications system.

scribes the MU-MIMO system considered in the paper. Sec. III reviews the leakage-based criterion and its solution. Sec. IV extends the method to MIMO systems with Alamouti coding. Sec. V designs the precoders for the multi-stream-per-user case. Sec. VI shows how to modify the design in order to account for channel estimation errors at the transmitter, and Sec. VII presents simulation results.

II. System Model

Consider a downlink multi-user environment with a base station communicating with K users. The base station employs N transmit antennas and each user could be equipped with multiple antennas as well. Let M_i denote the number of receive antennas at the ith user. A block diagram of the system is shown in Figure 1, where $s_i(n)$ denotes the transmitted data intended for user i at time n. The scalar symbol $s_i(n)$ is multiplied by an $N \times 1$ beamforming vector $\mathbf{w}_i$ prior to transmission over the channel. In this way, the overall $N \times 1$ transmitted vector at time n is given by

$$\mathbf{x}(n) = \sum_{k=1}^{K} \mathbf{w}_k s_k(n) \qquad (N \times 1) \tag{1}$$

The data $s_i(n)$ and the beamforming vectors $\mathbf{w}_k$ are assumed to be normalized as follows:

$$\mathsf{E}|s_k(n)|^2 = 1, \quad \|\mathbf{w}_k\|^2 = 1$$

for $k = \{1, \ldots, K\}$.

The $N \times 1$ vector $\mathbf{x}(n)$ is broadcast over the channel. Assuming a narrow-band (single-path) channel, the received vector of size $M_i \times 1$ at the ith user at time n is given by

$$\begin{aligned} \mathbf{y}_i(n) &= \mathbf{H}_i \mathbf{x}(n) + \mathbf{v}_i(n) \qquad (M_i \times 1) \\ &= \mathbf{H}_i \sum_{k=1}^{K} \mathbf{w}_k s_k(n) + \mathbf{v}_i(n) \end{aligned} \tag{2}$$

where the entries of the $M_i \times N$ channel matrix $\mathbf{H}_i$ are denoted by

$$\mathbf{H}_i = \begin{bmatrix} h_i^{(1,1)} & \ldots & h_i^{(1,N)} \\ \vdots & \ddots & \vdots \\ h_i^{(M_i,1)} & \ldots & h_i^{(M_i,N)} \end{bmatrix} \qquad (M_i \times N) \tag{3}$$

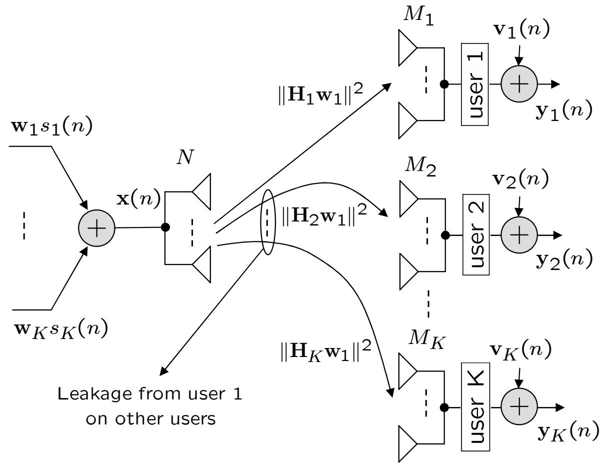


Fig. 2. A block diagram depicting the leakage from user 1 on other users.

with $h_i^{(k,l)}$ representing the channel coefficient from the lth antenna at the base station to the kth receiver antenna at user i. The elements of $\mathbf{H}_i$ are assumed to be complex Gaussian variables with zero-mean and unit-variance, so that $\mathsf{E}\mathsf{Tr}(\mathbf{H}_i\mathbf{H}_i^*) = M_i N$. Furthermore, the additive noise vector $\mathbf{v}_i(n)$ is assumed to have independent complex Gaussian elements with variance σ_i^2 and is spatially white, i.e.,

$$\mathsf{E}\left[\mathbf{v}_i(n)\mathbf{v}_j^*(n)\right] = \sigma_i^2 \mathbf{I}_{M_i} \delta_{ij}$$

where $\mathbf{I}_{M_i}$ is the $M_i \times M_i$ identity matrix. Since the random quantities $\mathbf{H}_i$, $s_i(n)$, and $\mathbf{v}_i(n)$ are assumed independent, we shall plot all the BER curves in this paper versus $1/\sigma_i^2$. The quantity $1/\sigma_i^2$ represents the SNR per receive antenna for systems without precoding; we shall refer to $1/\sigma_i^2$ as the received SNR throughout the paper and the simulation results are plotted against this quantity.

We assume initially that the channel matrices $\mathbf{H}_i$, $i = \{1, \ldots, K\}$, are available at the base station (e.g., either through reverse channel estimation in time-division-duplex (TDD) or feedback in frequency-division-duplex (FDD)). We also assume that the channel matrix $\mathbf{H}_i$ is known at the corresponding receiver i, but is not required to be known by the other users. Furthermore, we assume a slow-fading wireless channel with packet-based transmission where the channel is quasi-static over a packet length, and changes independently between consecutive transmissions. Later in Sec. VI we show how to modify the design in order to account for channel estimation errors at the transmitter.

III. Multi-user Beamforming and Leakage

We start from the received signal (2) by user i and drop the time index n for notational simplicity so that

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{w}_i s_i + \sum_{k=1,k\neq i}^{K} \mathbf{H}_i \mathbf{w}_k s_k + \mathbf{v}_i \qquad (M_i \times 1) \quad (4)$$

where the second term is the co-channel interference (CCI) caused by the multi-user nature of the system. The signal-to-interference-plus-noise ratio (SINR) at the *input* of the receiver is given by

$$\text{SINR}_i = \frac{\|\mathbf{H}_i \mathbf{w}_i\|^2}{M_i \sigma_i^2 + \sum_{k=1,k\neq i}^{K} \|\mathbf{H}_i \mathbf{w}_k\|^2} \quad (5)$$

One could use the SINR expression in (5) for $i = \{1, \ldots, K\}$ as an optimization criterion for determining the $\{\mathbf{w}_i\}_{i=1}^{K}$, i.e., the beamforming vectors $\{\mathbf{w}_i\}_{i=1}^{K}$ would be determined so as to maximize the SINR for each user i. However, this criterion generally results in a challenging optimization problem to with K coupled variables $\{\mathbf{w}_i\}$ [4], [13].

To avoid solving the coupled problem, in prior work on downlink multi-user MIMO systems, the major focus has been on cancelling the CCI term perfectly by using zero-forcing (ZF) schemes. For example in [5], [8], the criterion for choosing the beamforming vectors $\mathbf{w}_i$, $i = \{1, \ldots, K\}$, has been to enforce the conditions

$$\mathbf{H}_i \mathbf{w}_k = \mathbf{0} \qquad \text{for all } i, k = \{1, \ldots, K\}, i \neq k. \quad (6)$$

This solution results in good performance since it completely cancels the CCI at every receiver. However, this solution is sensitive to unmodeled interferences and other sources of distortion. Moreover, choosing the $\{\mathbf{w}_k\}$ according to (6) imposes a strong condition on the system configuration in terms of the number of antennas that are needed. Specifically, in order for the problem (6) to be well posed (i.e., in order for solutions $\mathbf{w}_k$ to exist), one needs to require

$$N > \max_i \left\{ \sum_{k=1,k\neq i}^{K} M_k \right\} \quad (7)$$

That is, the number of transmit antennas essentially needs to be as large as the number of all receive antennas combined. Thus the scheme (6) requires an increase in the number of base station antennas as the number of users or the number of receive antennas per user increase. Also, the ZF solution can lead to a small signal-to-noise-ratio since it ignores the noise power in finding $\mathbf{w}_i$. For these reasons, we shall rely on an alternative criterion that relaxes the requirement (7) and that takes the noise contribution into account when choosing $\mathbf{w}_i$. The criterion is based on defining a so-called signal-to-leakage-plus-noise ratio (SLNR) as advaned in [11] and used in [12]. It leads to a closed form characterization of the optimal $\{\mathbf{w}_i\}$ in terms of generalized eigenvalue problems. Moreover, the scheme does not require the dimensionality condition (7).

