Joint Compensation of Transmitter and Receiver Impairments in OFDM Systems

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***Abstract*— The implementation of OFDM-based systems suffers from impairments such as in-phase and quadrature-phase (IQ) imbalances in the front-end analog processing. Such imbalances are caused by the analog processing of the radio frequency (RF) signal and can be present at both the transmitter and receiver. The resulting IQ distortion limits the achievable operating SNR at the receiver and the achievable data rates. In this paper, the effect of both the transmitter and receiver IQ imbalances in an OFDM system is studied and algorithms are developed to compensate for such distortions in the digital domain. The algorithms include post-FFT least-squares and adaptive equalization, as well as a pre-distortion scheme at the transmitter and a pre-FFT correction at the receiver.**

***Index Terms*— Compensation algorithms for analog impairments, orthogonal frequency division multiplexing (OFDM), transmitter and receiver in-phase and quadrature-phase (IQ) imbalances.**

I. INTRODUCTION

A Limiting issue in the implementation of wireless systems is the impairment associated with analog processing due to component imperfections. Most of the impairments cannot be efficiently nor entirely eliminated in the analog domain due to power-area-cost trade-offs. Therefore, efficient compensation schemes in the digital baseband domain are desirable for wireless transceivers.

A major source of impairments in high-frequency wireless system implementations is the imbalance between the In-phase (I) and Quadrature-phase (Q) branches; or equivalently, the real and imaginary parts of the complex signal [1]. This imbalance can be introduced at *both* the transmitter (during frequency up-conversion) and the receiver (during frequency down-conversion). Both the up-conversion and down-conversion are implemented in the analog domain by what is known as complex up-conversion and complex down-conversion (for more information see [1], [2]). To perform the complex frequency conversion, both the *sine* and *cosine* oscillating waveforms are required. The IQ imbalance is basically any mismatch between the I and Q branches from the ideal case, i.e., from the exact 90^o phase difference and equal amplitudes. The performance of a receiver can be severely limited by such IQ imbalances at the transmitter and receiver.

The effect of *receiver* IQ imbalances on OFDM systems and the resulting performance degradation have been investigated in [3], [4]. Several compensation algorithms have been proposed in [5]–[9]. Developing receiver algorithms in the digital domain that compensate for IQ imbalances in integrated wireless systems can lead to more efficient systems in terms of area-power-cost [5]. In the the recent works [5], [9] by the authors, compensation algorithms for OFDM receivers with IQ imbalances have been developed for both cases of SISO and MIMO communications. All these previous studies have focused on the problem of IQ imbalances at the *receiver only*.

In this paper, we first develop a framework that models the effect of IQ imbalances at *both* the transmitter *and* receiver of an OFDM system. Two compensation approaches are then presented. In one approach, both the transmitter and receiver distortions are compensated jointly at the receiver after the FFT operation. In the other approach, the transmitter imbalance is compensated at the transmitter by using a proposed pre-distorter, while the receiver imbalance is compensated at the receiver. The latter approach addresses the transmitter error vector magnitude (EVM) in standardized OFDM systems.

The paper is organized as follows. The next section describes the model used for transmitter and receiver IQ imbalances. Section III formulates the effect of IQ imbalances on an OFDM system. Joint compensation schemes at the receiver are presented in Section IV. A pre-distortion scheme at the transmitter is discussed in Section V. Simulation results are shown in Section VI and conclusions are given in Section VII.

II. FORMULATION OF IQ IMBALANCES

Let $y(t)$ represent the received baseband complex signal before being distorted by the IQ imbalance at the receiver. The distorted signal in the time domain can be modeled as [3], [4]:

$$y_d(t) = \mu_r y(t) + \nu_r y^*(t) \tag{1}$$

where the distortion parameters, μ_r and ν_r, are related to the amplitude and phase imbalances between the I and Q branches in the RF/Analog demodulation process at the receiver. A simplified model for the distortion parameters can be written as [4]:

$$\begin{aligned} \mu_r &= \cos(\theta_r/2) + j\alpha_r \sin(\theta_r/2) \\ \nu_r &= \alpha_r \cos(\theta_r/2) - j \sin(\theta_r/2) \end{aligned} \tag{2}$$

where θ_r and α_r are respectively the phase and amplitude imbalance between the I and Q branches at the receiver. The phase imbalance is any phase deviation from the ideal 90^o

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between the I and Q branches. The amplitude imbalance is defined as:

$$\alpha_r = \frac{a_I - a_Q}{a_I + a_Q}$$

where a_I and a_Q are the gain amplitudes on the I and Q branches. When stated in dB, the amplitude imbalance is computed as $10\log(1+\alpha_r)$. For instance, an amplitude imbalance of 0dB corresponds to the ideal case of $\alpha_r = 0$. The values of θ_r and α_r are not known at the receiver since they are caused by manufacturing inaccuracies in the analog components.

A similar expression can be used to model IQ imbalances at the transmitter. Let $s(t)$ represent the transmitted baseband complex signal before being distorted by IQ imbalances. Then the distorted baseband signal in the time domain will be given by–see Fig. 1:

$$s_d(t) = \mu_t s(t) + \nu_t s^*(t) \tag{3}$$

where the distortion parameters μ_t and ν_t are defined as in (2). The design of OFDM receivers in the presence of both transmitter and receiver IQ imbalances is discussed next.

III. OFDM Signals With IQ Imbalances

We extend the approach of [5], [9] to the case in which IQ imbalances are present at *both* the transmitter and the receiver. Thus recall that in OFDM systems, a block of data is transmitted as an OFDM symbol. Assuming a block size equal to N (where N is a power of 2), the transmitted block of data is denoted by

$$\mathbf{s} \triangleq \text{col}\{\mathbf{s}(1), \mathbf{s}(2), \ldots, \mathbf{s}(N)\} \tag{5}$$

Each block is passed through the IDFT operation:

$$\bar{\mathbf{s}} = \mathbf{F}^*\mathbf{s} \tag{6}$$

where $\mathbf{F}$ is the unitary discrete Fourier transform (DFT) matrix of size N defined by

$$[\mathbf{F}]_{ik} \triangleq \tfrac{1}{\sqrt{N}} \exp \tfrac{-j2\pi ik}{N}, \quad j = \sqrt{-1}$$
$$i,k = \{0, 1, \ldots, N-1\}$$

