

# An Uplink DS-CDMA Receiver Using a Robust Post-Correlation Kalman Structure

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**Abstract**—Kalman filtering has been proposed in the literature for wireless channel estimation, however, it is not sufficiently robust to uncertainties in the channel auto-correlation model as well as to multiple access interference (MAI). This paper presents a receiver structure for direct-sequence code-division multiple-access (DS-CDMA) systems by using robust Kalman estimation and post-correlation (i.e., symbol rate) processing for channel estimation. The proposed structure is also generalized to incorporate multiple-antenna combining and interference cancellation techniques. The resulting receiver outperforms earlier structures in the presence of channel modeling uncertainties, MAI, and low-received signal-to-noise ratio. The enhancement in performance is achieved at the same order of complexity as a standard Kalman-based receiver.

**Index Terms**—Antenna-rake combiner, channel auto-correlation model, channel tracking, post-correlation state-space model, robust Kalman estimation, uplink code-division multiple-access (CDMA).

## I. INTRODUCTION

CHANNEL estimation in code-division multiple-access (CDMA) systems has a significant impact on the overall performance of the receiver. Estimating and tracking the channel, however, is challenging in the presence of fast-varying multipaths and low signal-to-interference-noise ratio (SINR). There has been extensive work in the literature on different architectures to address this challenge [1]–[4]. One such approach presented in [5]–[9] is to apply Kalman and extended Kalman filtering to estimate and track the channel, assuming an auto-correlation model for the channel variations. In these previous works, it has been assumed that either the auto-correlation model is given, or that it can be computed from the Doppler frequency using Jake’s model [13]. For this reason, these solutions tend to be sensitive to uncertainties in the channel auto-correlation model. For instance, a small error in estimating the Doppler frequency can lead to significant degradation in the Kalman estimation. Such an error can make the Kalman-based receiver perform even worse than a conventional Rake receiver (as will be shown in the simulation results). Obtaining reliable channel estimation at the receiver becomes even more difficult in a multipath multiuser scenario, where there can be different Doppler frequencies corresponding to different channel taps

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and users. In this case, a standard Kalman-based receiver would need to have accurate estimates of the Doppler frequencies (or the auto-correlation model) for the different taps and users. In this letter, we develop a robust Kalman estimation procedure that can tolerate uncertainties in the assumed auto-correlation model or Doppler frequencies, and thereby outperform the conventional receiver. Actually, the algorithm outperforms the conventional receiver in a multipath multiuser environment even if only one Doppler frequency (for instance, the maximum Doppler frequency) is used in the proposed structure to model all channel taps. This solution will be achieved at the same order of complexity as in a standard Kalman estimation.

In Kalman-based estimation, a known sequence (pilot) is needed to perform the estimation. Any interfering signal in addition to this known sequence results in error in the channel estimation. For instance, in CDMA systems [wavelength CDMA (WCDMA) and CDMA2000], this pilot sequence is transmitted simultaneously with other traffic, synchronization, or data channels. These parallel channels behave as interfering signals and they can significantly degrade the Kalman estimation results when chip-rate estimation is performed (e.g., as in [10]). For this reason, our structure will be a post-correlation (i.e., a symbol rate) structure for robust Kalman estimation. This structure is able to cancel out the interference from other parallel channels, and it also attenuates the multiple access interference (MAI). The increased SINR results in an improved Kalman estimation. The effect of the remaining attenuated MAI terms (due to the cross-correlation terms between the PN sequences) is reduced by treating them as uncertainties in the robust algorithm. The proposed structure will also be generalized to incorporate antenna combining and interference cancellation techniques.

The paper is organized as follows. The next section introduces the channel model and its auto-correlation properties. In Section III, an uplink multiuser CDMA system is presented and the post-correlation Kalman estimation is formulated. Sections IV and V incorporate antenna combining and interference cancellation techniques into the proposed structure. Section VI explains the uncertainty model and the robust estimation. Section VII includes simulation results for the proposed receiver structure and compares it with the true channel and a conventional estimator. Conclusions are given in Section VIII.

## II. CHANNEL MODEL

Consider a base-station receiving signals from  $N$  active users within a cell. Without loss of generality, we assume initially that the base-station uses a single antenna receiver, even though the results presented here can be generalized to any receiver

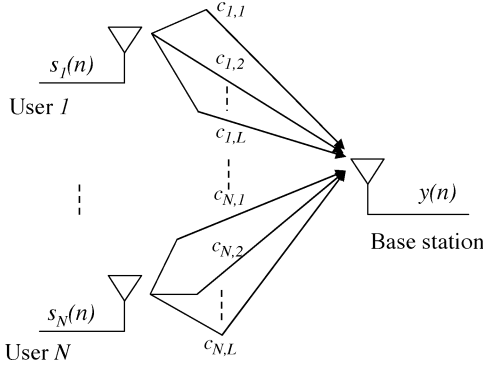


Fig. 1. Multiuser multipath channel.

dimension (see Section IV). Assuming the maximum number of channel taps to be  $L$ , the received signal at time  $n$  is

$$y(n) = \sum_{i=1}^N \sum_{k=1}^L c_{i,k}(n) s_i(n - \tau_{i,k}) + v(n) \quad (1)$$

where  $s_i(n)$  is the transmitted sequence by the  $i$ th user, and  $c_{i,k}(n)$  and  $\tau_{i,k}$  are, respectively, the  $k$ th channel tap and the channel delay from the  $i$ th user to the base station. Here, time is defined in terms of the chip rate.<sup>1</sup> The multipath channel propagation model is shown in Fig. 1.