The following is a summary of the leakage-based solution from [11] [12]. Start from (4) and note that the power of the desired signal component for user i is given by $\|\mathbf{H}_i \mathbf{w}_i\|^2$. At the same time, the power of the interference that is caused by user i on the signal received by some other user k is given by $\|\mathbf{H}_k \mathbf{w}_i\|^2$. We thus define a quantity, called *leakage* for user i, as the total power leaked from this user to all other users–see Figure 2:

$$\sum_{k=1,k\neq i}^{K} \|\mathbf{H}_k \mathbf{w}_i\|^2$$

For each user i, we would like its signal power, $\|\mathbf{H}_i \mathbf{w}_i\|^2$, to be large compared to the noise power at its receiver (i.e., $M_i \sigma_i^2$). We would also like $\|\mathbf{H}_i \mathbf{w}_i\|^2$ to be large compared to the power leaked from user i to all other users, i.e., $\sum_{k=1,k\neq i}^{K} \|\mathbf{H}_k \mathbf{w}_i\|^2$. These considerations motivate us to introduce a figure of merit in terms of so-called signal-to-leakage-noise ratio (SLNR) defined as

$$\text{SLNR}_i = \frac{\|\mathbf{H}_i \mathbf{w}_i\|^2}{M_i \sigma_i^2 + \sum_{k=1,k\neq i}^{K} \|\mathbf{H}_k \mathbf{w}_i\|^2} \quad (8)$$

Using this concept of leakage, we can formulate an optimization problem which instead of dealing with the total interference of all users on user i as in (5), it deals with the total interfering power that user i causes on all other users. Specifically, we would like to select beamforming vectors $\mathbf{w}_i$, $i = \{1, \ldots, K\}$, such that (8) is maximized over $\mathbf{w}_i$ and subject to $\|\mathbf{w}_i\|^2 = 1$.

The SLNR expression in (8) can be rewritten as

$$\text{SLNR}_i = \frac{\|\mathbf{H}_i \mathbf{w}_i\|^2}{M_i \sigma_i^2 + \|\tilde{\mathbf{H}}_i \mathbf{w}_i\|^2} \quad (9)$$

where

$$\tilde{\mathbf{H}}_i = [\mathbf{H}_1 \cdots \mathbf{H}_{i-1} \mathbf{H}_{i+1} \cdots \mathbf{H}_K]^T \qquad \left(\sum_{k\neq i} M_k \times N \right) \quad (10)$$

is an extended channel matrix that excludes $\mathbf{H}_i$ only. It was shown in [12] that the solution is given by

$$\boxed{\mathbf{w}_i^o \propto \text{max. eigenvector}\left(\left(M_i \sigma_i^2 \mathbf{I} + \tilde{\mathbf{H}}_i^* \tilde{\mathbf{H}}_i \right)^{-1} \mathbf{H}_i^* \mathbf{H}_i \right)} \quad (11)$$

in terms of the eigenvector corresponding to the largest eigenvalue of the matrix $\left(M_i \sigma_i^2 \mathbf{I} + \tilde{\mathbf{H}}_i^* \tilde{\mathbf{H}}_i \right)^{-1} \mathbf{H}_i^* \mathbf{H}_i$. The norm of $\mathbf{w}_i^o$ is adjusted to $\|\mathbf{w}_i^o\|^2 = 1$.

For comparison purposes, we also mention the zero-forcing solution for the choice of $\mathbf{w}_i$ from [5], [8], namely,

$$\mathbf{w}_i = \mathbf{G}_i \mathbf{u}_i \quad (12)$$

where $\mathbf{G}_i = \mathbf{I} - \tilde{\mathbf{H}}_i^{\dagger} \tilde{\mathbf{H}}_i$ and $\mathbf{u}_i \propto$ max. eigenvector$(\mathbf{H}_i \mathbf{G}_i)$, and $\tilde{\mathbf{H}}_i^{\dagger}$ is the pseudo inverse of $\tilde{\mathbf{H}}_i$; again the norm of $\mathbf{w}_i$ is normalized to unity. Note that $\mathbf{H}_i \mathbf{G}_i$ reduces to zero if $\tilde{\mathbf{H}}_i$ is a tall matrix suggesting that $\mathbf{w}_i = 0$. This explains why the zero-forcing solution is only applicable when the dimension condition [5] is satisfied. It is worth noting that the computational complexity of the ZF solution (12) and the leakage-based solution (11) are similar, namely, $O(N^3)$.

Observe that the vector $\mathbf{w}_i^o$ that optimizes the SLNR is not optimal relative to the SINR criterion (5), which is the criterion that is usually used to evaluate system performance. As mentioned before, optimizing (5) over $\mathbf{w}_i$ is challenging and we are therefore using the alternative SLNR criterion (8).

IV. Multi-user Beamforming with Alamouti Coding

We now show how to extend the leakage-based technique to MIMO systems that employ Alamouti coding [14], [15], [16]. In this case, we would need to design beamforming (or precoding) *matrices* $\mathbf{W}_i$ as opposed to beamforming (or precoding) vectors $\mathbf{w}_i$. This is because the symbols will now be transmitted in pairs over two consecutive time instants.

Thus refer to Figure 3, which shows a block diagram of the system. We have dropped the time index n for notational simplicity. In this figure, $s_{i,1}$ and $s_{i,2}$ denote the transmitted data intended for user i; they are space-time coded using the Alamouti scheme [16] before transmission. Let us denote the transmitted pair of data and the transmitted coded block by

$$\mathbf{s}_i = \begin{bmatrix} s_{i,1} \\ s_{i,2} \end{bmatrix} \quad \text{and} \quad \mathbf{S}_\text{i} = \begin{bmatrix} s_{i,1} & -s_{i,2}^* \\ s_{i,2} & s_{i,1}^* \end{bmatrix} \tag{13}$$

Each matrix $\mathbf{S}_i$ satisfies

$$\mathbf{S}_i\mathbf{S}_i^* = (|s_{k,1}|^2 + |s_{k,2}|^2)\mathbf{I}_2 \tag{14}$$

For each user i, the matrix $\mathbf{S}_i$ is multiplied by an $N \times 2$ beamforming matrix $\mathbf{W}_i$ before being transmitted over the channel. In this way, the overall $N \times 2$ transmitted block is given by

$$\mathbf{X} = \sum_{k=1}^{K} \mathbf{W}_k\mathbf{S}_k \qquad (N \times 2) \tag{15}$$

The data $\{s_{i,1}, s_{i,2}\}$ and the beamforming coefficients $\mathbf{W}_i$ are assumed to be normalized as follows:

$$\mathsf{E}|s_{k,1}|^2 = \mathsf{E}|s_{k,2}|^2 = 1, \quad \mathsf{Tr}(\mathbf{W}_k^*\mathbf{W}_k) = 1$$

for $k = \{1, \ldots, K\}$.

The received block of size $M_i \times 2$ at the ith user is given by

$$\begin{aligned} \mathbf{Y}_i &= \mathbf{H}_i\mathbf{X} + \mathbf{V}_i \qquad (M_i \times 2) \\ &= \mathbf{H}_i\mathbf{W}_i\mathbf{S}_i + \mathbf{H}_i \sum_{k=1,k\neq i}^{K} \mathbf{W}_k\mathbf{S}_k + \mathbf{V}_i \end{aligned} \tag{16}$$

In order to derive the optimum precoding matrices $\mathbf{W}_i$, we proceed as follows. Let us denote the entries of the $M_i \times 2$ matrix $\mathbf{Y}_i$ by

$$\mathbf{Y}_k \triangleq \begin{bmatrix} y_k^{(1,1)} & y_k^{(1,2)} \\ y_k^{(2,1)} & y_k^{(2,2)} \\ \vdots & \vdots \\ y_k^{(M_i,1)} & y_k^{(M_i,2)} \end{bmatrix} \qquad (M_i \times 2)$$

where $y_k^{(i,j)}$ represents the received signal by user i at its jth antenna at block time intervals $j = \{1, 2\}$. Furthermore, let $\mathbf{F}_k = \mathbf{H}_i\mathbf{W}_k$ and denote its entries by

$$\mathbf{F}_k \triangleq \begin{bmatrix} f_k^{(1,1)} & f_k^{(1,2)} \\ f_k^{(2,1)} & f_k^{(2,2)} \\ \vdots & \vdots \\ f_k^{(M_i,1)} & f_k^{(M_i,2)} \end{bmatrix} \qquad (M_i \times 2)$$