A cyclic prefix of length P is added to each transformed block of data and then transmitted through the channel–see Fig. 1. Due to IQ imbalances at the transmitter, as modeled by (3), the distorted transmitted vector is given by:

$$\boxed{\bar{\mathbf{s}}_d = \mu_t \bar{\mathbf{s}} + \nu_t \text{conj}(\bar{\mathbf{s}})} \tag{7}$$

where the notation conj(.) denotes a column vector (or matrix) whose entries are the complex conjugates of its argument. An FIR model with $L+1$ taps is assumed for the channel, i.e.,

$$\mathbf{h} = \text{col}\{h_0, h_1, \ldots,\ h_L\} \tag{8}$$

with $L \le P$ in order to preserve the orthogonality between tones. At the receiver, the received samples corresponding to the transmitted block $\bar{\mathbf{s}}$ are collected into a vector, after discarding the received cyclic prefix samples. The received block of data *before* being distorted by receiver IQ imbalances is given by [5]:

$$\boxed{\bar{\mathbf{y}} = \mathbf{H}^c \bar{\mathbf{s}}_d + \bar{\mathbf{v}}} \tag{9}$$

where $\mathbf{H}^c$ is an $N \times N$ circulant matrix whose first row is

$$\begin{bmatrix} h_0 & h_1 & \cdots & h_L & 0 & \ldots & 0 \end{bmatrix} \tag{10}$$

and $\bar{\mathbf{v}}$ is additive white noise at the receiver. It is known that $\mathbf{H}^c$ can be diagonalized by the N-point DFT matrix as

$$\mathbf{H}^c = \mathbf{F}^*\mathbf{\Lambda}\mathbf{F} \tag{11}$$

where

$$\mathbf{\Lambda} = \text{diag}\{\lambda\} \tag{12}$$

and the vector λ is related to $\mathbf{h}$ via

$$\lambda = \sqrt{N}\ \mathbf{F}^* \begin{bmatrix} \mathbf{h} \\ \mathbf{0}_{(N-(L+1))\times 1} \end{bmatrix} \tag{13}$$

Substituting (6) and (7) into (9) leads to

$$\bar{\mathbf{y}} = \mathbf{H}^c \left[\mu_t \bar{\mathbf{s}} + \nu_t \text{conj}(\bar{\mathbf{s}})\right] + \bar{\mathbf{v}} \tag{14}$$

or

$$\bar{\mathbf{y}} = \mathbf{H}^c \left[\mu_t \mathbf{F}^*\mathbf{s} + \nu_t \text{conj}(\mathbf{F}^*\mathbf{s})\right] + \bar{\mathbf{v}} \tag{15}$$

The received block of data $\bar{\mathbf{y}}$ after being distorted by receiver IQ imbalances will be transformed into (using (1)):

$$\boxed{\bar{\mathbf{z}} = \mu_r \bar{\mathbf{y}} + \nu_r \text{conj}(\bar{\mathbf{y}})} \tag{16}$$

Now remember that the N-point DFT of the complex conjugate of a sequence is related to the DFT of the original sequence through a mirrored relation. For notational simplicity, we denote the operation which gives the DFT of the complex conjugate of a vector by the superscript $\#$, i.e., for a vector X of size N we write

$$X = \begin{bmatrix} X(1) \\ X(2) \\ \vdots \\ X(N/2) \\ X(N/2+1) \\ X(N/2+2) \\ \vdots \\ X(N) \end{bmatrix} \Longrightarrow X^{\#} = \begin{bmatrix} X^*(1) \\ X^*(N) \\ \vdots \\ X^*(N/2+2) \\ X^*(N/2+1) \\ X^*(N/2) \\ \vdots \\ X^*(2) \end{bmatrix} \tag{17}$$

so that

$$X = \mathbf{F}x \Longrightarrow X^{\#} = \mathbf{F}\text{conj}\,(x) \tag{18}$$

It can be verified similarly that

$$x = \mathbf{F}^* X \Longrightarrow x^{\#} = \mathbf{F}^*\text{conj}\,(X) \tag{19}$$

Now substituting (11) into (15) gives

$$\begin{aligned} \bar{\mathbf{y}} &= \mathbf{F}^*\mathbf{\Lambda}\mathbf{F}\left[\mu_t \mathbf{F}^*\mathbf{s} + \nu_t \text{conj}(\mathbf{F}^*\mathbf{s})\right] + \bar{\mathbf{v}} \\ &= \mu_t \mathbf{F}^*\mathbf{\Lambda}\mathbf{s} + \nu_t \mathbf{F}^*\mathbf{\Lambda}\mathbf{F}\text{conj}(\mathbf{F}^*\mathbf{s}) + \bar{\mathbf{v}} \\ &= \mu_t \mathbf{F}^*\mathbf{\Lambda}\mathbf{s} + \nu_t \mathbf{F}^*\mathbf{\Lambda}\mathbf{s}^{\#} + \bar{\mathbf{v}} \\ &= \mathbf{F}^*\text{diag}\{\lambda\}\left(\mu_t \mathbf{s} + \nu_t \mathbf{s}^{\#}\right) + \bar{\mathbf{v}} \end{aligned} \tag{20}$$

where we used the fact that $\mathbf{F}\text{conj}(\mathbf{F}^*\mathbf{s}) = (\mathbf{F}\mathbf{F}^*\mathbf{s})^{\#} = \mathbf{s}^{\#}$. Moreover, using (14), we can write

$$\text{conj}(\bar{\mathbf{y}}) = \text{conj}(\mathbf{H}^c)\left[\mu_t^* \text{conj}(\bar{\mathbf{s}}) + \nu_t^*(\bar{\mathbf{s}})\right] + \text{conj}(\bar{\mathbf{v}}) \tag{21}$$

where $\text{conj}(\mathbf{H}^c)$ is again a circulant matrix defined in terms of $\text{conj}(\mathbf{h})$ as in (10) so that

$$\text{conj}(\mathbf{H}^c) = \mathbf{F}^*\text{diag}\left\{\lambda^{\#}\right\}\mathbf{F} \tag{22}$$

$$\mathbf{\Gamma}_k = \begin{bmatrix} \mu_r\mu_t\lambda(k) + \nu_r\nu_t^*\lambda^*(N-k+2) & \mu_r\nu_t\lambda(k) + \nu_r\mu_t^*\lambda^*(N-k+2) \\ \nu_r^*\mu_t\lambda(k) + \mu_r^*\nu_t^*\lambda^*(N-k+2) & \nu_r^*\nu_t\lambda(k) + \mu_r^*\mu_t^*\lambda^*(N-k+2) \end{bmatrix} \tag{4}$$

