Optimal Multiband Joint Detection for Spectrum Sensing in Cognitive Radio Networks

Zhi Quan, *Student Member, IEEE*, Shuguang Cui, *Member, IEEE*, Ali H. Sayed, *Fellow, IEEE*, and H. Vincent Poor, *Fellow, IEEE*

***Abstract*—Spectrum sensing is an essential functionality that enables cognitive radios to detect spectral holes and to opportunistically use under-utilized frequency bands without causing harmful interference to legacy (primary) networks. In this paper, a novel wideband spectrum sensing technique referred to as *multiband joint detection* is introduced, which jointly detects the primary signals over multiple frequency bands rather than over one band at a time. Specifically, the spectrum sensing problem is formulated as a class of optimization problems, which maximize the *aggregated opportunistic throughput* of a cognitive radio system under some constraints on the interference to the primary users. By exploiting the hidden convexity in the seemingly nonconvex problems, optimal solutions can be obtained for multiband joint detection under practical conditions. The situation in which individual cognitive radios might not be able to reliably detect weak primary signals due to channel fading/shadowing is also considered. To address this issue by exploiting the spatial diversity, a cooperative wideband spectrum sensing scheme refereed to as *spatial-spectral joint detection* is proposed, which is based on a linear combination of the local statistics from multiple spatially distributed cognitive radios. The cooperative sensing problem is also mapped into an optimization problem, for which suboptimal solutions can be obtained through mathematical transformation under conditions of practical interest. Simulation results show that the proposed spectrum sensing schemes can considerably improve system performance. This paper establishes useful principles for the design of distributed wideband spectrum sensing algorithms in cognitive radio networks.**

***Index Terms*—Cognitive radio, cooperative sensing, hypothesis testing, multiband joint detection, nonlinear optimization, spectrum sensing.**

Manuscript received November 04, 2007; accepted October 02, 2008. First published November 07, 2008; current version published February 13, 2009. This research was supported in part by the National Science Foundation under Grants ANI-0338807, CNS-06-25637, ECS-0601266, ECS-0725441, CNS-0721935, CCF-0726740, and by the Department of Defense under Grant HDTRA-07-1-0037. Part of this work was presented at the IEEE Conference on Communications, Beijing, China, May 2008, and at the IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Las Vegas, NV, April 2008.

Z. Quan and A. H. Sayed are with the Department of Electrical Engineering, University of California, Los Angeles, CA 90095 USA (e-mail: quan@ee.ucla.edu; sayed@ee.ucla.edu).

S. Cui is with the Department of Electrical and Computer Engineering, Texas A&M University, College Station, TX 77843 USA (e-mail: cui@ece.tamu.edu).

H. V. Poor is with the Department of Electrical Engineering, Princeton University, Princeton, NJ 08544 USA (e-mail: poor@princeton.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSP.2008.2008540

I. Introduction

Traditional wireless networks are regulated by fixed spectrum allocation policies to operate in certain time frames, over certain frequency bands, and within certain geographical regions. This regulation results in situations in which some radio bands are overcrowded while other bands remain moderately or rarely occupied. In order to improve spectral utilization, cognitive radio (CR) technology has been proposed as a potential communication paradigm [1]. Cognitive radios are defined by the Federal Communications Commission (FCC) [2] as radio systems that continuously perform spectrum sensing, dynamically identify unused spectrum, and then operate in those spectral holes where the licensed (primary) radio systems are idle. In CR networks, secondary users are allowed to use some portions of licensed radio bands opportunistically provided that they do not cause harmful interference to the primary users in these frequency bands. CR is an important component of the IEEE 802.22 standard being developed for wireless regional area networks, which involves a cognitive radio based air interface to operate in a licence-exempt way over the TV broadcast bands. This new communication paradigm, also referred to as the dynamic spectrum access (DSA) or neXt Generation (XG) network [3], can dramatically improve spectral utilization.

Effective spectrum sensing needs to detect weak primary radio signals of possibly-unknown formats reliably [4]. Generally, spectrum sensing techniques can be classified into three broad categories: energy detection [5], [6], matched filtering (coherent) detection [7], and feature detection [8]. Energy detection has been shown to be optimal if the cognitive devices have no *a priori* information about the features of the primary signals except local noise statistics [9]. When the CRs have some knowledge about the primary signal features such as preambles, pilots, and synchronization symbols, the optimal detector usually applies the matched filter structure to maximize the probability of detection. If the modulation schemes of the primary signals are known, then the cyclostationary feature detector can differentiate primary signals from the local noise by exploiting certain periodicity exhibited by the mean and autocorrelation of the corresponding modulated signals. Since noncoherent energy detection is simple and able to determine spectrum-occupancy information quickly, we will adopt it as the building block for constructing the proposed wideband spectrum sensing techniques.