Equation (1) can be rewritten in vector form. Collecting the channel taps for user  $i$  into a column vector  $c_{i,n}$

$$c_{i,n} = \text{col} \{c_{i,1}(n) \ c_{i,2}(n) \ \dots \ c_{i,L}(n)\} \quad (2)$$

and defining a vector of length  $NL$  containing all channel taps for all users at time  $n$

$$h_n = \text{col} \{c_{1,n} \ c_{2,n} \ \dots \ c_{N,n}\} \quad (3)$$

we can rewrite (1) as

$$y(n) = s_n h_n + v(n) \quad (4)$$

where the  $NL$  row data vector  $s_n$  contains the transmitted data by all users from time  $n - (L - 1)$  to  $n$

$$s_n = [s_1(n) \ \dots \ s_1(n - L + 1) \ \dots \ s_N(n) \ \dots \ s_N(n - L + 1)].$$

The goal is to track the vector  $h_n$ , estimate the channel taps, and recover the data in  $s_n$ .

### A. Autocorrelation Model

We use the model from [12] where all channel taps are assumed to be independent, and where the variability of the wireless channel over time is reflected in the autocorrelation function of a complex Gaussian process. It is shown in [13] that the theoretical power spectral density function associated with either

<sup>1</sup>We use  $n$  to refer to time instants relative to the chip rate. On the other hand, we shall use  $m$  to refer to time instants relative to the symbol rate.

the in-phase or quadrature portion of each channel tap has the well-known U-shaped bandlimited form

$$S(f) = \begin{cases} \frac{1}{\pi f_d \sqrt{1 - (\frac{f}{f_d})^2}}, & |f| \leq f_d \\ 0, & \text{elsewhere} \end{cases} \quad (5)$$

where  $f_d$  is the maximum Doppler frequency. The corresponding normalized discrete-time autocorrelation of each tap is  $R[n] = J_0(2\pi f_d T |n|)$ , where  $J_0(\cdot)$  is the zeroth-order Bessel function of the first kind, and  $T$  is the sampling period.

### B. State-Space Model

The time variation of the vector process  $\{h_m\}$  at symbol rate can be approximated by the following AR process of order  $q$  [15]:

$$h_m = \sum_{l=1}^q A(l) h_{m-l} + G_0 w_{m-1} \quad (6)$$

where  $w_{m-1}$  is a zero-mean i.i.d circular complex Gaussian vector process with unit variance. Assuming Gaussian wide-sense stationary uncorrelated scattering fading (WSSUS), the  $NL \times NL$  matrices  $A(l)$ ,  $l = 1, \dots, q$ , and  $G_0$  turn out to be diagonal. For the selection of their entries, various criteria of optimality can be adopted, such as requiring the AR model of order  $q$  in (6) to provide a “best-fit” to the real channel auto-correlation function of (5). In this letter, we adopt the AR model presented in [14] by solving a set of  $q$  Yule–Walker equations.

The multichannel AR model (6) can be rewritten in state-space form as

$$x_{m+1} = F x_m + G w_m \quad (7)$$

where

$$x_m = \text{col} \{h_m \ h_{m-1} \ \dots \ h_{m-q+1}\} \quad (8)$$

$$F = \begin{bmatrix} A(1) & A(2) & \dots & A(q-1) & A(q) \\ I_{NL} & & & & \\ & I_{NL} & & & \\ & & \ddots & & \\ & & & I_{NL} & 0_{NL} \end{bmatrix} \quad (9)$$

$$G = \begin{bmatrix} G_0 \\ 0_{(q-1)NL \times NL} \end{bmatrix}. \quad (10)$$

Note that this model depends only on the Doppler frequency and on the sampling rate at the receiver. The measurement equation that we shall use is not based on (4) since  $\{s_n\}$  is not known. Instead, we shall derive an alternative measurement equation that uses estimates from the pilot symbols [see (17)].

### C. CDMA Uplink Signals

In this letter, we use a simplified model for the CDMA2000 physical layer where only the pilot and traffic channels are modeled. However, the results can be generalized for any number of traffic channels and any other CDMA standard, e.g., WCDMA.

The baseband equivalent of the transmitted data from mobile user  $i$  [used as  $s_i(n)$  in (1)] can be written at chip rate as

$$s_i = (pW_p + jd_iW_d)PN_i. \quad (11)$$

Note that (11) is describing the transmitted signal at the chip rate as a function of pilot and data symbols. Since this relationship (a transform from symbol rate to chip rate) is somewhat complicated to represent in matrix form, this equation is only depicting such relation and does not provide the transmitted sequence in matrix form. In (11),  $p$  and  $d_i$  are the pilot and data symbols, respectively,  $W_p$  and  $W_d$  are the orthogonal Walsh-codes for the pilot and data channels, and  $PN_i$  is the effective PN-sequence used by the  $i$ th user

$$PN_i = PN_{i,L}(PN_I + jPN_Q) \quad (12)$$

where  $PN_{i,L}$  is the long pseudorandom noise (PN) code for user  $i$  and is different for each user and  $PN_I$  and  $PN_Q$  are the in-phase and quadrature PN-codes intended to differentiate among cells and are the same within each cell. The pilot portion of the transmitted signal, i.e.,  $pW_pPN_i$  will be used for training purposes in the Kalman estimation step, since it is known at the base station. The effective PN-code for pilot and data channels are defined for user  $i$  as  $PN_i^p = W_pPN_i$  and  $PN_i^d = W_dPN_i$ , respectively.