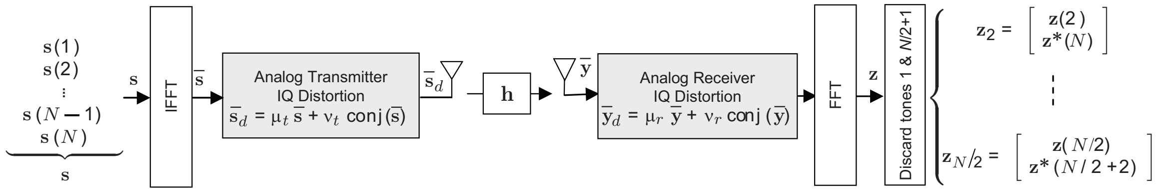


Fig. 1. An OFDM system with both transmit and receive IQ imbalances and the notation used in the derivations. Note that the addition and removal of the cyclic prefix are not shown in this figure for ease of notations.

where

$$\mathbf{F}^* \begin{bmatrix} \text{conj}(\mathbf{h}) \\ \mathbf{0}_{(N-(L+1))\times 1} \end{bmatrix} = \lambda^{\#} \tag{23}$$

Substituting the above into (21) results in

$$\begin{aligned} \text{conj}(\bar{\mathbf{y}}) &= \mathbf{F}^*\text{diag}\left\{\lambda^{\#}\right\}\mathbf{F}\left[\mu_t^*\text{conj}(\bar{\mathbf{s}}) + \nu_t^*\bar{\mathbf{s}}\right] + \text{conj}(\bar{\mathbf{v}}) \\ &= \mathbf{F}^*\text{diag}\left\{\lambda^{\#}\right\}\left(\mu_t^*\mathbf{s}^{\#} + \nu_t^*\mathbf{s}\right) + \text{conj}(\bar{\mathbf{v}}) \end{aligned} \tag{24}$$

where we substituted $\mathbf{F}\bar{\mathbf{s}} = \mathbf{s}$ and $\mathbf{F}\text{conj}(\bar{\mathbf{s}}) = \mathbf{s}^{\#}$ using (6) and (18).

After applying the DFT operation to the received block of data $\bar{\mathbf{z}}$ given by (16) (as is done in an OFDM receiver) and using (20) and (24), we obtain

$$\begin{aligned} \mathbf{z} &\triangleq \mathbf{F}\bar{\mathbf{z}} \\ &= \mu_r\text{diag}\{\lambda\}\left(\mu_t\mathbf{s} + \nu_t\mathbf{s}^{\#}\right) + \\ &\quad \nu_r\text{diag}\left\{\lambda^{\#}\right\}\left(\mu_t^*\mathbf{s}^{\#} + \nu_t^*\mathbf{s}\right) + \mu_r\mathbf{v} + \nu_r\mathbf{v}^{\#} \end{aligned} \tag{25}$$

where the vector $\mathbf{v}$ is given by $\mathbf{v} = \mathbf{F}\bar{\mathbf{v}}$ and the vector $\mathbf{v}^{\#}$ is defined according to the transformation (17). After rearranging terms,

$$\boxed{\begin{aligned} \mathbf{z} = &\left(\mu_r\mu_t\text{diag}\{\lambda\} + \nu_r\nu_t^*\text{diag}\left\{\lambda^{\#}\right\}\right)\mathbf{s} + \\ &\left(\mu_r\nu_t\text{diag}\{\lambda\} + \nu_r\mu_t^*\text{diag}\left\{\lambda^{\#}\right\}\right)\mathbf{s}^{\#} + \\ &\left(\mu_r\mathbf{v} + \nu_r\mathbf{v}^{\#}\right) \end{aligned}} \tag{26}$$

This result gives the exact input-output relation in an OFDM system with both transmitter and receiver IQ imbalances as a function of the channel taps $\{\lambda\}$ and the distortion parameters μ_r, μ_t, ν_r, and ν_t. Note that (26) collapses to the input-output relation derived in [5] for $\mu_t = 1$ and $\nu_t = 0$, as a special case where ideal IQ branches were assumed at the transmitter.

As seen from (26), the vector $\mathbf{z}$ is no longer related only to the transmitted block $\mathbf{s}$ through a diagonal matrix, as is the case in an OFDM system with ideal I and Q branches. There is also a contribution from $\mathbf{s}^{\#}$. Note however that the system of equations defined by (26) can be reduced to independent 2×2 systems of equations as follows [5]:

$$\mathbf{z}_k = \mathbf{\Gamma}_k\mathbf{s}_k + \mathbf{v}_k \tag{27}$$

where

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{z}(k) \\ \mathbf{z}^*(N-k+2) \end{bmatrix}, \ \mathbf{s}_k = \begin{bmatrix} \mathbf{s}(k) \\ \mathbf{s}^*(N-k+2) \end{bmatrix} \tag{28}$$

and

$$\mathbf{v}_k = \begin{bmatrix} \mathbf{v}(k) \\ \mathbf{v}^*(N-k+2) \end{bmatrix}$$

for $k = \{2, \ldots, N/2\}$ and the 2×2 matrix $\mathbf{\Gamma}_k$ is given by (4).

The objective is to recover $\mathbf{s}_k$ from $\mathbf{z}_k$ in (27) for $k = \{2, \ldots, N/2\}$ or, equivalently, $\mathbf{s}$ from $\mathbf{z}$. Several algorithms, adaptive and otherwise, for estimating channel/disortion parameters and recovering the $\mathbf{s}_k$ for the special case with ideal transmitter ($\mu_t = 1$ and $\nu_t = 0$) were proposed in [5]. The main difference here in relation to [5] is in the form of the matrix $\mathbf{\Gamma}_k$, which contains contributions from IQ distortions from *both* the transmitter *and* receiver. In the sequel, we extend some of the schemes of [5] to the more general case of imbalances at both the transmitter and receiver.