A. Related Work

Previous studies on spectrum sensing in CR networks have focused primarily on cooperation among multiple secondary users [4], [10], [11] using distributed detection approaches [12], [13], but are limited to the detection of signals over a single frequency band. The scheme based on voting rules [14] is one of the simplest suboptimal solutions, which counts the number of nodes that vote for the presence of the signal and compares it against a given threshold. In [15], a fusion rule known as the OR logic operation was used to combine decisions from several secondary users. In [16], two decision-combining approaches were studied: hard decision with the AND logic operation, and soft decision using the likelihood ratio test [12]. It was shown that the soft decision combination of spectrum sensing results yields gains over hard decision combining. In [17], the authors exploited the fact that summing signals from two secondary users can increase the signal-to-noise ratio (SNR) and detection reliability if the signals are correlated. In [18], a generalized likelihood ratio test for detecting the presence of cyclostationarity using multiple cyclic frequencies was proposed and evaluated using Monte Carlo simulations. Another two cooperative spectrum sensing algorithms based on the likelihood ratio test can be found in [19] and [20]. Along with these works, we have developed an optimal cooperation strategy [21] based on a linear combination of local statistics from multiple cognitive radios. Other suboptimal solutions for linear cooperation such as maximal radio combining and maximal deflection coefficient combining can be found respectively in [21]–[23].

On the other hand, the literature of wideband spectrum sensing for cognitive radio networks is rather limited at this time. An existing approach is to use a tunable narrowband bandpass filter at the radio frequency (RF) front-end to search one narrow frequency band at a time [24], over which existing narrowband spectrum sensing techniques can be applied. In order to search over multiple frequency bands at a time, the RF front-end needs a wideband architecture and spectrum sensing usually operates over an estimate of the power spectral density (PSD) of the wideband signal. In [25] and [26], the multiresolution features of the wavelet transform were used to estimate the PSD over a wide frequency range. However, no prior work attempts to make decisions over multiple frequency bands jointly, which is essential for implementing efficient cognitive radio networks. A survey of existing spectrum sensing techniques can be found in [35].

B. Contribution

The contribution of this paper is twofold. First, we introduce the multiband joint detection framework for wideband spectrum sensing in a single CR. Within this framework, we jointly optimize a bank of multiple narrowband detectors to improve the *aggregate opportunistic throughput* of a cognitive radio system while limiting the interference to the primary communication system. In particular, we formulate the design of wideband spectrum sensing into a class of optimization problems. The objective is to maximize the aggregate opportunistic throughput in an interference-limited cognitive radio network. By exploiting the hidden convexity of the seemingly nonconvex problems, we show that the optimization problem can be reformulated into a convex program under practical conditions. The multiband joint detection strategy allows cognitive radios to best take advantage of the unused frequency bands and limit the resulting interference.

In addition, we consider the situation in which spectrum sensing is compromised by destructive channel conditions between the target-under-detection and the detecting cognitive radios, where it is hard to distinguish between a white spectrum and a weak signal attenuated by deep fading. We propose a cooperative wideband spectrum sensing scheme that exploits the spatial diversity among multiple CRs to improve the sensing reliability. The cooperation is based on a linear combination of local statistics from spatially distributed cognitive radios [21], [23], where these signals are assigned different weights according to their individual positive contributions to the joint sensing. In such a scenario, we view the design of distributed wideband spectrum sensing as a spatial–spectral joint detection (SSJD) problem, which is further formulated into an optimization problem with the objective of maximizing the aggregate opportunistic throughput under constraints on the interference to primary users. Through mathematical reformulation, we derive two sets of suboptimal but efficient solutions for the optimization problem, which can considerably improve the sensing performance.

The rest of the paper is organized as follows. In Section II, we describe the system model for wideband spectrum sensing. In Section III, we develop multiband joint detection algorithms for spectrum sensing, which seek to maximize the aggregate opportunistic throughput of a CR system. The spatial–spectral joint detection strategy is formulated in Section IV, where we derive two efficient solutions to optimize the cooperation among a network of CRs. The advantages of the proposed spectrum sensing algorithms are illustrated by simulations in Section V, and conclusions are drawn in Section VI.

II. System Model

A. Wideband Spectrum Sensing

Consider a primary communication system (e.g., multicarrier modulation based) operating over a wideband channel that is divided into K nonoverlapping narrowband subbands. In a particular geographical region and within a particular time interval, some of the K subbands might not be used by the primary users and are available for opportunistic spectrum access.

We model the detection problem on subband k as one of choosing between a hypothesis $\mathcal{H}_{0,k}$ ("0"), which represents the absence of primary signals, and an alternative hypothesis $\mathcal{H}_{1,k}$ ("1"), which represents the presence of primary signals. An example where only some of the K bands are occupied by primary users is depicted in Fig. 1. The underlying hypothesis vector is a binary representation of the subbands that are allowed for or prohibited from opportunistic spectrum access.

The crucial task of spectrum sensing is to sense the K subbands and identify spectral holes for opportunistic use. For simplicity, we assume that the upper-layer protocols, e.g., the medium access control (MAC) layer, can guarantee that all cognitive radios stay silent during the detection interval such that the only spectral power remaining in the air is emitted by

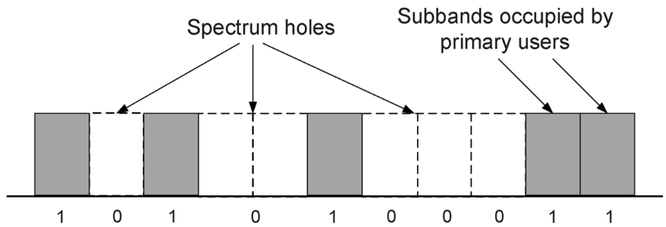

Fig. 1. Schematic illustration of the occupancy of a multiband channel.

the primary users. In this paper, instead of considering a single band at a time, we propose to use a multiband joint detection technique, which jointly takes into account the detection of primary users across multiple frequency bands.