Then, expression (16) can be rewritten as

$$\mathbf{Y}_i = \mathbf{F}_i\mathbf{S}_i + \sum_{k=1,k\neq i}^{K} \mathbf{F}_k\mathbf{S}_k + \mathbf{V}_i \tag{17}$$

Exploiting the orthogonal Alamouti structure (14) of the transmitted matrices $\mathbf{S}_i$ for $i = \{1, \cdots, K\}$, expression (17) can be rearranged in vector form as follows:

$$\mathbf{z}_i = \mathbf{A}_i\mathbf{s}_i + \sum_{k=1,k\neq i}^{K} \mathbf{A}_k\mathbf{s}_k + \mathbf{r}_i \qquad (2M_i \times 1) \tag{18}$$

where the entries of the $M_i \times 2$ matrix $\mathbf{Y}_i$ ($\mathbf{V}_i$) have been rearranged into the $2M_i \times 1$ vector $\mathbf{z}_i$ ($\mathbf{r}_i$) as

$$\mathbf{z}_i = \left[\; \mathbf{y}_i^{(1,1)}\mathbf{y}_i^{(1,2)*} \middle| \; \mathbf{y}_i^{(2,1)}\mathbf{y}_i^{(2,2)*} \middle| \; \cdots\cdots \middle| \; \mathbf{y}_i^{(M_i,1)}\mathbf{y}_i^{(M_i,2)*} \;\right]^T$$

Likewise, for $\mathbf{r}_i$. Moreover, the entries of $\mathbf{A}_k$ are obtained by rearranging the entries of $\mathbf{F}_k$ as

$$\mathbf{A}_k = \left[\begin{array}{cc} f_k^{(1,1)} & f_k^{(1,2)} \\ f_k^{(1,2)*} & -f_k^{(1,1)*} \\ \hline f_k^{(2,1)} & f_k^{(2,2)} \\ f_k^{(2,2)*} & -f_k^{(2,1)*} \\ \hline \vdots & \vdots \\ \vdots & \vdots \\ \hline f_k^{(M_i,1)} & f_k^{(M_i,2)} \\ f_k^{(M_i,2)*} & -f_k^{(M_i,1)*} \end{array}\right] \qquad (2M_i \times 2)$$

The SINR at the input of the receiver of user i is then given by

$$\text{SINR}_i = \frac{\|\mathbf{A}_i\|_F^2}{2M_i\sigma_i^2 + \sum_{k=1,k\neq i}^{K} \|\mathbf{A}_k\|_F^2} \tag{19}$$

It can now be verified that due to the special structure of the matrix $\mathbf{A}_i$, it holds that

$$\mathbf{A}_i^*\mathbf{A}_i = \left(\sum_{j=1}^{M_i} \alpha_j\right)\mathbf{I}_2 = \|\mathbf{F}_i\|_F^2\mathbf{I}_2$$

where the positive scalars $\{\alpha_j\}$ are given by

$$\alpha_j = \left|f_i^{(j,1)}\right|^2 + \left|f_i^{(j,2)}\right|^2$$

Since $\|\mathbf{A}_i\|_F^2 = \mathsf{Tr}(\mathbf{A}_i^*\mathbf{A}_i)$ it follows that

$$\|\mathbf{A}_i\|_F^2 = 2\|\mathbf{F}_i\|_F^2 \tag{20}$$

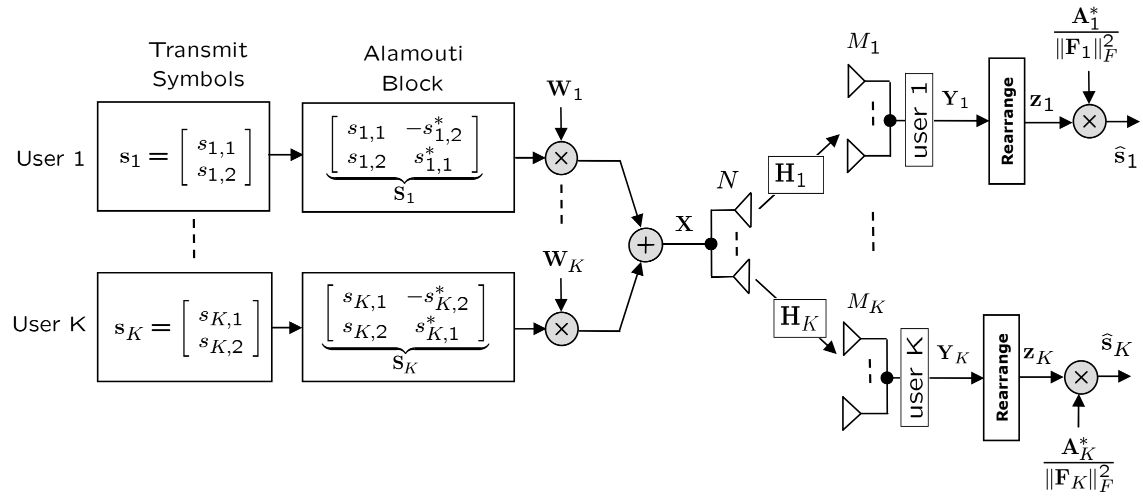


Fig. 3. Block diagram of the multi-user beamforming system with OSTBC.

Substituting (20) into (19) gives

$$\begin{aligned}\mathrm{SINR}_i &= \frac{2\|\mathbf{F}_i\|_F^2}{2M_i\sigma_i^2 + 2\sum_{k=1,k\neq i}^{K}\|\mathbf{F}_k\|_F^2} \\ &= \frac{\|\mathbf{H}_i\mathbf{W}_i\|_F^2}{M_i\sigma_i^2 + \sum_{k=1,k\neq i}^{K}\|\mathbf{H}_i\mathbf{W}_k\|_F^2}\end{aligned} \tag{21}$$

Again, one could consider selecting the $\mathbf{W}_i$, $i=\{1,\ldots,K\}$, in order to maximize this SINR expression for each user i. However, following this procedure would result in a challenging optimization problem with K coupled matrix variables $\{\mathbf{W}_i\}$. Alternatively, as we indicated in Sec. III, we consider maximizing the signal-to-leakage-noise ratio (SLNR).

Problem Statement: *Select $N\times 2$ beamforming matrices $\mathbf{W}_i$, $i=\{1,\ldots,K\}$, such that the following signal-to-leakage-plus-noise ratio (SLNR) is maximized for every user, i.e.,*

$$\mathbf{W}_i^o = \arg\max_{\mathbf{W}_i\in\mathbb{C}^{N\times 2}} \underbrace{\frac{\|\mathbf{H}_i\mathbf{W}_i\|_F^2}{M_i\sigma_i^2 + \sum_{k=1,k\neq i}^{K}\|\mathbf{H}_k\mathbf{W}_i\|_F^2}}_{\text{SLNR for user } i}$$

$$\textit{subject to } \mathsf{Tr}(\mathbf{W}_i^*\mathbf{W}_i)=1,\quad i=\{1,\ldots,K\}. \tag{22}$$

It can be verified that the SLNR expression in (22) can be written as

$$\begin{aligned}\mathrm{SLNR}_i &= \frac{\|\mathbf{H}_i\mathbf{W}_i\|_F^2}{M_i\sigma_i^2 + \mathsf{Tr}\left(\mathbf{W}_i^*\tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\mathbf{W}_i\right)} \\ &= \frac{\mathsf{Tr}(\mathbf{W}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{W}_i)}{\mathsf{Tr}\left[(\mathbf{W}_i^*(M_i\sigma_i^2\mathbf{I}+\tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i)\mathbf{W}_i\right]}\end{aligned} \tag{23}$$

where we used $\mathsf{Tr}(\mathbf{W}_i^*\mathbf{W}_i)=1$. In order to maximize the above expression for SLNR, we first denote the individual column vectors of $\mathbf{W}_i$ by

$$\mathbf{W}_i = \left(\ \mathbf{w_1}\ \middle|\ \mathbf{w_2}\ \right)$$

with $\|\mathbf{w}_1\|^2+\|\mathbf{w}_2\|^2=1$. Then the SLNR has the form

$$\mathrm{SLNR}_i = \frac{\mathbf{w}_1^*\mathbf{C}\mathbf{w}_1 + \mathbf{w}_2^*\mathbf{C}\mathbf{w}_2}{\mathbf{w}_1^*\mathbf{D}\mathbf{w}_1 + \mathbf{w}_2^*\mathbf{D}\mathbf{w}_2} \tag{24}$$

where $\mathbf{C}=\mathbf{H}_i^*\mathbf{H}_i$ and $\mathbf{D}=M_i\sigma_i^2\mathbf{I}+\tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i$. Applying the Rayleigh-Ritz quotient result [11] we have that

$$\frac{\mathbf{w}_1^*\mathbf{C}\mathbf{w}_1}{\mathbf{w}_1^*\mathbf{D}\mathbf{w}_1} \le \lambda_{\max}(\mathbf{C},\mathbf{D}) \tag{25}$$

in terms of the maximum generalized eigenvalue of $\mathbf{C}$ and $\mathbf{D}$, so that

$$\mathbf{w}_1^*\mathbf{C}\mathbf{w}_1 \le \lambda_{\max}(\mathbf{C},\mathbf{D})\,\mathbf{w}_1^*\mathbf{D}\mathbf{w}_1 \tag{26}$$