### III. RECEIVER STRUCTURE

Previous structures for data and channel estimation use the received signal  $y(n)$  directly [5], [10]. For CDMA systems, this will imply using the received CDMA chips to form the measurement equation. This chip-rate approach has the following drawbacks:

- 1) the signal-to-interference-noise-ratio (SINR) at the chip level can be as low as  $-20$  dB in a multiuser environment, which makes it difficult to track channels in such noisy environments;
- 2) the training sequence at the chip rate (pilot portion of  $s_i$ ) is corrupted by unknown traffic channels (data portion of  $s_i$  and of other users) since different traffic channels are transmitted simultaneously, as shown by (11), in current CDMA standards (CDMA2000 and WCDMA).

To overcome these difficulties, we propose a post-correlation structure to derive the measurement equation. In this architecture, the received pilot symbols (rather than the received CDMA chips) are used in the estimation step.

#### A. Channel Estimation and Tracking

At the base station, the received pilot symbols from all users are estimated using each user's PN-code and the Walsh code of the pilot channel. Since channel estimates for all multipaths are needed at the Rake combiner, as is briefly discussed in Section III-B, the pilot symbols are estimated for each channel multipath. The  $m$ th estimated pilot symbol for user  $i$ , using multipath  $j$ , is calculated as follows:

$$\hat{p}_{i,j}(m) = \frac{1}{PG} \sum_{n=\tau_{i,j}+(m+1)PG-1}^{\tau_{i,j}+(m+1)PG-1} y(n) [PN_i(n)W_p(n)]^* \quad (13)$$

where PG is the processing gain used on the pilot channel and  $\tau_{i,j}$  is the delay from user  $i$  to the base station on multipath  $j$ . Note that the estimated pilot symbols in (13) are scaled down by a factor PG in order to have the same magnitude as the transmitted symbols. It is assumed that all PN-codes and channel delays ( $\tau_{i,j}$ ) are known at the base station before estimating the pilot and data symbols. These parameters are typically estimated by a multipath searcher at the base station before the data estimation starts (see [11] and the references therein). Replacing  $y(n)$  in (13) by (4) and assuming the channel taps are constant within each pilot symbol, we have the following:<sup>2</sup>

$$\begin{aligned} \hat{p}_{i,j}(m) &= c_{i,j}(m)p_i(m) + v \\ &+ \sum_{\substack{l=1 \\ l \neq j}}^L \rho_{(i,i,j,l)} c_{i,l}(m)(p_{i,l}(m) + jd_{i,l}(m)) \\ &+ \sum_{\substack{k=1 \\ k \neq i}}^N \sum_{l=1}^L \rho_{(i,k,j,l)} c_{k,l}(m)(p_{k,l}(m) + jd_{k,l}(m)) \end{aligned} \quad (14)$$

where  $p_i(m)$  is the transmitted pilot symbol by user  $i$ , and  $p_{k,l}$  and  $d_{k,l}$  are the transmitted pilot and data symbols by user  $k$  and received through multipath  $l$  at the same time as  $p_{i,j}$  is received

$$\begin{aligned} p_{k,l}(m) &= p_k \left( m - fl \left( \frac{(\tau_{k,l} - \tau_{i,j})}{PG} \right) \right) \\ d_{k,l}(m) &= d_k \left( m - fl \left( \frac{(\tau_{k,l} - \tau_{i,j})}{PG} \right) \right) \end{aligned}$$

where  $fl(\cdot)$  is the truncation function. The multipath index  $l$  is added for the scenario that the channel maximum delay is larger than the symbol period. Moreover,  $\rho_{(i,k,j,l)}$  is the cross correlation between the PN-codes of users  $i$  and  $k$ , received on multipaths  $j$  and  $l$ , respectively. Collecting the estimated pilot symbols from all users and multipaths ( $i = 1, \dots, N$  and  $j = 1, \dots, L$ ) into a vector  $b_m$  and using the definition (3) we get

$$b_m = R_m h_m + u_m \quad (15)$$

where

$$b_m = \left[ \underbrace{\hat{p}_{1,1}(m) \dots \hat{p}_{1,L}(m)}_{\text{user 1}} \dots \underbrace{\hat{p}_{N,1}(m) \dots \hat{p}_{N,L}(m)}_{\text{user N}} \right]^T \quad (16)$$

$$R_m = p(m)I + U$$

where  $u_m$  in (15) contains the noise components after the correlation for different users and multipaths;  $R_m$ ,  $I$ , and  $U$  are size  $(NL \times NL)$  matrices;  $I$  is the identity matrix; and  $p(m)$  is the transmitted pilot symbol which is the same for all users. In (16), the matrix  $U$  contains the cross correlation terms with its diagonal elements equal to zero. We now complete the derivation of the symbol-rate measurement equation by collecting  $q$  consecutive realizations of (15) into a column vector  $y_m$

$$y_m = H_m x_m + v_m \quad (17)$$

<sup>2</sup>It is assumed that the received signal is passed through a matched pulse-shaping filter before the correlation is performed on the received signal. The previous derivations hold for any pulse-shaping filter, given that the right sample within the chip is selected after pulse shaping at the receiver.