B. Received Signal

Consider a multipath fading environment, where $h(l)$, $l = 0, 1, \ldots, L-1$, represents the discrete-time channel impulse response between the primary transmitter and a CR receiver with L denoting the number of resolvable paths. The received baseband signal at the RF front-end can be written as

$$r(n) = \sum_{l=0}^{L-1} h(l)s(n-l) + v(n) \tag{1}$$

where $s(n)$ represents the primary transmitted signal (with cyclic prefix) at time n and $v(n)$ is additive complex white Gaussian noise with zero mean and variance σ_v^2, i.e., $v(n) \sim \mathcal{CN}(0, \sigma_v^2)$. In a multipath fading environment, the wideband channel exhibits frequency-selective features [27] and its discrete frequency response can be obtained through a K-point fast Fourier transform (FFT) $(K \geq L)$:

$$H_k = \frac{1}{\sqrt{K}} \sum_{n=0}^{L-1} h(n)e^{-j2\pi nk/K}, \quad k = 1, 2, \ldots, K. \tag{2}$$

In the frequency domain, the received signal at each subchannel can be represented by its discrete Fourier transform (DFT):

$$\begin{aligned} R_k &= \frac{1}{\sqrt{K}} \sum_{n=0}^{K-1} r(n)e^{-j2\pi nk/K} \\ &= H_k S_k + V_k, \quad k = 1, 2, \ldots, K \end{aligned} \tag{3}$$

where S_k is the primary transmitted signal at subchannel k and

$$V_k = \frac{1}{\sqrt{K}} \sum_{n=0}^{K-1} v(n)e^{-j2\pi nk/K}, \quad k = 1, 2, \ldots, K \tag{4}$$

is the received noise represented in the frequency domain. The random variables $\{V_k\}$ are independent and normally distributed with zero means and variances σ_v^2, i.e., $V_k \sim \mathcal{CN}(0, \sigma_v^2)$, since $v(n) \sim \mathcal{CN}(0, \sigma_v^2)$ and the DFT is a linear unitary operation. Without loss of generality, we assume that the transmitted signal S_k, channel gain H_k, and additive noise V_k are independent of each other.

Since the IEEE 802.22 consumer premise equipment (CPE) is generally used in a fixed wireless network in the TV bands [28], it is reasonable to assume that the channels between the primary transmitter and secondary receivers change slowly such that they can be assumed to be constant during each operation period of interest. Our sensing algorithm needs to know only the noise power σ_v^2 and the squared values of the channel frequency responses $\{|H_k|^2\}$, which can be estimated in practice. Specifically, σ_v^2 can be calibrated in a given band that is known for sure to be idle (e.g., TV channel 37 is currently always empty) [29]. Accordingly, $\{|H_k|^2\}$ can be learned *a priori* during a period when the primary transmitter was known for sure to be working. This *a priori* information is obtainable since most current TV stations transmit pilot signals periodically at a fixed power level.

C. Signal Detection in Individual Bands

We start from signal detection in a single narrowband subband, which will constitute a building block for our multiband joint detection procedure. Following [21], [23], to decide whether the kth subband is occupied or not, we test the following binary hypotheses:

$$\begin{aligned} \mathcal{H}_{0,k} &: R_k = V_k \\ \mathcal{H}_{1,k} &: R_k = H_k S_k + V_k, \end{aligned} \tag{5}$$

where R_k is the secondary received signal, S_k is the primary transmitted signal, and H_k is the channel gain between the primary transmitter and the secondary receiver.

For each subband k, we compute the summary statistic as the sum of received signal energy over an interval of M samples, i.e.,

$$Y_k \triangleq \sum_{n=1}^{M} |R_k(n)|^2, \quad k = 1, 2, \ldots, K \tag{6}$$

and the decision rule is chosen as

$$Y_k \underset{\mathcal{H}_{0,k}}{\overset{\mathcal{H}_{1,k}}{\gtrless}} \gamma_k, \quad k = 1, 2, \ldots, K \tag{7}$$

where γ_k is the decision threshold of subband k. For simplicity, we assume that the transmitted signal in each subband has unit power, i.e., $\mathbb{E}[|S_k|^2] = 1$; this assumption holds when primary radios adopt uniform power transmission strategies given no channel knowledge at the transmitter side. However, the development of the multiband joint detection algorithm does not rely on this assumption while only the knowledge of the received signal power and noise power is needed.