Likewise,

$$\mathbf{w}_2^*\mathbf{C}\mathbf{w}_2 \le \lambda_{\max}(\mathbf{C},\mathbf{D})\,\mathbf{w}_2^*\mathbf{D}\mathbf{w}_2 \tag{27}$$

It follows that

$$\begin{aligned}\mathrm{SLNR}_i &= \frac{\mathbf{w}_1^*\mathbf{C}\mathbf{w}_1 + \mathbf{w}_2^*\mathbf{C}\mathbf{w}_2}{\mathbf{w}_1^*\mathbf{D}\mathbf{w}_1 + \mathbf{w}_2^*\mathbf{D}\mathbf{w}_2} \\ &\le \frac{\lambda_{\max}(\mathbf{C},\mathbf{D})\,.(\mathbf{w}_1^*\mathbf{D}\mathbf{w}_1 + \mathbf{w}_2^*\mathbf{D}\mathbf{w}_2)}{\mathbf{w}_1^*\mathbf{D}\mathbf{w}_1 + \mathbf{w}_2^*\mathbf{D}\mathbf{w}_2} \\ &= \lambda_{\max}(\mathbf{C},\mathbf{D})\end{aligned} \tag{28}$$

Equality holds if we select

$$\begin{aligned}\mathbf{w}_1 &= \alpha_1.\max.\ \text{generalized eigenvector}(\mathbf{C},\mathbf{D}) \\ \mathbf{w}_2 &= \alpha_2.\max.\ \text{generalized eigenvector}(\mathbf{C},\mathbf{D})\end{aligned} \tag{29}$$

for any complex scalars $\{\alpha_1,\alpha_2\}$ chosen to enforce the condition $\|\mathbf{w}_1\|^2+\|\mathbf{w}_2\|^2=1$. That is, the beamforming vectors are chosen in proportion to the generalized eigenvector of $(\mathbf{C},\mathbf{D})$ that corresponds to the largest generalized eigenvalue. When $\mathbf{D}$ is full rank, as is the case here, then

$$\max.\ \text{gen. eigenvector}(\mathbf{C},\mathbf{D}) = \max.\ \text{eigenvector}(\mathbf{D}^{-1}\mathbf{C}) \tag{30}$$

so that, for user i,

$$\begin{aligned}&\mathbf{w}_1^o = \mathbf{w}_2^o = \\ &\alpha\max.\ \text{eigenvector}\left(\left(M_i\sigma_i^2\mathbf{I}+\tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\right)^{-1}\mathbf{H}_i^*\mathbf{H}_i\right)\end{aligned} \tag{31}$$

Again, for comparison purposes, the zero-forcing solution for $\mathbf{W}_i$ is [8]:

$$\mathbf{W}_i = \mathbf{G}_i \mathbf{U}_i \tag{32}$$

where

$$\mathbf{G}_i = \mathbf{I} - \tilde{\mathbf{H}}_i^{\dagger} \tilde{\mathbf{H}}_i$$

$$2 \text{ columns of } \mathbf{U}_i \propto 2 \text{ largest eigenvectors } (\mathbf{H}_i \mathbf{G}_i)$$

i.e., to the two eigenvectors corresponding to the two largest eigenvalues.

V. Multi-user Beamforming for Multiple Streams

In Sec. III, we reviewed the SLNR-based multi-user beamforming scheme for the case of a single-stream per user as used in [11], [12], and in Sec. IV we extended the solution to systems that employ orthogonal space-time coding, such as Alamouti coding. In this section, we further extend the multi-user beamforming SLNR-based scheme to the multi-stream case. A block diagram of the system is shown in Fig. 4.

In the multi-stream case, an $m \times 1$ vector $\mathbf{s}_i(n)$ is transmitted to user i at time n such that

$$\mathbf{s}_i(n) = [s_{i1}(n), s_{i2}(n), \cdots, s_{im}(n)]^T$$

The vector $\mathbf{s}_i(n)$ is multiplied by the $N \times m$ beamforming matrix $\mathbf{W}_i$ prior to transmission over the channel. The overall $N \times 1$ transmitted vector at time n is given by

$$\mathbf{x}(n) = \sum_{k=1}^{K} \mathbf{W}_k \mathbf{s}_k(n) \qquad (N \times 1) \tag{33}$$

The data vectors $\mathbf{s}_i(n)$ and the beamforming matrices $\mathbf{W}_k$ are assumed to be normalized as follows:

$$\mathsf{E}\mathbf{s}_k(n)\mathbf{s}_k^*(n) = \frac{1}{m}\mathbf{I}, \quad \mathsf{Tr}(\mathbf{W}_i^* \mathbf{W}_i) = m, \quad i = \{1, \ldots, K\} \tag{34}$$

for $k = \{1, \ldots, K\}$. These power constraints ensure that the power transmitted per user is normalized to 1. In addition to the power constraint in (34), there should be another design constraint that leads to decoupling among the multiple streams during decoding in order to prevent inter-stream-interference. Note that in Sec. IV, the structure of the Alamouti code is such that it results in decoupling among the symbols at the decoder. Subsequently, the beamforming scheme used in conjunction with the Alamouti code does not suffer from inter-stream-interference. In order to derive the design constraint, we shall examine the decoded signal vector $\hat{\mathbf{s}}_i$ at user i.

The received $M_i \times 1$ vector at user i at time n

$$\mathbf{y}_i(n) = \mathbf{H}_i \sum_{k=1}^{K} \mathbf{W}_k \mathbf{s}_k(n) + \mathbf{v}_i(n) \tag{35}$$

Dropping the time index n for compactness of notation, and rearranging, equation (35) can be written as:

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{W}_i \mathbf{s}_i + \mathbf{H}_i \sum_{k \neq i}^{K} \mathbf{W}_k \mathbf{s}_k + \mathbf{v}_i \tag{36}$$

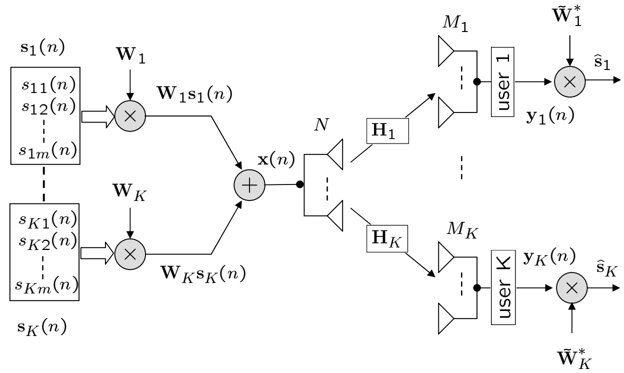


Fig. 4. A block diagram of the multi-user beamforming system with multiple streams per user.

The decoded signal vector is thus given by

$$\begin{aligned}\hat{\mathbf{s}}_i &= \tilde{\mathbf{W}}_i^* \mathbf{y}_i \\ &= \tilde{\mathbf{W}}_i^* \mathbf{H}_i \mathbf{W}_i \mathbf{s}_i + \tilde{\mathbf{W}}_i^* \left(\mathbf{H}_i \sum_{k \neq i}^{K} \mathbf{W}_k \mathbf{s}_k + \mathbf{v}_i \right)\end{aligned} \tag{37}$$

for some (linear) receiver matrix $\tilde{\mathbf{W}}_i$ to be chosen.