IV. Joint TX/RX Compensation at the Receiver

A. Least-Squares Compensation

The least-squares estimate of $\mathbf{s}_k$, $k = \{2, \ldots, N/2\}$, denoted by $\hat{\mathbf{s}}_k$, is given by [10]:

$$\hat{\mathbf{s}}_k = (\delta\mathbf{I} + \mathbf{\Gamma}_k^*\mathbf{\Gamma}_k)^{-1}\mathbf{\Gamma}_k^*\mathbf{z}_k \tag{31}$$

where the parameter $\delta > 0$ is added for regularization. In order to implement the solution (31), the channel information (λ) and the distortion parameters (μ_t,ν_t,μ_r,ν_r) are required. Training symbols are required to enable the receiver to estimate those values. Thus note that we may use equation (27) for channel estimation by rewriting it as:

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{s}(k) & 0 & \mathbf{s}^*(N-k+2) & 0 \\ 0 & \mathbf{s}(k) & 0 & \mathbf{s}^*(N-k+2) \end{bmatrix} \times \begin{bmatrix} \mu_r\mu_t\lambda(k) + \nu_r\nu_t^*\lambda^*(N-k+2) \\ \nu_r^*\mu_t\lambda(k) + \mu_r^*\nu_t^*\lambda^*(N-k+2) \\ \mu_r\nu_t\lambda(k) + \nu_r\mu_t^*\lambda^*(N-k+2) \\ \nu_r^*\nu_t\lambda(k) + \mu_r^*\mu_t^*\lambda^*(N-k+2) \end{bmatrix} + \mathbf{v}_k \tag{32}$$

Assuming n_{Tr} OFDM symbols are transmitted for training, then n_{Tr} realizations of the above equation can be collected to perform the least-squares estimation of the elements forming $\mathbf{\Gamma}_k$. The estimated $\mathbf{\Gamma}_k$ can then be substituted into (31) for data estimation. The same training data used for channel estimation in standard OFDM systems can be used in this scheme as the training symbols for joint channel and distortion estimation.

$$\mathbf{R}_{\tilde{\mathbf{s}}_k}(1,1) = \frac{\sigma_v^2}{|\lambda(k)|^2} \frac{(|\mu_r|^2+|\nu_r|^2)(|\mu_t|^2+|\frac{\lambda(k)}{\lambda(N-k+2)}|^2|\nu_t|^2)+4\mathrm{Re}(\frac{\lambda^*(k)}{\lambda^*(N-k+2)}\mu_r^*\mu_t^*\nu_r\nu_t^*)}{(|\mu_r|^2-|\nu_r|^2)^2(|\mu_t|^2-|\nu_t|^2)^2} \tag{29}$$

$$\text{Loss in SNR} = -10\log\left(\frac{(|\mu_r|^2+|\nu_r|^2)(|\mu_t|^2+|\frac{\lambda(k)}{\lambda(N-k+2)}|^2|\nu_t|^2)+4\mathrm{Re}(\frac{\lambda^*(k)}{\lambda^*(N-k+2)}\mu_r^*\mu_t^*\nu_r\nu_t^*)}{(|\mu_r|^2-|\nu_r|^2)^2(|\mu_t|^2-|\nu_t|^2)^2}\right) \tag{30}$$

A performance analysis on the achievable SNR using this compensation scheme is now given and compared to that of a receiver with ideal IQ branches. Using (27) and the corresponding least-squares estimate given by (31), the error in the estimation of $\mathbf{s}_k$ is given by (assuming $\delta \approx 0$):

$$\tilde{\mathbf{s}}_k \triangleq \hat{\mathbf{s}}_k - \mathbf{s}_k = \mathbf{\Gamma}_k^{-1}\mathbf{v}_k \tag{33}$$

and, consequently, the covariance matrix of the error vector is

$$\mathbf{R}_{\tilde{\mathbf{s}}_k} \triangleq \mathsf{E}\left(\tilde{\mathbf{s}}_k\tilde{\mathbf{s}}_k^*\right) = \sigma_v^2(\mathbf{\Gamma}_k^*\mathbf{\Gamma}_k)^{-1} \tag{34}$$

where $\mathbf{R}_{\mathbf{v}_k}$ was substituted by $\sigma_v^2\mathbf{I}_{2\times 2}$. Now let us consider the $(1,1)$ element of $\mathbf{R}_{\tilde{\mathbf{s}}_k}$, which denotes the error variance in estimating $\mathbf{s}(k)$. It can be verified that the $(1,1)$ element of $\mathbf{R}_{\tilde{\mathbf{s}}_k}$ is given by (29).

Note that the expression (29) collapses to

$$\frac{\sigma_v^2}{|\lambda(k)|^2} \tag{35}$$

for a receiver with ideal transmitter and receiver IQ branches ($\mu_r = \mu_r = 1$ and $\nu_r = \nu_t = 0$), as is expected. The difference between the error variance given by (29) for a receiver with least-squares equalization and the error variance given by (35) for a receiver with ideal IQ branches is defined as the loss in SNR (in dB) and is given by (30) where μ_r, μ_t, ν_r, and ν_t are related to the physical imbalances α_r, α_t, θ_r, and θ_t via (2). A similar expression can be derived for the error variance in estimating the other element of $\mathbf{s}_k$, namely, $\mathbf{s}(N-k+2)$. Note that the above expression also depends on the ratio between the channel values $\lambda(k)$ and $\lambda(N-k+2)$. To better illustrate the dependence of the SNR loss on the physical distortion parameters (as an upper bound on the achievable performance), we evaluate the expression (30) for several values of distortion parameters assuming $\lambda(k) \approx \lambda(N-k+2)$, i.e.,

$$\text{Loss in SNR} = -10\log\left(\frac{(|\mu_r|^2+|\nu_r|^2)(|\mu_t|^2+|\nu_t|^2)+4\mathrm{Re}(\mu_r^*\mu_t^*\nu_r\nu_t^*)}{(|\mu_r|^2-|\nu_r|^2)^2(|\mu_t|^2-|\nu_t|^2)^2}\right) \tag{36}$$

The above expression is evaluated for different values of transmitter and receiver distortion parameters–see Table I at the top of the next page. For instance, the SNR degradation in the presence of both transmitter and receiver imbalances (for reasonably large values of $\theta_r = 2.0^o$, $\alpha_r = 1.0$dB, $\theta_t = 2.0^o$, and $\alpha_r = 1.0$dB) is lower bounded by ~ 0.5dB.