According to the central limit theorem [30], for large M, the statistics $\{Y_k\}_{k=1}^{K}$ are approximately normally distributed [5] with means

$$\mathbb{E}[Y_k] = \begin{cases} M\sigma_v^2, & \mathcal{H}_{0,k} \\ M\left(\sigma_v^2 + |H_k|^2\right), & \mathcal{H}_{1,k} \end{cases} \tag{8}$$

and variances

$$\mathrm{Var}(Y_k) = \begin{cases} 2M\sigma_v^4, & \mathcal{H}_{0,k} \\ 2M\left(\sigma_v^2 + 2|H_k|^2\right)\sigma_v^2, & \mathcal{H}_{1,k}. \end{cases} \tag{9}$$

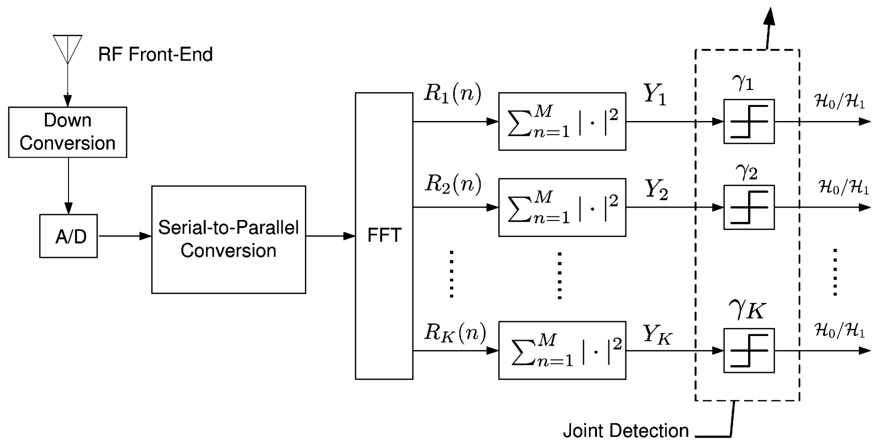

Fig. 2. Schematic representation of multiband joint detection for wideband spectrum sensing in cognitive radio systems.

Thus, we can write these approximate statistics compactly as $Y_k \sim \mathcal{N}(\mathbb{E}[Y_k], \mathrm{Var}(Y_k))$.

Using the decision rule in (7), the probabilities of false alarm and detection in the kth subband can be approximately expressed as

$$P_f^{(k)}(\gamma_k) = \Pr\left(Y_k > \gamma_k | \mathcal{H}_{0,k}\right) = Q\left(\frac{\gamma_k - M\sigma_v^2}{\sigma_v^2\sqrt{2M}}\right) \tag{10}$$

and

$$P_d^{(k)}(\gamma_k) = \Pr\left(Y_k > \gamma_k | \mathcal{H}_{1,k}\right) = Q\left[\frac{\gamma_k - M\left(\sigma_v^2 + |H_k|^2\right)}{\sigma_v\sqrt{2M\left(\sigma_v^2 + 2|H_k|^2\right)}}\right]. \tag{11}$$

Note that the SNR of such an energy detector is defined as $\mathrm{SNR} = |H_k|^2/\sigma_v^2$, which plays an important role in determining the detection performance. The choice of the threshold γ_k leads to a tradeoff between the probability of false alarm and the probability of missed detection, $P_m^{(k)}(\gamma_k) = 1 - P_d^{(k)}(\gamma_k)$. Specifically, a higher threshold will result in a smaller probability of false alarm, but a larger probability of miss, and vice versa.

The probabilities of false alarm and miss have unique implications for CR networks. Low probabilities of false alarm are necessary to maintain high spectral utilization in CR systems, since a false alarm would prevent the unused spectral segments from being accessed by secondary users. On the other hand, the probability of missed detection measures the interference of secondary users to the primary users, which should be limited in opportunistic spectrum access. These implications are based on a typical assumption that if primary signals are detected, the secondary users will not use the corresponding channel, and if no primary signals are detected, then the corresponding frequency band will be used by secondary users.

III. Multiband Joint Detection

In this section, we present the multiband joint detection framework for wideband spectrum sensing [31], as illustrated in Fig. 2. The design objective is to find the optimal threshold vector $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_K]^T$ so that the cognitive radio system can make efficient use of the unused spectral segments without causing harmful interference to the primary users. For a given threshold vector $\boldsymbol{\gamma}$, the probabilities of false alarm and detection can be compactly represented as

$$\boldsymbol{P}_f(\boldsymbol{\gamma}) = \left[P_f^{(1)}(\gamma_1), P_f^{(2)}(\gamma_2), \ldots, P_f^{(K)}(\gamma_K)\right]^T \tag{12}$$

and

$$\boldsymbol{P}_d(\boldsymbol{\gamma}) = \left[P_d^{(1)}(\gamma_1), P_d^{(2)}(\gamma_2), \ldots, P_d^{(K)}(\gamma_K)\right]^T. \tag{13}$$

Similarly, the probabilities of missed detection can be written in a vector as

$$\boldsymbol{P}_m(\boldsymbol{\gamma}) = \left[P_m^{(1)}(\gamma_1), P_m^{(2)}(\gamma_2), \ldots, P_m^{(K)}(\gamma_K)\right]^T. \tag{14}$$

The vector $\boldsymbol{P}_m$ can be expressed as $\boldsymbol{P}_m(\boldsymbol{\gamma}) = \mathbf{1} - \boldsymbol{P}_d(\boldsymbol{\gamma})$, where $\mathbf{1}$ denotes the all-one vector.