Based on the available knowledge of $\mathbf{H}_i$, $\mathbf{W}_i$ and σ_i^2 at user i, a matched filter can be used at the receiver where

$$\tilde{\mathbf{W}}_i^* = \frac{\mathbf{W}_i^* \mathbf{H}_i^*}{m \|\mathbf{H}_i \mathbf{W}_i\|_F} \tag{38}$$

Substituting from (38) into (37), we get

$$\begin{aligned}\hat{\mathbf{s}}_i =& \frac{1}{m \|\mathbf{H}_i \mathbf{W}_i\|_F} \mathbf{W}_i^* \mathbf{H}_i^* \mathbf{H}_i \mathbf{W}_i \mathbf{s}_i \\ &+ \frac{\mathbf{W}_i^* \mathbf{H}_i^*}{m \|\mathbf{H}_i \mathbf{W}_i\|_F} \left(\mathbf{H}_i \sum_{k \neq i}^{K} \mathbf{W}_k \mathbf{s}_k + \mathbf{v}_i \right)\end{aligned} \tag{39}$$

In order for the multiple streams to be decoupled, the decoded signal should take the form

$$\hat{\mathbf{s}}_i = \alpha \mathbf{D}_i \mathbf{s}_i + \frac{\mathbf{W}_i^* \mathbf{H}_i^*}{m \|\mathbf{H}_i \mathbf{W}_i\|_F} \left(\mathbf{H}_i \sum_{k \neq i}^{K} \mathbf{W}_k \mathbf{s}_k + \mathbf{v}_i \right) \tag{40}$$

where $\mathbf{D}_i$ is some diagonal matrix. This leads to the following design constraint:

$$\mathbf{W}_i^* \mathbf{H}_i^* \mathbf{H}_i \mathbf{W}_i = \mathbf{D}_i \quad \text{(a diagonal matrix)} \tag{41}$$

Using the signal-to-leakege-plus-noise ratio (SLNR) as the optimization criterion, the SLNR of user i is given by:

$$\mathrm{SLNR}_i = \frac{\mathsf{E}[\mathbf{s}_i^* \mathbf{W}_i^* \mathbf{H}_i^* \mathbf{H}_i \mathbf{W}_i \mathbf{s}_i]}{M_i \sigma_i^2 + \mathsf{E}[\sum_{k \neq i} \sum_{j \neq i} \mathbf{s}_i^* \mathbf{W}_i^* \mathbf{H}_k^* \mathbf{H}_j \mathbf{W}_i \mathbf{s}_i]} \tag{42}$$

Evaluating the expectations in (42), the cross terms will disappear since $\mathsf{E}s_{i,n}^* s_{i,m}^* = 0$ for $n \neq m$, and $\mathsf{E}s_{k,n}^* s_{j,m}^* = 0$ for $k \neq j$. Thus, the SLNR expression in (42) can be simplified to:

$$\begin{aligned}\mathrm{SLNR_i} &= \frac{(1/m)\mathsf{Tr}(\mathbf{W}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{W}_i)}{M_i\sigma_i^2 + \sum_{k\neq i}(1/m)\mathsf{Tr}(\mathbf{W}_i^*\mathbf{H}_k^*\mathbf{H}_j\mathbf{W}_i)} \\ &= \frac{\mathsf{Tr}(\mathbf{W}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{W}_i)}{mM_i\sigma_i^2 + \mathsf{Tr}(\mathbf{W}_i^*\tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\mathbf{W}_i)} \\ &= \frac{\mathsf{Tr}(\mathbf{W}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{W}_i)}{\mathsf{Tr}[\mathbf{W}_i^*\left(M_i\sigma_i^2 + \tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\right)\mathbf{W}_i]}\end{aligned} \tag{43}$$

where $\tilde{\mathbf{H}}_i$ is defined in (10) and where we used $\mathsf{Tr}(\mathbf{W}_i^*\mathbf{W}_i) = m$.

Problem Statement: *Select $N \times m$ beamforming matrices $\mathbf{W}_i$, $i = \{1, \ldots, K\}$, such that the following signal-to-leakage-plus-noise ratio (SLNR) is maximized for every user, i.e.,*

$$\mathbf{W}_i^o = \arg \max_{\mathbf{W}_i \in \mathbb{C}^{N\times m}} \underbrace{\frac{\mathsf{Tr}(\mathbf{W}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{W}_i)}{\mathsf{Tr}[\mathbf{W}_i^*\left(M_i\sigma_i^2 + \tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\right)\mathbf{W}_i]}}_{\text{SLNR for user } i} \tag{44}$$

subject to $\mathsf{Tr}(\mathbf{W}_i^*\mathbf{W}_i) = m,$ *and* $\mathbf{W}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{W}_i = \mathbf{D}_i \quad i = \{1, \ldots, K\}$.

Note that $\mathbf{H}_i^*\mathbf{H}_i$ is Hermitian and $\left(M_i\sigma_i^2\mathbf{I} + \tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\right)$ is Hermitian and positive definite. Both matrices are $N \times N$. Therefore, from the definition of generalized eigensapces, there exists an invertible $N \times N$ matrix $\mathbf{T}_i$ such that

$$\begin{aligned}\mathbf{T}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{T}_i &= \mathbf{\Lambda}_i \\ \mathbf{T}_i^*\left(M_i\sigma_i^2\mathbf{I} + \tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\right)\mathbf{T}_i &= \mathbf{I}\end{aligned} \tag{45}$$

for some $N \times N$ diagonal matrix $\mathbf{\Lambda}_i$ with nonnegative entries. We assume the entries of $\mathbf{\Lambda}_i$ are listed in decreasing order, namely,

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_m \geq \ldots \geq \lambda_N \geq 0$$

The range space of $\mathbf{T}_i$ defines the generalized eigenspace of the pair $\left\{\mathbf{H}_i^*\mathbf{H}_i, \left(M_i\sigma_i^2\mathbf{I} + \tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\right)\right\}$. Likewise, the $\{\lambda_i\}$ coincide with the corresponding generalized eigenvalues.

Introduce the change of variables $\mathbf{W}_i = \mathbf{T}_i\mathbf{X}_i$, where $\mathbf{X}_i$ is $N \times m$. Since $\mathbf{T}_i$ is non-singular, there is a one-to-one correspondence between $\mathbf{W}_i$ and $\mathbf{X}_i$. Substituting into the cost function (44) we get

$$\begin{aligned}&\frac{\mathsf{Tr}(\mathbf{W}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{W}_i)}{\mathsf{Tr}[\mathbf{W}_i^*\left(M_i\sigma_i^2 + \tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\right)\mathbf{W}_i]} \\ &\quad = \frac{\mathsf{Tr}(\mathbf{X}_i^*\mathbf{T}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{T}_i\mathbf{X}_i)}{\mathsf{Tr}[\mathbf{X}_i^*\mathbf{T}_i^*\left(M_i\sigma_i^2 + \tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\right)\mathbf{T}_i\mathbf{X}_i]} \\ &\quad = \frac{\mathsf{Tr}(\mathbf{X}_i^*\mathbf{\Lambda}_i\mathbf{X}_i)}{\mathsf{Tr}[\mathbf{X}_i^*\mathbf{X}_i]}\end{aligned}$$

We will seek first a matrix $\mathbf{X}_i$ that maximizes the above ratio. Introduce the SVD of $\mathbf{X}_i$, namely,

$$\mathbf{X}_i = \mathbf{U}_i \begin{bmatrix} \mathbf{\Sigma}_i \\ \mathbf{0} \end{bmatrix} \mathbf{V}_i^*$$

where $\mathbf{U}_i$ and $\mathbf{V}_i$ are unitary and $\mathbf{\Sigma}_i$ is $m \times m$ with positive entries $\{\kappa_i\}$. Then

$$\frac{\mathsf{Tr}(\mathbf{X}_i^*\mathbf{\Lambda}_i\mathbf{X}_i)}{\mathsf{Tr}[\mathbf{X}_i^*\mathbf{X}_i]} = \frac{\mathsf{Tr}\left(\begin{bmatrix} \mathbf{\Sigma}_i & \mathbf{0} \end{bmatrix} \mathbf{U}_i^*\mathbf{\Lambda}_i\mathbf{U}_i \begin{bmatrix} \mathbf{\Sigma}_i \\ \mathbf{0} \end{bmatrix}\right)}{\sum_{i=1}^m \kappa_i^2}$$