where

$$y_m = \begin{bmatrix} b_m \\ b_{m-1} \\ \vdots \\ b_{m-q+1} \end{bmatrix} \quad (18)$$

$$x_m = \begin{bmatrix} h_m \\ h_{m-1} \\ \vdots \\ h_{m-q+1} \end{bmatrix} \quad (19)$$

$$H_m = \begin{bmatrix} R_m & & & \mathbf{0} \\ & R_{m-1} & & \\ & & \ddots & \\ \mathbf{0} & & & R_{m-q+1} \end{bmatrix} \quad (19)$$

and  $v_m$  in (17) is defined similar to (18) using  $\{u_m, \dots, u_{m-q+1}\}$ . The reason we collect only  $q$  consecutive realizations is because we are using an AR model of order  $q$  to model the channel variations. Equations (7) and (17) define the desired state and measurement equations at symbol rate

$$\begin{cases} x_{m+1} = Fx_m + Gw_m \\ y_m = H_m x_m + v_m \end{cases} \quad (20)$$

where  $m$  is the pilot symbol index. Note that  $H_m$  is a block diagonal matrix with its diagonal elements equal to  $p(m)$  and the nondiagonal elements representing the cross correlation terms, as shown in (16). The exact values of the nondiagonal elements depend on many parameters, e.g., the PN-codes, the relative delays of different multipaths and the transmitted data symbols by all users. Therefore, the nondiagonal elements of the  $\{R_m\}$  are not practically available at the base station. However, we know that the nondiagonal elements have lower power than the diagonal elements by a factor of PG due to the properties of PN-codes.

### B. Rake Combiner

Once the channel taps are estimated by the robust Kalman filter of Section VI, a Rake receiver uses the estimated taps  $\{\hat{c}_{i,j}(m)\}$  to combine the estimated data symbols from different multipaths (Rake fingers), see Fig. 2. Since the scope of the paper is primarily channel estimation techniques, we shall use the well known maximal-ratio-combining (MRC) Rake combiner to evaluate the performance of the proposed algorithm [18]. Let  $\hat{d}_{i,j}(m)$  denote the  $m$ th estimated data symbol for user  $i$  using multipath  $j$ , and let  $\hat{c}_{i,m}$  denote the estimated channel vector for user  $i$  at the symbol rate, defined as

$$\hat{c}_{i,m} = \text{col} \{ \hat{c}_{i,1}(m) \hat{c}_{i,2}(m) \dots \hat{c}_{i,L}(m) \} \quad (21)$$

similar to the definition given in (2) at the chip rate. Then, the MRC Rake receiver output for the  $m$ th data symbol of user  $i$  is given by

$$\hat{d}_i(m) = \hat{c}_{i,m}^* \hat{d}_{i,m} \quad (22)$$

$$\hat{d}_{i,m} = \text{col} \{ \hat{d}_{i,1}(m) \hat{d}_{i,2}(m) \dots \hat{d}_{i,L}(m) \}. \quad (23)$$

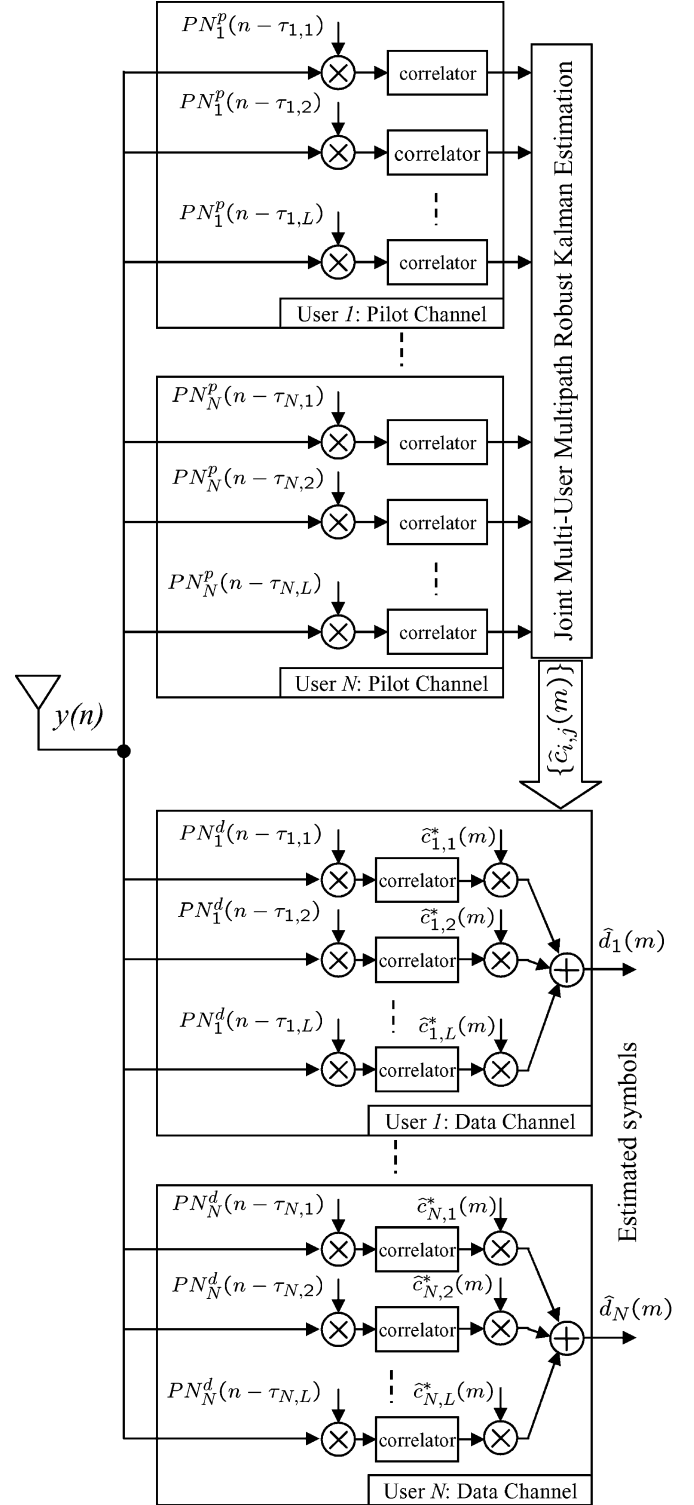


Fig. 2. Receiver structure.