Consider a CR device sensing the K narrowband subbands to take use of the unused ones for opportunistic transmission. Let r_k denote the throughput achievable over the kth subband if used by secondary users, and $\boldsymbol{r} = [r_1, r_2, \ldots, r_K]^T$. If the transmit power and the channel gains between secondary users are known, $\boldsymbol{r}$ can be estimated using the Shannon capacity formula [27]. Since $1 - P_f^{(k)}$ measures the opportunistic spectral utilization of subband k, the aggregate opportunistic throughput of the CR system can be defined as

$$R(\boldsymbol{\gamma}) = \boldsymbol{r}^T\left[\mathbf{1} - \boldsymbol{P}_f(\boldsymbol{\gamma})\right] \tag{15}$$

which is a function of the threshold vector $\boldsymbol{\gamma}$. Due to the inherent tradeoff between $P_f^{(k)}(\gamma_k)$ and $P_m^{(k)}(\gamma_k)$, maximizing the sum rate $R(\boldsymbol{\gamma})$ will result in large $\boldsymbol{P}_m(\boldsymbol{\gamma})$, hence causing harmful interference to primary users.

However, the interference to primary users should be limited in a CR network. For a wideband primary communication system, the effect of interference induced by CR devices can be characterized by a relative priority factor for each primary user transmitting over the corresponding subbands, i.e., $\boldsymbol{c} = [c_1, c_2, \ldots, c_K]^T$, where c_k indicates the cost incurred if the primary user in subband k is interfered with. In a special case where the jth primary user is equally important, we may have $\boldsymbol{c} = \mathbf{1}$. Suppose that J primary users share a portion of the K subbands and each primary user occupies a subset S_j of subbands. The aggregate interference to primary user j can be expressed as

$$\sum_{i\in S_j} c_i P_m^{(i)}(\gamma_i). \tag{16}$$

This expression models, for example, the situation arising in a multiuser orthogonal frequency division multiplexing (OFDM) system, where various primary users have different levels of priority. Alternatively, c_k can be defined as a function of the bandwidth of subband k since in some applications each particular subband does not have to occupy an equal amount of bandwidth.

To summarize, our objective is to find the optimal thresholds $\{\gamma_k\}_{k=1}^K$ for the K subbands in order to collectively maximize the aggregate opportunistic throughput subject to some interference constraints for each primary user. As such, the opportunistic rate optimization problem in the context of a multiuser primary system can be formulated as

$$\begin{aligned} \max_{\boldsymbol{\gamma}} \quad & R(\boldsymbol{\gamma}) \qquad \text{(P1)} \\ \text{s.t.} \quad & \sum_{i\in S_j} c_i P_m^{(i)}(\gamma_i) \le \varepsilon_j, \quad j=1,2,\ldots,J \\ & \boldsymbol{P}_m(\boldsymbol{\gamma}) \preceq \boldsymbol{\alpha} \qquad (17) \\ & \boldsymbol{P}_f(\boldsymbol{\gamma}) \preceq \boldsymbol{\beta} \qquad (18) \end{aligned}$$

where the constraint (17) limits the interference in each subband with $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_K]^T$, and the constraint (18) dictates that each subband should be able to achieve a minimum opportunistic spectral utilization given by $[1-\beta_1, 1-\beta_2, \ldots, 1-\beta_K]^T$. For a single-user primary system where all the subbands are used by one primary user, we have $J = 1$.

Intuitively, some factors need to be considered in the multiband joint detection. First, the subband with a higher opportunistic rate r_k should have a higher threshold γ_k (i.e., a smaller probability of false alarm) such that it can be best used by CRs. Second, the subband that carries a higher priority primary user should have a lower threshold γ_k (i.e., a smaller probability of missed detection) in order to prevent opportunistic access by secondary users. Third, a little compromise on those subbands carrying less important primary users might boost the opportunistic rate considerably. Thus, in the determination of the optimal threshold vector, it is necessary to strike a balance among the channel conditions, the opportunistic throughput, and the relative priority of each subband.

The objective and constraint functions in (P1) are generally nonconvex, making it difficult to efficiently solve for the global optimum. In most cases, suboptimal solutions or heuristics have to be used. However, we find that this seemingly nonconvex problem can be made convex by exploiting hidden convexity properties and reformulating the problem.

The fact that the Q-function is monotonically nonincreasing allows us to transform the constraints in (17) and (18) into linear constraints. Specifically, from (17), we obtain

$$1 - P_d^{(k)}(\gamma_k) \le \alpha_k \quad k=1,2,\ldots,K. \tag{19}$$

Substituting (11) into (19) gives

$$\gamma_k \le \gamma_{\max,k}, \quad k=1,2,\ldots,K \tag{20}$$

where

$$\gamma_{\max,k} \triangleq M\left(\sigma_v^2 + |H_k|^2\right) + \sigma_v \sqrt{2M\left(\sigma_v^2 + 2|H_k|^2\right)} Q^{-1}(1-\alpha_k). \tag{21}$$