Let $\{\mathbf{u}_i\}$ denote the columns of the unitary matrix $\mathbf{U}_i$ with entries $\{u_{ji}\}$. Then

$$\frac{\mathsf{Tr}\left(\begin{bmatrix} \mathbf{\Sigma}_i & \mathbf{0} \end{bmatrix} \mathbf{U}_i^*\mathbf{\Lambda}_i\mathbf{U}_i \begin{bmatrix} \mathbf{\Sigma}_i \\ \mathbf{0} \end{bmatrix}\right)}{\sum_{i=1}^m \kappa_i^2} = \frac{\sum_{i=1}^m \kappa_i^2 \left(\sum_{j=1}^N \lambda_l |u_{ji}|^2\right)}{\sum_{i=1}^m \kappa_i^2} \tag{46}$$

where, due to the unitarity of $\mathbf{U}_i$, we have, for each i,

$$0 \leq |u_{ji}|^2 \leq 1 \quad \text{and} \quad \sum_{j=1}^N |u_{ji}|^2 = 1$$

It is now immediate to see that we can maximize the numerator in (46) by setting

$$u_{jj} = 1 \quad \text{and} \quad u_{ji} = 0 \ \text{ for } \ j \neq i \ \text{ and } \ j = 1, \ldots, m$$

which is attained by the $N \times m$ choice

$$\mathbf{X}_i = \begin{bmatrix} \mathbf{I}_{m\times m} \\ \mathbf{0} \end{bmatrix}$$

The argument so far shows that the choice

$$\mathbf{W}_i = \mathbf{T}_i \begin{bmatrix} \mathbf{I}_{m\times m} \\ \mathbf{0} \end{bmatrix} \tag{47}$$

i.e., in terms of the leading m columns of the eigenspace $\mathbf{T}_i$, maximizes the cost function (44). Actually, this choice also satisfies the constraints as follows. First note that

$$\begin{aligned}\mathbf{W}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{W}_i &= \begin{bmatrix} \mathbf{I}_{m\times m} & \mathbf{0} \end{bmatrix} \mathbf{T}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{T}_i \begin{bmatrix} \mathbf{I}_{m\times m} \\ \mathbf{0} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{I}_{m\times m} & \mathbf{0} \end{bmatrix} \mathbf{\Lambda}_i \begin{bmatrix} \mathbf{I}_{m\times m} \\ \mathbf{0} \end{bmatrix}\end{aligned}$$

which is diagonal, as desired. Moreover, the expression (47) for $\mathbf{W}_i$ can be scaled to ensure $\mathsf{Tr}(\mathbf{W}_i^*\mathbf{W}_i) = m$.

In Sec. VII, simulation results are presented for the multi-stream case comparing the performance of our proposed solution to the iterative ZF solution in [17].

VI. Channel Estimation Errors

The multiuser beamforming scheme of the prior sections assumes the availability of the channel state information (CSI) at the transmitter. The CSI can be obtained either through reverse channel estimation in time-division-duplex (TDD) or feedback in frequency-division-duplex (FDD). Channel estimation errors will degrade the performance of the system. In this section, we re-visit the SLNR optimization problem in the presence of channel uncertainties. In case of channel

$$\mathbf{w}_i^o \propto \max \quad \text{eigenvector}\left(\left[M_i\sigma_i^2\mathbf{I} + \tilde{\mathbf{H}}'^*_i\tilde{\mathbf{H}}'_i + \left(\sum_{k\neq i}^{K} M_k - M_i\right)\sigma_c^2\mathbf{I}\right]^{-1}\left(\mathbf{H}'^*_i\mathbf{H}'_i + M_i\sigma_c^2\mathbf{I}\right)\right) \tag{48}$$

mean feedback, the channel estimate that is available at the transmitter will be modeled as

$$\mathbf{H}_i^{'} = \mathbf{H}_i + \mathbf{\Theta}_i \tag{49}$$

where the channel uncertainty is represented by the $N \times M_i$ matrix $\mathbf{\Theta}_i$; its elements are assumed to be i.i.d. zero-mean complex Gaussian with variance σ_c^2 and are also spatially white. Furthermore, the random quantities $\mathbf{H}_i$, $s_i(n)$, and $\mathbf{\Theta}_i$ are assumed independent. In the same manner, the extended channel matrix that is available at the transmitter is modeled as

$$\tilde{\mathbf{H}}_i^{'} = \tilde{\mathbf{H}}_i + \mathbf{\Gamma}_i \tag{50}$$

where $\mathbf{\Gamma}_i$ is an $N \times (\sum_{k\neq i}^{K} M_k - M_i)$ matrix whose elements have the same statistics as the elements of $\mathbf{\Theta}_i$, i.e., with variance σ_c^2.

Let us consider the case studied in Sec. III where no space-time coding is used. The SLNR expression that is used bythe transmitter to evaluate the beamforming vector, will be for user i

$$\text{SLNR}_i = \frac{\mathsf{E}(\mathbf{w}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{w}_i|\mathbf{H}_i^{'})}{M_i\sigma_i^2\mathbf{I} + \mathsf{E}(\mathbf{w}_i^*\tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\mathbf{w}_i|\tilde{\mathbf{H}}_i^{'})} \tag{51}$$

where the expectation is conditional on knowledge of $\mathbf{H}_i^{'}$ and $\tilde{\mathbf{H}}_i^{'}$ by the transmitter. Using (49) and (50) we get

$$\text{SLNR}_i = \frac{\mathbf{w}_i^*(\mathbf{H}_i^{'*}\mathbf{H}_i^{'} + M_i\sigma_c^2\mathbf{I})\mathbf{w}_i}{\mathbf{w}_i^*\left(M_i\sigma_i^2\mathbf{I} + \tilde{\mathbf{H}}'^*_i\tilde{\mathbf{H}}'_i + \left(\sum_{k\neq i}^{K} M_k - M_i\right)\sigma_c^2\mathbf{I}\right)\mathbf{w}_i} \tag{52}$$

Following the same argument used to optimize (8), the optimum beamforming vector $\mathbf{w}_i^o$ is given by (48) where the proportionality constant is chosen to normalize the norm of $\mathbf{w}_i^o$ to unity. Simulation results in Sec. VII show the improvement in performance when using this solution for the beamforming vector as opposed to the solution using the channel estimates alone.

VII. Simulation Results

A single-path quasi-static MIMO channel is used in the simulations with its elements generated as zero-mean and unit-variance independent and identically distributed (i.i.d) complex Gaussian random variables. All simulations are conducted using a QPSK transmit constellation and the results are averaged over 2000 channel realizations for BER curves. The noise variance per receive antenna is assumed the same for all users, $\sigma_1^2 = \ldots = \sigma_K^2 = \sigma^2$, and the BER curves are plotted versus $1/\sigma^2$ as the SNR: $1/\sigma^2$ functions as the SNR per receive antenna since the MIMO channel and precoding coefficients are all normalized, as explained in Sec. II. To better compare the performance of different schemes, both BER results and outage curves are shown for different antenna configurations.

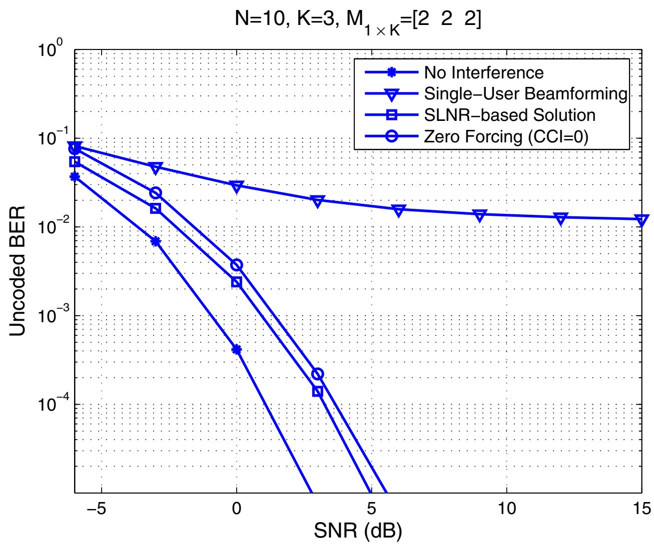


Fig. 5. Uncoded BER results for user 1 assuming $N = 10$ transmit antennas and $K = 3$ users, each equipped with $M_i = 2$ receive antennas. The dimension condition (7) is satisfied with a good margin.