### IV. RECEIVER WITH MULTIPLE ANTENNAS

The receiver structure proposed in the previous section can be generalized to the case of multiple antennas at the base station. Let  $M$  denote the number of antennas at the receiver. Then, (2) can be rewritten for the  $k$ th receive antenna by adding the antenna index  $k$  as

$$c_{i,k,n} = \text{col} \{ c_{i,k,1}(n) c_{i,k,2}(n) \dots c_{i,k,L}(n) \} \quad (24)$$

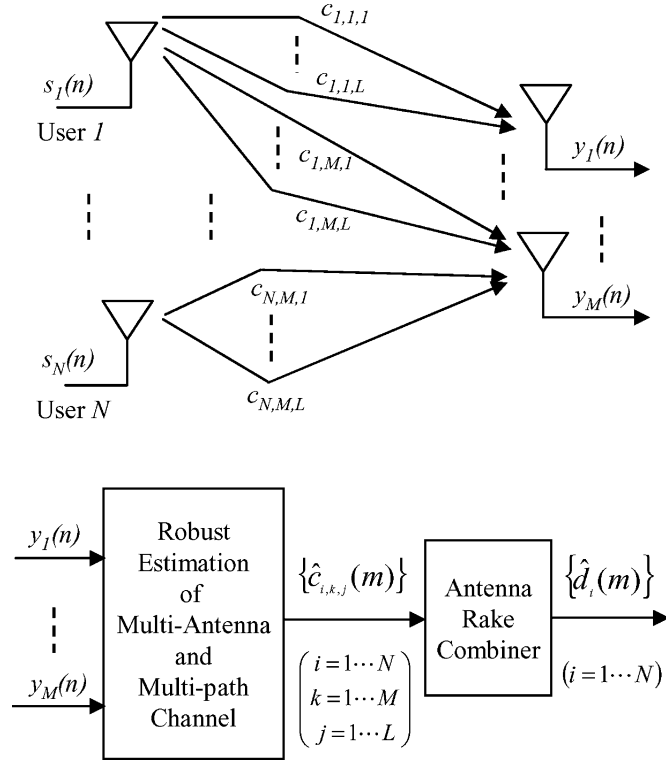


Fig. 3. Scenario with multiple receive antennas.

where  $c_{i,k,n}$  represents the channel from user  $i$  to the  $k$ th antenna at the receiver, as shown in Fig. 3. The channel taps for the same user on different receive antennas are then collected into a column vector  $c_{i,n}$

$$c_{i,n} = \text{col}\{c_{i,1,n} \ c_{i,2,n} \ \dots \ c_{i,M,n}\}. \quad (25)$$

This definition plays the role of (2), except that  $c_{i,n}$  in the above equation is of length  $M \times L$ , compared to length  $L$  in (2). The column vector  $h_n$  is defined as

$$h_n = \text{col}\{c_{1,n} \ c_{2,n} \ \dots \ c_{N,n}\} \quad (26)$$

containing  $N \times M \times L$  channel taps, from all users to all receive antennas. Similar to the structure proposed for the single antenna case, a receiver can be used to estimate and track the channel taps for all users and to all receive antennas. An extra dimension (due to the receive multiple antennas) is added to the formulation, increasing the sizes from  $N \times L$  to  $N \times M \times L$ , see Fig. 3.

#### A. Antenna-Rake Combiner

Once the channel taps for different multipaths and antennas are estimated (using the robust procedure presented in Section VI), an antenna-Rake combiner is used to combine the estimated data symbols from different multipaths and antennas. A two-dimensional (2-D) antenna and multipath combiner uses the estimated channel taps to provide the final data symbol estimates. Referring to definitions (24) and (25), the  $m$ th data symbol estimate for user  $i$  at the output of the 2-D maximal-ratio antenna-rake combiner is given by

$$\hat{d}_i(m) = \sum_{k=1}^M \hat{c}_{i,k}^* \hat{d}_{i,k,m} = \hat{c}_{i,m}^* \hat{d}_{i,m} \quad (27)$$

where  $\hat{d}_{i,k,m}$  contains the estimated data symbols for user  $i$  on antenna  $k$ , using multipaths  $\{1, \dots, L\}$ , and

$$\hat{d}_{i,m} = \text{col}\{\hat{d}_{i,1,m} \ \hat{d}_{i,2,m} \ \dots \ \hat{d}_{i,M,m}\}$$

#### V. MAI CANCELLATION

The proposed receiver structure can be followed by an MAI cancellation technique to improve the data estimates in a secondary stage. It is important to note the following.

- 1) The base station performs the channel estimation process for all users, multipaths, and antennas simultaneously. Therefore, an initial estimate for  $h_m$  is available at the base station.
- 2) The base station receives and decodes the signals from all users simultaneously. The data estimates  $\hat{d}_i(m)$  given in (27) for all users ( $i = 1, \dots, N$ ) are available at the base station.