B. Adaptive Equalization

The adaptive estimation of $\mathbf{s}(k)$ and $\mathbf{s}^*(N-k+2)$ in (28) can be attained as follows:

$$\begin{aligned}\hat{\mathbf{s}}(k) &= \mathbf{w}_k\mathbf{z}_k \\ \hat{\mathbf{s}}^*(N-k+2) &= \mathbf{w}_{N-k+2}\mathbf{z}_k\end{aligned} \tag{37}$$

where $\mathbf{w}_k$ and $\mathbf{w}_{N-k+2}$ are 1×2 equalization vectors updated according to an adaptive algorithm (for instance LMS or some other adaptive form) for $k = \{2,\ldots,N/2\}$ [10]. To better illustrate the update equations, we introduce the time (or iteration) index i. As a result, let $\mathbf{w}_k^{(i)}$ and $\mathbf{w}_{N-k+2}^{(i)}$ represent the equalization vectors at time instant i. Furthermore, let $\mathbf{z}_k^{(i)}$ represent the vector $\mathbf{z}_k$ defined in (28) at time instant i. Now, the equalization coefficients for $k = \{2,\ldots,N/2\}$ are updated according to the LMS rules:

$$\mathbf{w}_k^{(i+1)} = \mathbf{w}_k^{(i)} + \mu_{\text{LMS}}\left(\mathbf{z}_k^{(i)}\right)^* e_k^{(i)} \tag{38}$$

$$\mathbf{w}_{N-k+2}^{(i+1)} = \mathbf{w}_{N-k+2}^{(i)} + \mu_{\text{LMS}}\left(\mathbf{z}_k^{(i)}\right)^* e_{N-k+2}^{(i)} \tag{39}$$

where $e_k^{(i)} = d_k^{(i)} - \mathbf{w}_k^{(i)}\mathbf{z}_k^{(i)}$ is the error signal generated at iteration i for the tone index k using a training symbol $d_k^{(i)}$, where the training symbol $d_k^{(i)}$ can be different for different tone indices k. A similar relation holds for $e_{N-k+2}^{(i)}$. Moreover, μ_{LMS} is the LMS step-size parameter.

An important property of the schemes proposed in this section is that they compensate for both transmit and receive imbalances jointly at the *receiver*. In other words, the transmitter is not necessarily required to achieve good IQ matching. This is an advantage for proprietary systems where the transmitter and the receiver are designed by the same manufacturer, since it can significantly relax the design specification on the transmitter. However, this is not desired for standardized systems where the transmitters and receivers may be designed and manufactured by different manufacturers. In such systems, the transmitted signal's distortion has to be below a certain level specified by the standard, namely the error vector magnitude (EVM), so that receivers by other manufacturers can correctly decode it. In this case, the transmitter has to meet a certain level of IQ matching. To address this issue, we suggest below a scheme at the transmitter, referred to as *pre-distortion*, such that the final transmitted signal is sufficiently close to an ideal transmitter.

V. Post-IFFT and Pre-FFT Compensation

Recalling (1) as the model for the distorted signal, let us define the following operation [5]:

$$y_c(t) \triangleq y_d(t) - \eta_r y_d^*(t) \tag{40}$$

where

$$\eta_r \triangleq \frac{\nu_r}{\mu_r^*} \tag{41}$$

Then it can be verified that

$$y_c(t) = \left(\mu_r - \frac{|\nu_r|^2}{\mu_r^*}\right)y(t) \tag{42}$$

TABLE I
LOSS IN SNR (IN DB) ACCORDING TO (36) EVALUATED FOR DIFFERENT VALUES OF TRANSMITTER AND RECEIVER DISTORTION PARAMETERS

	$\theta_t = 0^o$ $\alpha_t = 0$dB	$\theta_t = 0.4^o$ $\alpha_t = 0.2$dB	$\theta_t = 0.8^o$ $\alpha_t = 0.4$dB	$\theta_t = 1.2^o$ $\alpha_t = 0.6$dB	$\theta_t = 1.6^o$ $\alpha_t = 0.8$dB	$\theta_t = 2.0^o$ $\alpha_t = 1.0$dB
$\theta_r = 0^o, \alpha_r = 0$dB	0.00	0.01	0.03	0.06	0.11	0.18
$\theta_r = 0.4^o, \alpha_r = 0.2$dB	0.01	0.02	0.05	0.10	0.16	0.23
$\theta_r = 0.8^o, \alpha_r = 0.4$dB	0.03	0.05	0.10	0.15	0.22	0.30
$\theta_r = 1.2^o, \alpha_r = 0.6$dB	0.06	0.10	0.15	0.21	0.29	0.38
$\theta_r = 1.6^o, \alpha_r = 0.8$dB	0.11	0.16	0.22	0.29	0.37	0.47
$\theta_r = 2.0^o, \alpha_r = 1.0$dB	0.18	0.23	0.30	0.38	0.47	0.57

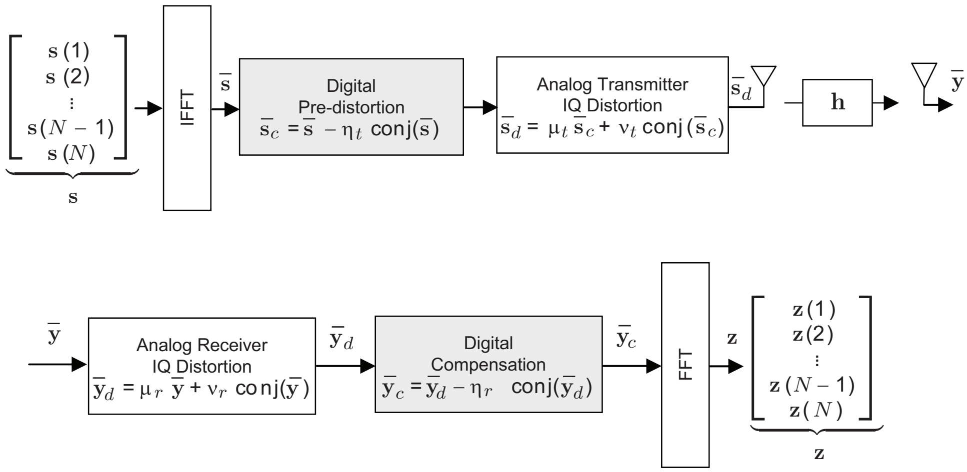


Fig. 2. An OFDM system with post-IFFT transmit and pre-FFT receive compensation for IQ imbalances. The gray blocks depict the compensation operations. Note that the addition and removal of the cyclic prefix are not shown in the figure for ease of notations.

This relation suggests that the receiver IQ distortion can be removed by using the above scaling given that the value of η_r is provided. Note that only the ratio between ν_r and μ_r^* is needed to calculate (40), and not the individual values.