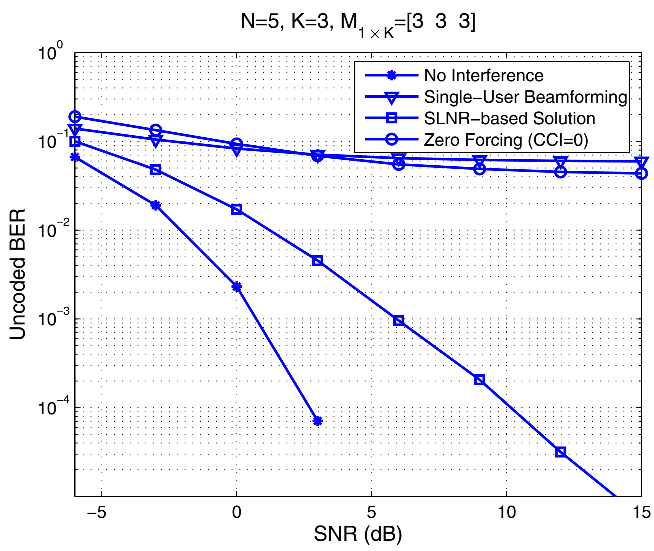
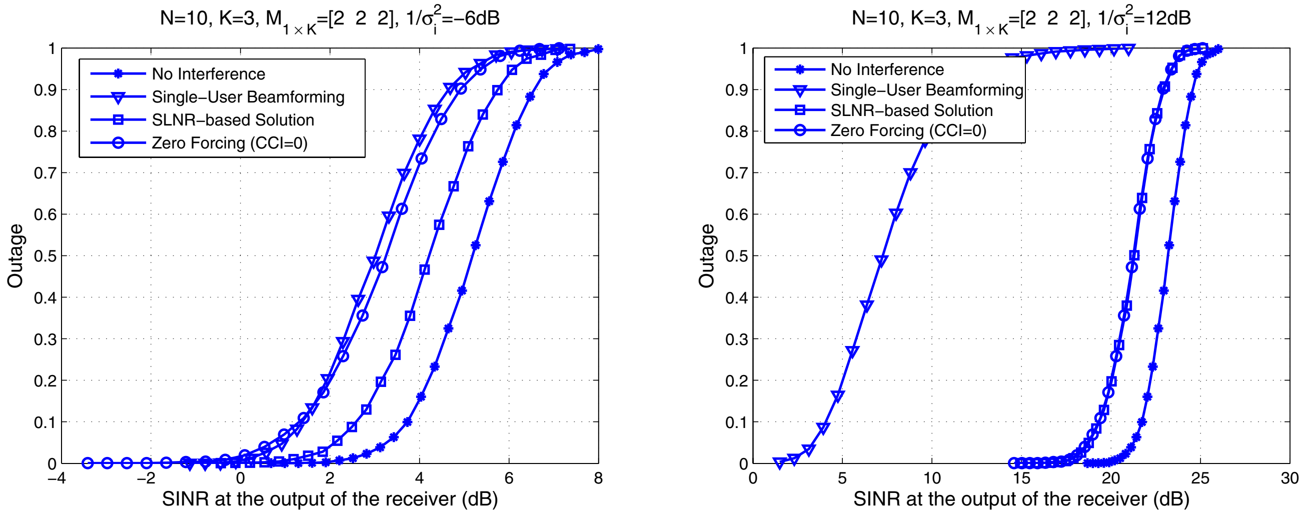


Fig. 7. Uncoded BER results for user 1 assuming $N = 5$ transmit antennas and $K = 3$ users, each equipped with $M_i = 3$ receive antennas. The dimension condition (7) is not satisfied.

A. *Beamforming for the single-stream case*

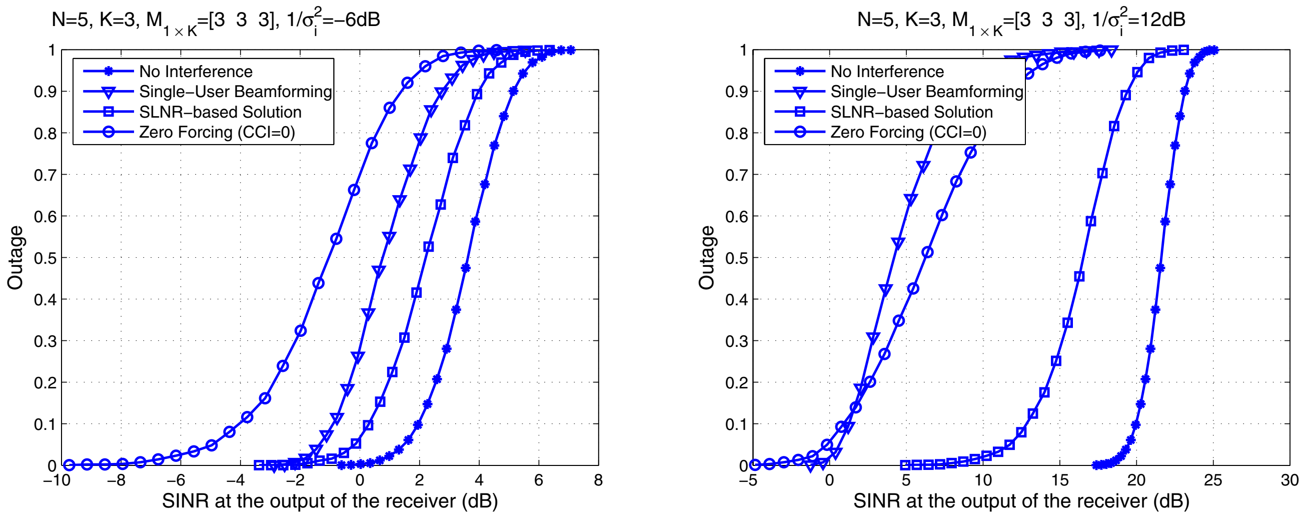
We examine the performance of the SLNR-based scheme (11) in comparison to the ZF scheme (12) and to two other extreme cases, namely, the no-interference case (a hypothetical scenario that refers to a single-user environment and is



(a) Per receive antenna SNR of $1/\sigma^2 = -6$dB

(b) Per receive antenna SNR of $1/\sigma^2 = 12$dB

Fig. 6. Outage comparison for $N = 10$ transmit antennas and $K = 3$ users, each equipped with $M_i = 2$ receive antennas.



(a) Per receive antenna SNR of $1/\sigma^2 = -6$dB

(b) Per receive antenna SNR of $1/\sigma^2 = 12$dB

Fig. 8. Outage comparison for $N = 5$ transmit antennas and $K = 3$ users, each equipped with $M_i = 3$ receive antennas.

plotted only as a comparison benchmark) and the single-user beamforming (which refers to a multi-user scenario where conventional single-user beamforming coefficients are used). We present simulation results for two cases; when the condition on the number of antennas in 7) is satisfied and when it is not satisfied.

When the condition (7) is satisfied ($N = 10, K = 3, M = \{2, 2, 2\}$), the BER results are shown in Figure 5. The proposed scheme outperforms the zero-forcing solution by 0.5-1dB. The outage curves for two different values of noise variance $1/\sigma^2 = -6, 12$dB are shown in Figure 6. As it can be seen in the figures, for large SNR values, the proposed scheme performs close to the zero forcing solution. However, at low SNR regimes where the noise variance becomes dominant in the SINR expression, the proposed scheme outperforms the zero forcing solution as it was explained.

When the condition (7) is not satisfied ($N = 5, K = 3, M = \{5, 5, 5\}$), the BER results in Figure 7 show that zero forcing fails for this configuration, as is expected, while the proposed scheme still results in acceptable BER values. The outage curves for two different values of noise variance $1/\sigma^2 = -6, 12$dB are shown in Figure 8.