This information can be used in a secondary stage to reduce the effect of interfering signals. The interfering signals are re-generated using the estimated data  $\{\hat{d}_i(m)\}$  and channel taps  $\{\hat{c}_{i,j}(m)\}$  for all users, and cancelled from the previous pilot symbol estimates  $\{\hat{p}_{i,j}(m)\}$  and a new channel estimation is obtained. Once the first channel and data estimates are available, the estimated pilot symbols used in the Kalman estimation (14) are corrected to

$$\begin{aligned} \hat{p}_{i,j}(m) &= \text{previous } \hat{p}_{i,j}(m) - \text{estimated MAI} \\ &= c_{i,j}(m)p_i(m) + v \\ &\quad + \sum_{\substack{l=1 \\ l \neq j}}^L \rho_{(i,i,j,l)} [(c_{i,l}(m) - \hat{c}_{i,l}(m))p_{i,l}(m) \\ &\quad + j(c_{i,l}(m)d_{i,l}(m) - \hat{c}_{i,l}(m)\hat{d}_{i,l}(m))] \\ &\quad + \sum_{\substack{k=1 \\ k \neq i}}^N \sum_{l=1}^L \rho_{(i,k,j,l)} [(c_{k,l}(m) - \hat{c}_{k,l}(m))p_{k,l}(m) \\ &\quad + j(c_{k,l}(m)d_{k,l}(m) - \hat{c}_{k,l}(m)\hat{d}_{k,l}(m))]. \end{aligned} \quad (28)$$

The new pilot symbol estimates are then used to update the (15)–(20) and conduct a new Kalman estimation for the channel taps.

#### VI. ROBUST KALMAN FILTER

Due to modeling errors in model (20), we rewrite the dynamics of the channel and measured signal at the receiver at symbol rate as follows:

$$x_{m+1} = (F + \delta F_m)x_m + Gw_m, \quad m \geq 0 \quad (29)$$

$$y_m = (H + \delta H_m)x_m + v_m. \quad (30)$$

In these equations, the nominal model  $\{F, G, H\}$  is time-invariant, and the matrices  $H_m$  of (20) are modeled as  $H_m = H + \delta H_m$ , with  $H = I$ . The uncertain matrix  $\delta H_m$  has zeros along its diagonal entries and quantities smaller than  $1/PG$  in

the off-diagonal entries. Moreover, the uncertain matrices  $\delta F_m$  are due to uncertainties in  $\{A(1), A(2), \dots, A(q)\}$ . We shall use the robust filter of [17]. For this purpose, we first note that the uncertainty in the product  $(H + \delta H_m)(F + \delta F_m)$  is given by

$$\delta(HF) = H\delta F_m + \delta H_m F + \delta H_m \delta F_m. \quad (31)$$

We are going to assume that  $\|\delta(HF)\| < \eta$ , for some  $\eta > 0$ , which is reasonable since both  $\delta H_m$  and  $\delta F_m$  are bounded. We can then model the uncertainties  $\delta(HF)$  as

$$\delta(HF) = M\Delta_m E_f \quad (32)$$

for some known  $\{M, E_f\}$  and for an arbitrary contraction  $\Delta_m$ . Any  $\{M, E_f\}$  that allows us to represent the uncertainty in the product  $HF$  as in (32) is an appropriate choice. Now, for the particular structure given in (32), if we choose  $M = \gamma I$  and  $E_f$  as a full rank matrix, then there always exists a contraction  $\Delta_m$  with appropriate  $\gamma$  such that any uncertainty in the product  $HF$  can be written as in (31). The value of  $\gamma$  can be chosen as follows. Choose any full rank  $E_f$  and let  $\|E_f\| = \sigma$ . Assume  $\|\delta(HF)\| < \eta$  and choose  $\gamma = \eta/\sigma$ . Then,  $\delta(HF) \leq \|M\|\|\Delta_m\|\|E_f\| = \eta\|\Delta_m\|$  and  $\Delta_m$  is a contraction. Let the noises  $\{w_m, v_m\}$  have covariance matrices  $\{Q, R\}$ , respectively, (the values for these matrices are given in the simulations section). To estimate the channel taps in the presence of uncertainties in the state matrices, we use the robust algorithm developed in [17] and listed in Table I along with the previously mentioned choices of  $M$  and  $E_f$ . A procedure to determine the parameter  $\alpha$  is given in the Appendix. The computational complexity of the proposed robust algorithm is  $\mathcal{O}(K^3)$  where  $K$  is the dimension of  $x_m$ . A summary of the steps to perform the robust estimation is given in Table I.

## VII. SIMULATION RESULTS

An uplink CDMA2000 environment is simulated to evaluate the performance of the architecture presented in the paper. The proposed architecture is compared to an ideal Rake combiner (knowing the exact channel taps) and a conventional Rake combiner, which uses an averaging technique for channel estimation. The performance of the link is simulated for different number of active users. Every user sends the pilot and data channels simultaneously using orthogonal Walsh codes. Simulated for high data rate applications, the processing gain on data channels is 64, which corresponds to a data rate of 19.2 kb/s for each user per channel, assuming a chip rate of 1.2288 MHz. On the base station side, the correlation length for the pilot channel (PG) is 256, and the averaging period used by the conventional Rake combiner is 10 symbols (a typical estimation length used in third-generation (3G) CDMA systems). Results are simulated for received chip rate SNR of  $-6$  to  $0$  dB. The covariance matrix  $Q$  is equal to the identity matrix because of the discussion following (6). The covariance matrix  $R$  is chosen as approximately  $(1/\text{PG}) \times (1/\text{SNR})$  since the power of the noise before correlation (at the antenna) is  $(1/\text{SNR})$  and it gets attenuated by a factor of  $(1/\text{PG})$  after the correlation. Without loss of generality, we have assumed that the transmitted sequence has unity power. The channel model used in the simulation has two independent taps for each user and is approximated by an AR