A similar approach can be used at the transmitter but in a different order. In this case, the transmitted signal is pre-distorted in the digital domain before transmission in such a way that the final transmitted signal is sufficiently free of distortion. Recalling (3), let us assume that the transmitted signal $s(t)$ is first pre-distorted according to

$$s_c(t) \triangleq s(t) - \eta_t s^*(t) \tag{43}$$

where

$$\eta_t \triangleq \frac{\nu_t}{\mu_t} \tag{44}$$

Then using $s_c(t)$ in (3) gives

$$s_d(t) = \left(\mu_t - \frac{|\nu_t|^2}{\mu_t^*}\right) s(t) \tag{45}$$

In this way, the transmitted signal is a multiple of the desired signal and is free of IQ distortion. A block diagram of an OFDM system using the above operations is depicted in Fig. 2.

The issue now becomes how to estimate the compensation parameters η_t and η_r. Let us revisit the channel matrix given by (4). Let $\rho_{11}(k)$, $\rho_{12}(k)$, $\rho_{21}(k)$, and $\rho_{22}(k)$ denote the four elements in $\mathbf{\Gamma}_k$, i.e,

$$\mathbf{\Gamma}_k = \begin{bmatrix} \rho_{11}(k) & \rho_{12}(k) \\ \rho_{21}(k) & \rho_{22}(k) \end{bmatrix} \tag{46}$$

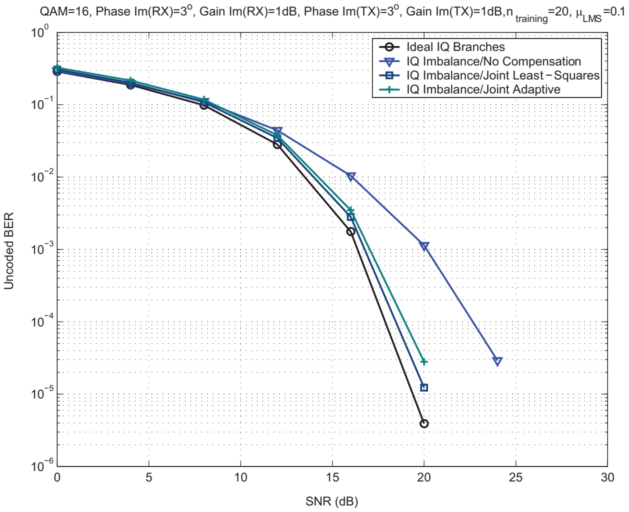
It can be verified that these elements can be reorganized as a function of η_t and η_r:

$$\begin{aligned} \rho_{11}(k) &= \beta(k) + \eta_r \eta_t^* \beta^*(N-k+2) \\ \rho_{12}(k) &= \eta_t \beta(k) + \eta_r \beta^*(N-k+2) \\ \rho_{21}(k) &= \eta_r^* \beta(k) + \eta_t^* \beta^*(N-k+2) \\ \rho_{22}(k) &= \eta_r^* \eta_t \beta(k) + \beta^*(N-k+2) \end{aligned} \tag{47}$$

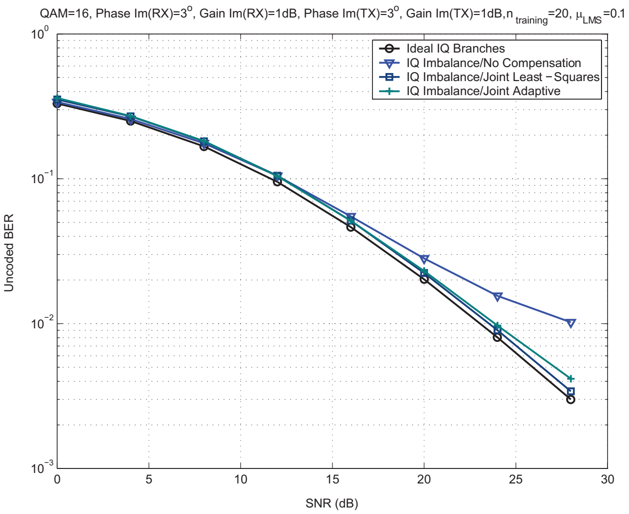
where

$$\begin{aligned} \beta(k) &\triangleq \mu_r \mu_t \lambda(k) \\ \beta(N-k+2) &\triangleq \mu_r \mu_t \lambda(N-k+2) \end{aligned} \tag{48}$$

Using the input-output relation (27) and by transmitting some training sequence, the channel matrix $\mathbf{\Gamma}_k$ can be estimated. A special pilot pattern is proposed in [5] that can be used here to simplify the process of estimating the elements of $\mathbf{\Gamma}_k$. Thus, assume that sufficiently accurate estimates of the elements of $\mathbf{\Gamma}_k$ are achievable. Denote them by $\hat{\rho}_{11}(k)$, $\hat{\rho}_{12}(k)$, $\hat{\rho}_{21}(k)$, and $\hat{\rho}_{22}(k)$. Once these estimates are available, they can be substituted in the system of equations (47) to solve for the four unknowns η_t, η_r, $\beta(k)$, and $\beta(N-k+2)$. There are two points here to consider: 1) The system of equations (47) can be repeated for different values of $k = \{2, \ldots, N/2\}$. Since η_t and η_r are independent of k, the estimates of η_t and η_r for different values of k can be averaged at the end for better estimates. 2) It can be verified that the four equations in (47) collapse to only two independent equations when $\beta(k) = \beta(N-k+2)$ or, equivalently, $\lambda(k) = \lambda(N-k+2)$, with three unknowns left η_t, η_r, and $\beta(k)$. In other words, with a flat (single-path) channel, not enough information is gathered

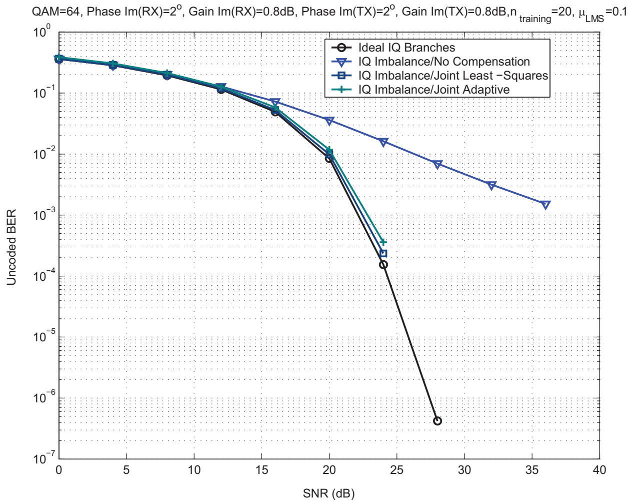


(a) AWGN flat channel (*non-fading*).