B. Beamforming for the OSTBC case

Figure 10 shows the SINR outage curves for the following antenna configuration: $N = 6, K = 3, M = \{3, 3, 3\}$ and $1/\sigma_i^2 = 15$ dB. The figure compares the beamforming scheme in (31) to the ZF scheme in (32) and to the two extreme cases of no-interference and of single-user beamforming. The figures shows that the SLNR-based scheme outperforms the ZF scheme, as expected when the condition in (7) is not satisfied.

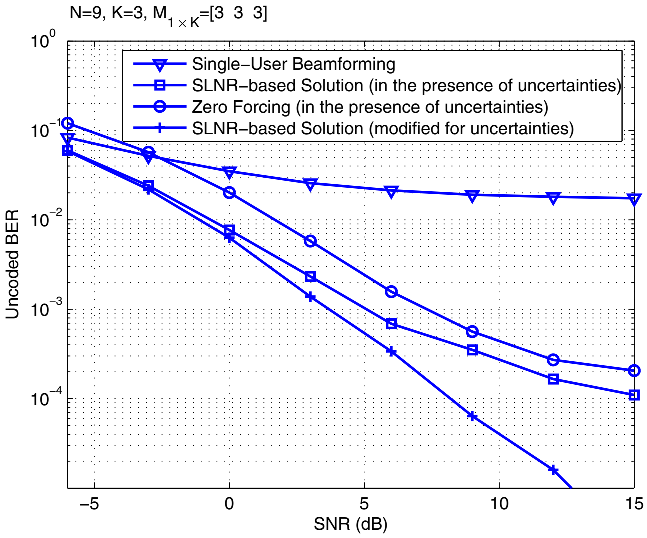


Fig. 9. Uncoded BER results averaged over 5000 channels realizations for user 1 assuming $N = 9$ transmit antennas and $K = 3$ users, each equipped with $M_i = 3$ receive antennas. The channels are assumed uncertain with $1/\sigma_c^2 = 10$dB.

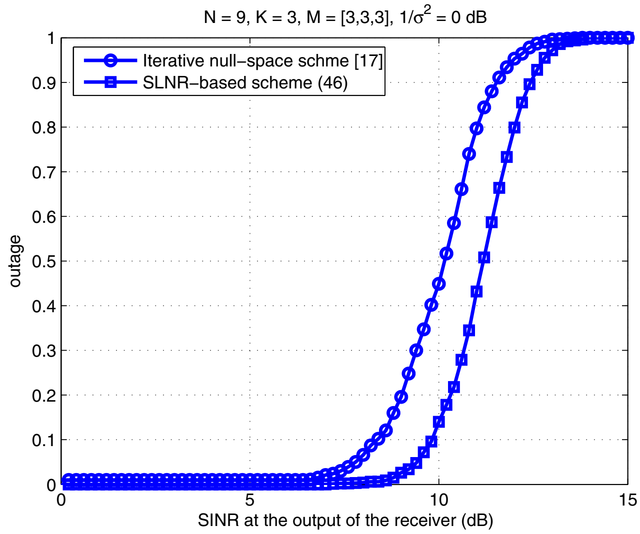


Fig. 11. Outage vs. SINR for one of 3 users each having 3 receive antennas for a system with 9 transmit antennas for $m = 2$ streams/user and $1/\sigma^2 = 0$ dB.

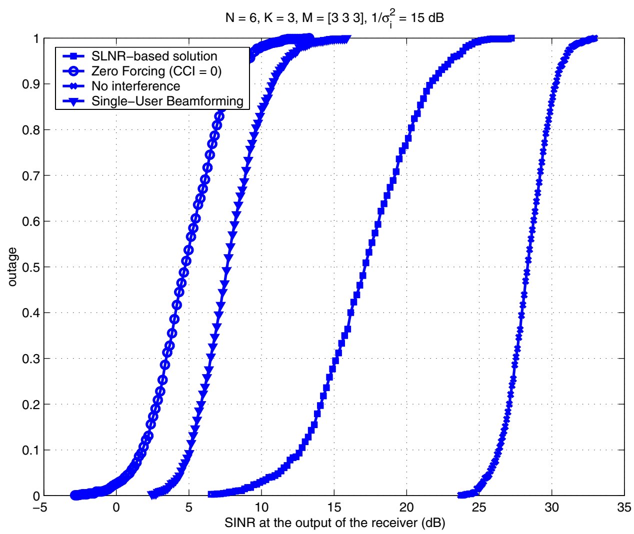


Fig. 10. Outage vs. SINR for one of 3 users each having 3 receive antennas for a system with 5 transmit antennas combined with OSTBC.

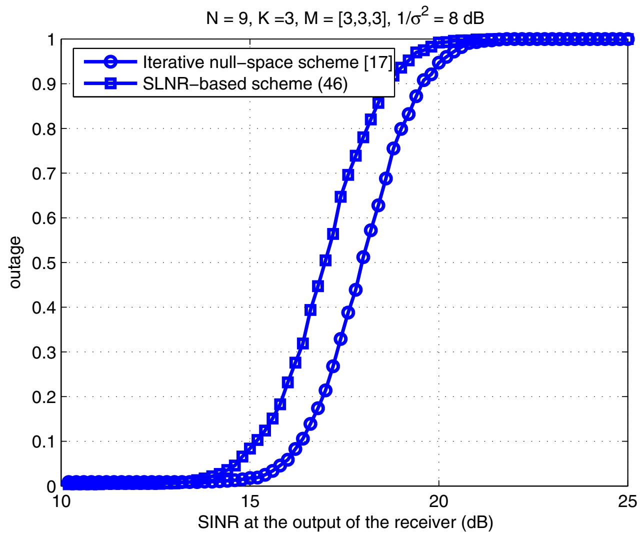


Fig. 12. Outage vs. SINR for one of 3 users each having 3 receive antennas for a system with 9 transmit antennas for $m = 2$ streams/user and $1/\sigma^2 = 8$ dB.

C. Beamforming for the multi-stream case

Figures 11 and 12 show the SINR outage curves for the following antenna configuration: $N = 9, K = 3, M = \{3, 3, 3\}$ and $1/\sigma_i^2 = 15$ dB for $m = 2$ streams/user for $1/\sigma^2 = 0$ dB and $1/\sigma^2 = 8$ dB respectively. The figures compare two schemes:

- 1. The SLNR-based scheme.
- 2. The iterative scheme proposed in [17] (referred to as the null-space scheme).

The figures shows that the SLNR-based scheme outperforms the iterative scheme in [17] in case of low SNR, i.e., noise limited scenario. This is because the SLNR-based scheme takes the noise variance into account when designing the beamforming vectors.

For higher SNR values, for instance at $1/\sigma^2 = 8$ dB, Figure 12 shows that the SLNR-based scheme performs within 1 dB of the iterative scheme in [17]. Note that the iterative scheme in [17] is orders of magnitude more complex than the SLNR-based scheme which provides a closed form solution.

D. $N - \sum_{k=1,k\neq i}^{K} M_k = 3$ *with Channel Estimation Errors*

The following parameters are used in this simulation. Transmit antennas $N = 9, K = 3, M = \{3, 3, 3\}$ and channel uncertainty of 10 dB (by that we mean $10\log_{10}(1/\sigma_c^2) =$ 10dB). Based on these parameters, the dimension condition (7) is satisfied by a small margin. The BER results are shown in Figure 9. The modified solution derived in Sec. VI outperforms the solution that would only use the instantaneous channel estimates without exploiting the information about the channel error variances.

VIII. Conclusions

In this paper, we considered a performance criterion based on optimizing the signal-to-leakage ratio (SLNR) for designing multi-user transmit beamforming vectors in a MIMO system. The decoupled nature of this criterion allows for a characterization of the solution to the multi-user beamforming problem in terms of generalized eigenvalue problems. The proposed design scheme does not impose a restriction on the number of available transmit antennas. The development and analysis in [11], [12] focused on the single stream case, which will be reviewed below in Sec. III. In this paper, we considered at least three extensions: (1) we investigate the incorporation of the leakage-based solution to MIMO systems that employ Alamouti coding, (2) we also apply the leakage-based solution to the case of multiple streams per user, and (3)finally, we examine the effect of channel estimation errors on system performance.

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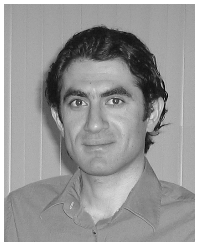

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