Similarly, the combination of (10) and (18) leads to

$$\gamma_k \ge \gamma_{\min,k}, \quad k=1,2,\ldots,K \tag{22}$$

where

$$\gamma_{\min,k} = \sigma_v^2 \left[M + \sqrt{2M} Q^{-1}(\beta_k)\right]. \tag{23}$$

Consequently, the original problem (P1) has the following equivalent form

$$\begin{aligned} \min \quad & \sum_{k=1}^K r_k P_f^{(k)}(\gamma_k) \qquad \text{(P2)} \\ \text{s.t.} \quad & \sum_{i\in S_j} c_i P_m^{(i)}(\gamma_i) \le \varepsilon_j, \quad j=1,2,\ldots,J \qquad (24) \\ & \gamma_{\min,k} \le \gamma_k \le \gamma_{\max,k}, \quad k=1,2,\ldots,K. \qquad (25) \end{aligned}$$

Although the constraint (25) is linear, the problem is still nonconvex. However, it can be transformed into a tractable convex optimization problem in the regime of low probabilities of false alarm and miss. To establish the transformation, we need the following results.

Lemma 1: The function $P_f^{(k)}(\gamma_k)$ is convex in γ_k if $P_f^{(k)}(\gamma_k) \le 1/2$.

Proof: Refer to Appendix A. ■

Lemma 2: The function $P_m^{(k)}(\gamma_k)$ is convex in γ_k if $P_m^{(k)}(\gamma_k) \le 1/2$.

Proof: Refer to Appendix B. ■

Recall that the nonnegative weighted sum of a set of convex functions is also convex [32]. The problem (P1) then becomes a convex program if we introduce the following conditions:

$$0 < \alpha_k \le 1/2 \text{ and } 0 < \beta_k \le 1/2, \quad k=1,2,\ldots,K. \tag{26}$$

This regime of probabilities of false alarm and missed detection is of practical interest for achieving rational opportunistic throughput and interference levels in CR networks.

Under the conditions in (26), the feasible set of problem (P2) is convex because the intersection of a convex set and a set of halfspaces is also convex. The optimization problem takes the form of minimizing a convex function subject to a convex constraint, and thus a local optimum is also the global optimum. Efficient numerical search algorithms such as the interior-point method can be used to find the optimal solution [32].

Alternatively, we can formulate the multiband joint detection problem into another optimization problem that minimizes the interference from CRs to the primary communication system

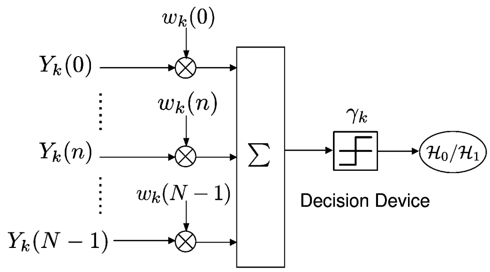

Fig. 3. Weighting cooperation for spectrum sensing in the kth subband.

subject to some constraints on the aggregate opportunistic throughput, i.e.,

$$\begin{aligned} \min_{\boldsymbol{\gamma}} \quad & \boldsymbol{c}^T \boldsymbol{P}_m(\boldsymbol{\gamma}) \qquad \text{(P3)} \\ \text{s.t.} \quad & \boldsymbol{r}^T [\mathbf{1} - \boldsymbol{P}_f(\boldsymbol{\gamma})] \geq \delta \\ & \boldsymbol{P}_m(\boldsymbol{\gamma}) \preceq \boldsymbol{\alpha} \\ & \boldsymbol{P}_f(\boldsymbol{\gamma}) \preceq \boldsymbol{\beta} \end{aligned}$$

where δ is the minimum required aggregate opportunistic throughput. Like (P1), this problem can be transformed into a convex optimization problem by enforcing the conditions in (26). The result will be illustrated numerically later in Section V.

IV. Spatial–Spectral Joint Detection

The detection performance of spectrum sensing is usually compromised by destructive channel conditions between the target-under-detection and the CRs, since it is hard to distinguish between a white spectrum and a weak signal attenuated by deep fading. In such scenarios, a network of cooperative CR devices, which experience different channel conditions from the target, would have a better chance of detecting the primary signal if they combine their sensing results. In this section, we present a cooperation framework for wideband spectrum sensing, within which CRs can exploit spatial diversity by exchanging local sensing results in order for the secondary network to obtain a more accurate estimate of the unused frequency bands [33].

Suppose that N spatially distributed CRs collaboratively sense a wide frequency band, aiming to find unused spectral segments for opportunistic communication. By combining the summary statistics from individual CRs, a fusion center, which could be one of the CRs, makes the final decision on the presence or absence of primary signals in each of the K subbands. We propose a linear weighting fusion scheme as illustrated in Fig. 3. It is assumed that there is a control channel, through which the summary statistics of individual secondary users are transmitted to the fusion center.