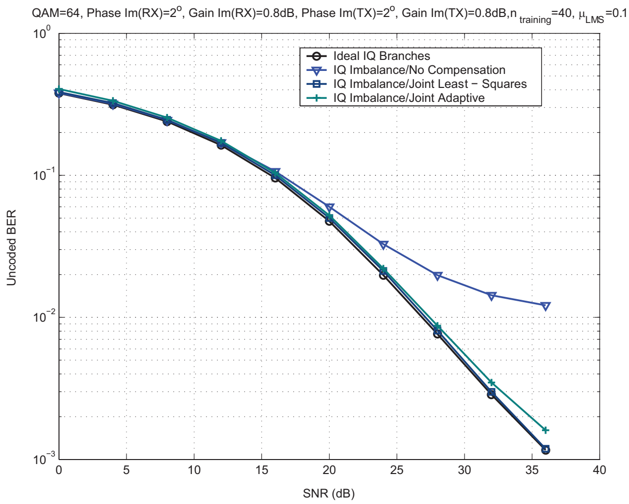


(b) 4-tap Rayleigh fading channel (*fading*).

Fig. 3. BER vs. SNR simulated for the following configuration: 16QAM constellation, training length of 20 OFDM symbols in least-squares and LMS solutions, LMS step-size of $\mu_{LMS} = 0.1$, transmitter phase imbalance of $\theta_t = 3^o$, transmitter amplitude imbalance of α_t=1dB, receiver phase imbalance of $\theta_r = 3^o$, and receiver amplitude imbalance of α_t=1dB. 'IQ Imbalance/Joint Least-Squares' refers to the compensation scheme given by (31) and 'IQ Imbalance/Joint Adaptive' refers to the scheme given by (37). In both schemes, the transmitter and receiver imbalances are compensated jointly at the receiver.



(a) AWGN flat channel (*non-fading*).



(b) 4-tap Rayleigh fading channel (*fading*).

Fig. 4. BER vs. SNR simulated for the following configuration: 64QAM constellation, training length of 40 OFDM symbols in least-squares and LMS solutions, LMS step-size of $\mu_{LMS} = 0.1$, transmitter phase imbalance of $\theta_t = 2^o$, transmitter amplitude imbalance of α_t=0.8dB, receiver phase imbalance of $\theta_r = 2^o$, and receiver amplitude imbalance of α_t=0.8dB. 'IQ Imbalance/Joint Least-Squares' refers to the compensation scheme given by (31) and 'IQ Imbalance/Joint Adaptive' refers to the scheme given by (37). In both schemes, the transmitter and receiver imbalances are compensated jointly at the receiver.

to estimate the unknown values. Therefore, it is necessary to have a multi-path channel in order to use (47) to estimate η_t and η_r.

A simpler scheme can be used to estimate η_t and η_r if a transmitter or receiver with sufficiently ideal IQ branches is available during the estimation process. In this case, an ideal transmitter can be used to estimate η_r or vice versa (an ideal receiver can be used to estimate η_t). If a receiver with ideal IQ branches is available, then substituting $\eta_r = 0$ in (47) results in

$$\begin{aligned}\rho_{11}(k) &= \beta(k)\\ \rho_{12}(k) &= \eta_t\beta(k)\\ \rho_{21}(k) &= \eta_t^*\beta^*(N-k+2)\\ \rho_{22}(k) &= \beta^*(N-k+2)\end{aligned} \tag{49}$$

and η_t can be estimated from

$$\frac{\rho_{12}(k)}{\rho_{11}(k)}, \quad \left(\frac{\rho_{21}(k)}{\rho_{22}(k)}\right)^* \tag{50}$$

If a transmitter with ideal IQ branches is available, then substituting $\eta_t = 0$ in (47) results in

$$\begin{aligned}\rho_{11}(k) &= \beta(k)\\ \rho_{12}(k) &= \eta_r\beta^*(N-k+2)\\ \rho_{21}(k) &= \eta_r^*\beta(k)\\ \rho_{22}(k) &= \beta^*(N-k+2)\end{aligned} \tag{51}$$

and η_r can be estimated from

$$\frac{\rho_{12}(k)}{\rho_{22}(k)}, \quad \left(\frac{\rho_{21}(k)}{\rho_{11}(k)}\right)^* \tag{52}$$

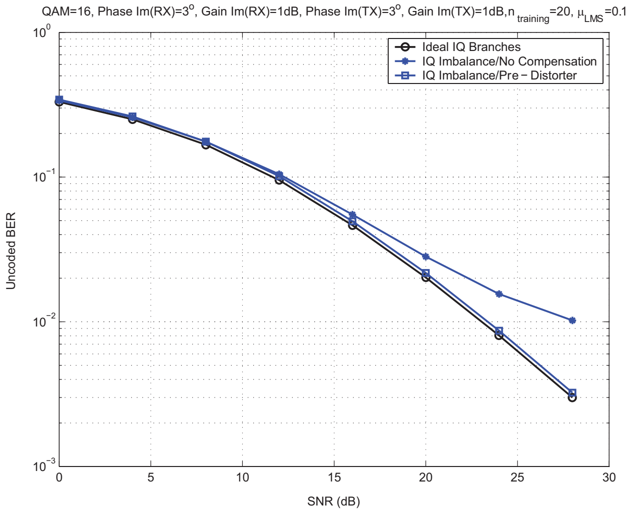


Fig. 5. BER vs. SNR simulated for the following configuration: 4-tap complex Gaussian *fading* channel, 16QAM constellation, training length of 20 OFDM symbols, transmitter phase imbalance of $\theta_t = 3^o$, transmitter amplitude imbalance of α_t=1dB, receiver phase imbalance of $\theta_r = 3^o$, and receiver amplitude imbalance of α_t=1dB. 'IQ Imbalance/Pre-Distorter' refers to the compensation scheme in defined by (40) and (43).