TABLE I  
ROBUST FILTERING ALGORITHM OF [17]

**Assumed uncertain model:** Eqs. (29)–(30). Also,  $\Pi_0 > 0$ ,  $R > 0$ ,  $Q > 0$  are given weighting matrices.

**Initial conditions:**  $\hat{x}_0 = 0$ ,  $P_0 = \Pi_0$ , and  $\hat{R} = R$ .

**Step 1a.** Using  $\{\hat{R}, H, P_m\}$  compute  $P_{m|m}$ :

$$P_{m|m} = P_m - P_m H^* (\hat{R}_m + H P_m H^*)^{-1} H P_m$$

**Step 1b.** If  $M = 0$ , then set  $\hat{\lambda} = 0$ . Otherwise, set  $\hat{\lambda} = (1 + \alpha)\lambda_t$

$$\lambda_t \triangleq \|M^* R^{-1} M\|$$

**Step 2.** Replace  $\{R, P_{m|m}, F\}$  by:

$$\hat{R} = R - \hat{\lambda}^{-1} M M^*$$

$$\hat{F}_m = F(I - \hat{\lambda} \hat{P}_{m|m} E_f^* E_f)$$

$$\hat{P}_{m|m} = P_{m|m} - P_{m|m} E_f^* (\hat{\lambda}^{-1} I + E_f P_{m|m} E_f^*)^{-1} E_f P_{m|m}$$

If  $\hat{\lambda} = 0$ , then set  $\hat{R} = R$ ,  $\hat{P}_{m|m} = P_{m|m}$ , and  $\hat{F}_m = F$ .

**Step 3.** Now update  $\{\hat{x}_m, P_m\}$  to  $\{\hat{x}_{m+1}, P_{m+1}\}$  as follows:

$$\hat{x}_{m+1} = \hat{F}_m \hat{x}_m + \hat{F}_m P_m H^* R_e^{-1} e_m$$

$$e_m = y_m - H \hat{x}_m$$

$$P_{m+1} = F P_m F^* - \bar{K}_m \bar{R}_e^{-1} \bar{K}_m^* + G Q G^*$$

$$\bar{K}_m = F P_m \bar{H}_m^*, \quad \bar{R}_e = I + \bar{H} P_m \bar{H}^*$$

$$\bar{H}^T = [ H^T \hat{R}^{-T/2} \quad \sqrt{\hat{\lambda}} E_f^T ]$$

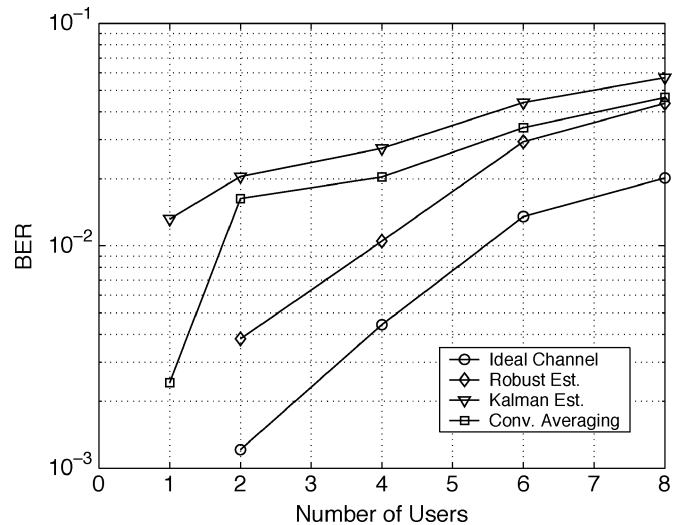


Fig. 4. Bit-error rate (BER) versus the number of users for different algorithms, SNR = 0 dB.

model of order  $q = 3$ . The maximum doppler frequency in the channel corresponds to a velocity of about 80 mi/h.  $M = I$  and  $E_f$  is a matrix that has zero on the diagonal elements and 0.1 on all the off-diagonal entries. The results for different number of users and different tracking techniques are shown in Figs. 4 and 5. In the figures, the legend “Ideal Channel” refers to the ideal case where the true channel values are used, and the legend “Conv. Avg.” stands for the averaging technique used in conventional Rake receivers. As shown in Figs. 4 and 5, the standard Kalman estimator performs even worse than conventional averaging techniques in the presence of uncertainties in the channel