Let $Y_k(n)$ denote the summary statistic of the nth secondary user in the kth subband. For each subband, the statistics from individual secondary users can be written in a vector as $\boldsymbol{Y}_k = [Y_k(1), Y_k(2), \ldots, Y_k(N)]^T$. The statistics across the K subbands can be compactly represented in matrix form as follows:

$$\boldsymbol{Y} = [\boldsymbol{Y}_1 \quad \boldsymbol{Y}_2 \quad \ldots \quad \boldsymbol{Y}_K] = \begin{bmatrix} Y_1(1) & Y_2(1) & \ldots & Y_K(1) \\ Y_1(2) & Y_2(2) & \ldots & Y_K(2) \\ \vdots & \vdots & \ddots & \vdots \\ Y_1(N) & Y_2(N) & \ldots & Y_K(N) \end{bmatrix}. \tag{27}$$

To exploit the spatial diversity, we linearly combine the summary statistics from spatially distributed CRs in each subband k to obtain a global test statistic

$$z_k = \sum_{n=1}^{N} w_k(n) Y_k(n) = \boldsymbol{w}_k^T \boldsymbol{Y}_k \tag{28}$$

where $\boldsymbol{w}_k = [w_k(1), w_k(2), \ldots, w_k(N)]^T$ are the combining coefficients for subband k, which can be compactly written as

$$\boldsymbol{W} = [\boldsymbol{w}_1 \quad \boldsymbol{w}_2 \quad \ldots \quad \boldsymbol{w}_K] = \begin{bmatrix} w_1(1) & w_2(1) & \ldots & w_K(1) \\ w_1(2) & w_2(2) & \ldots & w_K(2) \\ \vdots & \vdots & \ddots & \vdots \\ w_1(N) & w_2(N) & \ldots & w_K(N) \end{bmatrix}. \tag{29}$$

Note that $w_k(n) \geq 0$, for all k, n.

Since the elements in $\boldsymbol{Y}_k$ are normally distributed, the test statistics $z_k, k = 1, 2, \ldots, K$, are also normally distributed with means

$$\mathbb{E}[z_k] = \begin{cases} M\sigma_v^2 \boldsymbol{w}_k^T \mathbf{1}, & \mathcal{H}_{0,k} \\ M\boldsymbol{w}_k^T \left(\sigma_v^2 \mathbf{1} + \boldsymbol{G}_k\right), & \mathcal{H}_{1,k} \end{cases} \tag{30}$$

and variances

$$\operatorname{Var}(z_k) = \begin{cases} 2M\sigma_v^4 \boldsymbol{w}_k^T \boldsymbol{w}_k, & \mathcal{H}_{0,k} \\ 2M\sigma_v^2 \boldsymbol{w}_k^T \boldsymbol{\Sigma}_k \boldsymbol{w}_k, & \mathcal{H}_{1,k} \end{cases} \tag{31}$$

where

$$\boldsymbol{\Sigma}_k = \mathbb{E}\left[\boldsymbol{Y}_k \boldsymbol{Y}_k^T\right] = \sigma_v^2 \boldsymbol{I} + 2\operatorname{diag}(\boldsymbol{G}_k) \tag{32}$$

is a diagonal matrix assuming that the elements of $\boldsymbol{Y}_k$ are independent and

$$\boldsymbol{G}_k = \left[|H_k(1)|^2, |H_k(2)|^2, \ldots, |H_k(N)|^2\right]^T$$

are the squared magnitudes of the channel gains between the primary transmitter and secondary receivers on each subband. Please note that $\boldsymbol{\Sigma}_k$ becomes a nondiagonal matrix if the elements of $\boldsymbol{Y}_k$ are not independent, but the following derivation is still valid since $\boldsymbol{\Sigma}_k$ is a positive semidefinite matrix.

In order to decide the presence or absence of the primary signal in subband k, we use the following binary test:

$$z_k \underset{\mathcal{H}_{0,k}}{\overset{\mathcal{H}_{1,k}}{\gtrless}} \gamma_k, \quad k = 1, 2, \ldots, K. \tag{33}$$

Accordingly, the detection performance in terms of the probabilities of false alarm and detection are given approximately by

$$P_f^{(k)}(\boldsymbol{w}_k, \gamma_k) = Q\left(\frac{\gamma_k - M\sigma_v^2 \boldsymbol{w}_k^T \mathbf{1}}{\sigma_v^2 \sqrt{2M\boldsymbol{w}_k^T \boldsymbol{w}_k}}\right) \tag{34}$$

and

$$P_d^{(k)}(\boldsymbol{w}_k, \gamma_k) = Q\left[\frac{\gamma_k - M\boldsymbol{w}_k^T\left(\sigma_v^2 \mathbf{1} + \boldsymbol{G}_k\right)}{\sigma_v \sqrt{2M\boldsymbol{w}_k^T \boldsymbol{\Sigma}_k \boldsymbol{w}_k}}\right]. \tag{35}$$

In the design of an efficient distributed cooperative sensing system, the goal is to maximize the system performance measure of interest by controlling the weight coefficient matrix $\boldsymbol{W}$ and the threshold vector $\boldsymbol{\gamma}$. Just as we did in the previous section, we would like to maximize the opportunistic rate while satisfying some constraints on the interference to the primary communication system. Note that the aggregate opportunistic throughput of the K subbands is now a function of both the threshold vector $\boldsymbol{\gamma}$ and the weight coefficient matrix $\boldsymbol{W}$, i.e.,

$$R(\boldsymbol{W}, \boldsymbol{\gamma}) = \boldsymbol{r}^T\left[\mathbf{1} - \boldsymbol{P}_f(\boldsymbol{W}, \boldsymbol{\gamma})\right]. \tag{36}$$