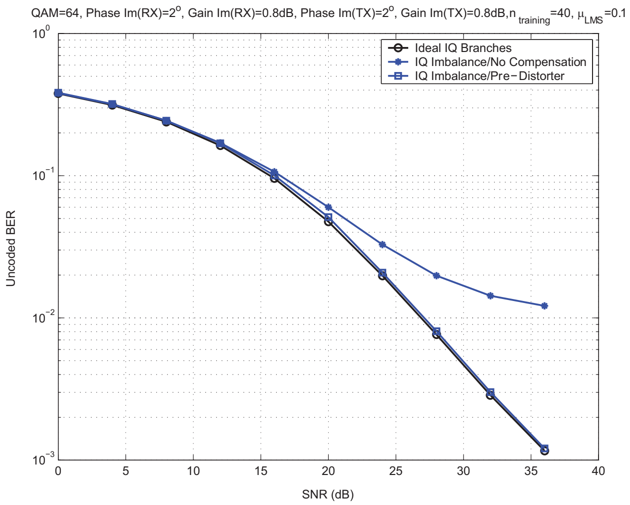


Fig. 6. BER vs. SNR simulated for the following configuration: 4-tap complex Gaussian *fading* channel, 64QAM constellation, training length of 40 OFDM symbols, transmitter phase imbalance of $\theta_t = 2^o$, transmitter amplitude imbalance of α_t=0.8dB, receiver phase imbalance of $\theta_r = 2^o$, and receiver amplitude imbalance of α_t=0.8dB. 'IQ Imbalance/Pre-Distorter' refers to the compensation scheme in defined by (40) and (43).

Once one of the correction coefficients η_r or η_t has been estimated, the other one can be obtained by using the system of equations (47).

Note that the schemes proposed in this section can be either implemented using the receiver at the receiving device or using the receiver available in the transmitting device. In the first case, the estimated parameters need to be sent back to the transmitter. The feedback overhead will be negligible since these parameters change at a very slow rate. In the latter case, the estimation is performed using the receiver available in the transmitting device and no feedback is required. In this case, the parameter estimation process is performed during the time that the device is not communicating. This process can be viewed as a self-calibration process by the device before it starts communicating.

VI. Simulations

A typical OFDM system (similar to IEEE802.11a) is simulated to evaluate the performance of the proposed schemes in comparison to an ideal OFDM system with no transmit-receive IQ imbalance and a receiver with no compensation scheme. The parameters used in the simulation are: OFDM symbol length of $N = 64$, cyclic prefix of $P = 16$. All the figures present simulation results for uncoded BER. To better depict the performance of the proposed schemes, each simulation configuration is repeated for two different channel profiles for the sake of comparison. : 1) additive white Gaussian noise (AWGN) channel with a single tap unity gain and 2) a multipath channel with $(L + 1) = 4$ taps where the taps are chosen independently with complex Gaussian distribution. Every channel realization is independent of the previous one and the BER results depicted are from averaging the BER curves over hundreds of independent channels. The simulation results are presented in the following two sub-sections for the algorithms presented in Section IV and Section V:

A. Joint TX/RX compensation at the Receiver

The BER versus SNR are simulated and shown in Figs. 3 and 4 for 16QAM and 64QAM constellations. In all figures, 'Ideal IQ Branches' legend refers to a receiver with perfect IQ branches and perfect channel knowledge and 'IQ Imbalance/No Compensation' refers to a system with IQ imbalance but no compensation scheme. 'IQ Imbalance/Joint Least-Squares' refers to the compensation scheme in subsection IV-A given by (31) where the matrix $\mathbf{\Gamma_k}$ is estimated using (32). 'IQ Imbalance/Joint Adaptive' refers to the scheme presented in sub-section IV-B given by (37). Each system configuration is repeated for an AWGN channel and a 4-tap complex Gaussian multipath channel for comparison purposes. The values used for phase and amplitude imbalances for both the transmitter and the receiver are typical values achievable in practical integrated circuit implementations.

B. Pot-IFFT Transmitter and Pre-FFT Receiver Compensations

The BER versus SNR are simulated and shown in Figs. 5 and 6 for 16QAM and 64QAM constellations. Only a 4-multipath Rayleigh fading channel is simulated in this part. 'IQ Imbalance/Pre-Distorter' refers to the compensation scheme in Section V defined by (40) and (43). An ideal receiver is assumed to estimate η_t using (50) and an ideal transmitter is assumed to estimate η_r using (52 as explained in Section V.

VII. Conclusions

The paper studied the problem of transmitter and receiver IQ imbalances in OFDM systems. An input-output relation is derived as a function of both transmit and receiver distortion parameters. The input-output relation is then used to develop compensation algorithms for the IQ imbalances in the digital domain. Different compensation schemes are presented. Two of the proposed algorithms are implemented at the receiver side and compensate for both transmitter and receiver distortions in one step. In another approach, the transmitter distortion is compensated through a pre-distortion operation and the receiver distortion is compensated for at the receiver.

REFERENCES

[1] B. Razavi, *RF Microelectronics*. Upper Saddle River, NJ: Prentice Hall, 1998.

[2] A. A. Abidi, "Direct-conversion radio transceivers for digital communications," *IEEE J. Solid-State Circuits*, vol. 30, pp. 1399–1410, Dec. 1995.

[3] A. Baier, "Quadrature mixer imbalances in digital TDMA mobile radio receivers," in *Proc. International Zurich Seminar on Digital Communications, Electronic Circuits and Systems for Communications*, Mar. 1990, pp. 147–162.

[4] C. L. Liu, "Impacts of I/Q imbalance on QPSK-OFDM-QAM detection," *IEEE Trans. Consumer Electron.*, vol. 44, pp. 984–989, Aug. 1998.

[5] A. Tarighat, R. Bagheri, and A. H. Sayed, "Compensation schemes and performance analysis of IQ imbalances in OFDM receivers," *IEEE Trans. Signal Processing*, vol. 53, no. 8, pp. 3257–3268, Aug. 2005.

[6] A. Schuchert, R. Hasholzner, and P. Antoine, "A novel IQ imbalance compensation scheme for the reception of OFDM signals," *IEEE Trans. Consumer Electron.*, vol. 47, pp. 313–318, Aug. 2001.

[7] S. Fouladifard and H. Shafiee, "Frequency offset estimation in OFDM systems in presence of IQ imbalance," in *Proc. International Conf. Commun.*, May 2003, pp. 2071–2075.

[8] J. Tubbax, B. Come, L. V. der Perre, S. Donnay, M. Engels, M. Moonen, and H. D. Man, "Joint compensation of IQ imbalance and frequency offset in OFDM systems," in *Proc. Radio and Wireless Conference*, Aug. 2003, pp. 39–42.

[9] A. Tarighat and A. H. Sayed, "MIMO OFDM receivers for systems with IQ imbalances," *IEEE Trans. Signal Processing*, vol. 53, no. 9, pp. 3583–3596, Sep. 2005.

[10] A. H. Sayed, *Fundamentals of Adaptive Filtering*. New York: Wiley, 2003.

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