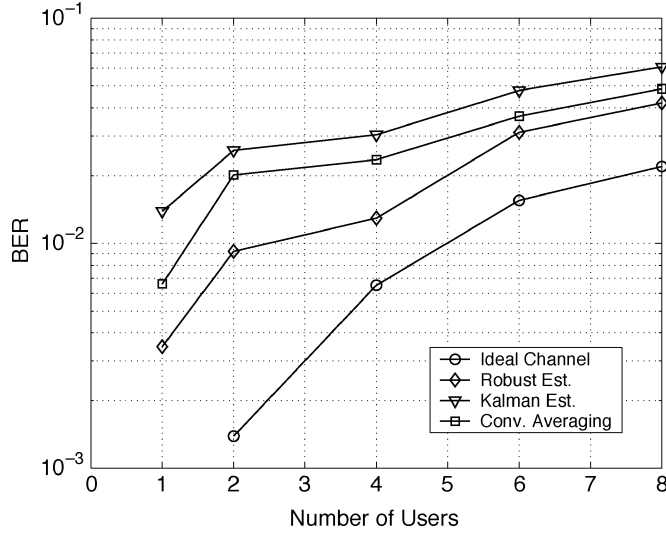


Fig. 5. BER versus the number of users for different algorithms, SNR = -6 dB.

modeling. The proposed robust tracking technique outperforms both the averaging algorithm used in CDMA systems and the standard Kalman estimation. The difference is more significant for small number of active users, which is the desired scenario in high data rate links.

### VIII. CONCLUSION

A symbol-rate post-correlation model-based channel tracking and symbol detection for uplink CDMA systems using robust Kalman estimation has been proposed. In order to deal with the uncertainties in the model that arise due to the unknown correlation between different PN sequences and errors in Doppler estimation, a robust tracking filter has been proposed in tandem with a RAKE receiver.

### APPENDIX

We now show how to choose the parameter  $\alpha$  that is needed in the algorithm of Table I. Let  $\tilde{x}_m = x_m - \hat{x}_m$ . Define the following matrices:

$$\mathcal{F} = \begin{pmatrix} F & 0 \\ F - F_p - F_p P H^T \hat{R}^{-1} H & F_p \end{pmatrix} \quad (33)$$

$$\mathcal{G} = \begin{pmatrix} G & 0 \\ G & -F_p P H^T \hat{R}^{-1} \end{pmatrix} \quad (34)$$

$$\delta \mathcal{F}_m = \begin{pmatrix} M \Delta_m E_f & 0 \\ M \Delta_m E_f & 0 \end{pmatrix} \quad (35)$$

where  $F_p = \hat{F}(I - P H^T R_e^{-1} H)$ ,  $\hat{F}$  is the steady-state value of  $\hat{F}_m$ , and  $P$  is the steady-state solution of

$$P_{m+1} = F P_m F^* - \bar{K}_m \bar{R}_e^{-1} \bar{K}_m^* + G Q G^*. \quad (36)$$

Then, under the detectability of  $\{F, \bar{H}\}$  and the stabilizability of  $\{F, G Q^{1/2}\}$ , it can be shown as in [17], that the estimation error of the robust filter satisfies

$$\lim_{i \rightarrow \infty} E \tilde{x}_m \tilde{x}_m^* \leq \mathcal{P}_{22} \quad (37)$$

where  $\mathcal{P}_{22}$  is the (2,2) block entry with the smallest trace among all (2,2) block entries of positive-definite matrices  $\mathcal{P}$  satisfying

$$\mathcal{P} - (\mathcal{F} + \delta \mathcal{F}_m) \mathcal{P} (\mathcal{F} + \delta \mathcal{F}_m)^* - \mathcal{G} \begin{pmatrix} Q & 0 \\ 0 & R \end{pmatrix} \mathcal{G}^* \geq 0 \quad (38)$$

for all contractive matrices  $\Delta_m$ . Now, (38) holds if there exists a scalar  $\omega \geq 0$  such that the following matrix inequality is feasible with  $\Sigma \leq \mathcal{P} - \mathcal{G} \bar{Q} \mathcal{G}^*$  (see [20, Lemma 2.1] and [21]):

$$\begin{pmatrix} \Sigma & \mathcal{F} & \bar{M} \\ \mathcal{F}^* & \mathcal{P}^{-1} - \omega \bar{E}^T \bar{E} & 0 \\ \bar{M}^T & 0 & \omega I \end{pmatrix} \geq 0 \quad (39)$$

where

$$\bar{Q} = \begin{pmatrix} Q & 0 \\ 0 & R \end{pmatrix} \quad (40)$$

$$\bar{M} = \begin{pmatrix} M & 0 \\ M & 0 \end{pmatrix}$$

$$\bar{E} = \begin{pmatrix} E_f & 0 \\ 0 & 0 \end{pmatrix}. \quad (41)$$

Now, we take a grid  $[\lambda_1, \lambda_2, \dots, \lambda_n]$  in the interval  $(\lambda_l, \lambda_l + \delta \lambda)$  where  $\delta \lambda$  is a given positive scalar (chosen by the user).  $\delta \lambda$  is typically chosen as twice or thrice the lower bound  $\lambda_l$ . For each  $\lambda$ , we choose an arbitrary  $\Sigma = \beta I$ ,  $\beta > 0$ , and solve the linear matrix inequality (39) over  $\omega$  and  $\mathcal{P}^{-1}$ . The resulting  $\mathcal{P}$  guarantees (38) if  $\Sigma \leq \mathcal{P} + \mathcal{G} \bar{Q} \mathcal{G}^*$ . Otherwise, we repeat the procedure with a different  $\Sigma = \beta I$ . Typically, beta is initialized to a low value say 1.0 initially and then increased until the inequalities are met. We evaluate the  $\mathcal{P}$  in (39) for each of the  $\lambda$  in the above grid and choose that  $\hat{\lambda} = \lambda$  that results in the smallest  $\mathcal{P}$ . From this value of  $\hat{\lambda}$ ,  $\alpha$  is found from  $\hat{\lambda} = (1 + \alpha) \lambda_l$ .

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