Consequently, the spatial–spectral joint detection problem is formulated as

$$\max_{\boldsymbol{W}, \boldsymbol{\gamma}} \quad R(\boldsymbol{W}, \boldsymbol{\gamma}) \qquad \text{(P4)}$$

$$\text{s.t.} \quad \boldsymbol{c}^T \boldsymbol{P}_m(\boldsymbol{W}, \boldsymbol{\gamma}) \le \varepsilon \tag{37}$$

$$\boldsymbol{P}_m(\boldsymbol{W}, \boldsymbol{\gamma}) \preceq \boldsymbol{\alpha} \tag{38}$$

$$\boldsymbol{P}_f(\boldsymbol{W}, \boldsymbol{\gamma}) \preceq \boldsymbol{\beta}. \tag{39}$$

Note that the formulation in (P4) is in the context of a single-user primary system and it can be easily extended to the case of a multiuser primary system as (P1) does. Finding the exact optimal solution for the above problem is difficult, since for any subband, the probabilities of false alarm and miss are neither convex nor concave functions of the weight coefficients $\boldsymbol{w}_k$ and the test threshold γ_k according to (34) and (35).

In the following, we will develop two efficient methods for solving for the weight coefficients $\boldsymbol{W}$ and the thresholds $\boldsymbol{\gamma}$, which lead to near-optimal solutions for (P4). For consistency, we still assume the practical conditions in (26) unless explicitly stated otherwise.

A. Joint Optimization

To jointly optimize $\boldsymbol{W}$ and $\boldsymbol{\gamma}$, we show that (P4) can be reformulated into an equivalent form with a convex feasible set and an objective function lower bounded by a concave function. Through maximizing the lower bound of the objective function, we are able to obtain a good approximation to the optimal solution of the original problem.

First, we show how to transform the nonconvex constraints in (38) and (39) into convex constraints by exploiting the monotonicity of the Q-function. Substituting (34) into the constraint (39), we have

$$Q^{-1}(\beta_k)\sqrt{2M\boldsymbol{w}_k^T \boldsymbol{w}_k} \le \frac{\gamma_k}{\sigma_v^2} - M\boldsymbol{w}_k^T \mathbf{1}, \quad k = 1, 2, \ldots, K \tag{40}$$

where $Q^{-1}(\beta_k) \ge 0$ since $\beta_k \le 1/2$. From (35), the constraint (38) can be expressed as

$$\sqrt{2M\boldsymbol{w}_k^T \boldsymbol{\Sigma}_k \boldsymbol{w}_k} \le \frac{\gamma_k - M\boldsymbol{w}_k^T\left(\sigma_v^2 \mathbf{1} + \boldsymbol{G}_k\right)}{\sigma_v Q^{-1}(1 - \alpha_k)} \tag{41}$$

for $k = 1, 2, \ldots, K$, since $\alpha_k \le 1/2$ and $Q^{-1}(1 - \alpha_k) \le 0$. It is implied by (40) and (41) that

$$M\sigma_v^2 \boldsymbol{w}_k^T \mathbf{1} \le \gamma_k \le M\boldsymbol{w}_k^T\left(\sigma_v^2 \mathbf{1} + \boldsymbol{G}_k\right). \tag{42}$$

Observing that the left-hand side on the constraint (40) is a convex function and the right-hand side is a linear function, (40) defines a convex set for $\{\gamma_k, \boldsymbol{w}_k\}$. Similarly, (41) is also a convex constraint.

Then, we reformulate problem (P4) by introducing a new variable

$$\mu_k = \sigma_v \sqrt{2M\boldsymbol{w}_k^T \boldsymbol{\Sigma}_k \boldsymbol{w}_k}. \tag{43}$$

Define $\gamma_k' = \gamma_k / \mu_k$ and $\boldsymbol{w}_k' = \boldsymbol{w}_k / \mu_k$. The constraints (40) and (41) can be written as

$$Q^{-1}(\beta_k)\sqrt{2M{\boldsymbol{w}_k'}^T \boldsymbol{w}_k'} \le \frac{\gamma_k'}{\sigma_v^2} - M\mathbf{1}^T \boldsymbol{w}_k' \tag{44}$$

and

$$\gamma_k' - M\left(\sigma_v^2 \mathbf{1} + \boldsymbol{G}_k\right)^T \boldsymbol{w}_k' \le \sigma_v Q^{-1}(1 - \alpha_k). \tag{45}$$

Note that (45) is actually a linear constraint.

The constraint (37) now becomes

$$\begin{aligned} &\boldsymbol{c}^T \boldsymbol{P}_m(\boldsymbol{W}, \boldsymbol{\gamma}) = \boldsymbol{c}^T\left[\mathbf{1} - \boldsymbol{P}_d(\boldsymbol{W}, \boldsymbol{\gamma})\right] \le \varepsilon \\ \Longleftrightarrow\ &\mathbf{1}^T \boldsymbol{c} - \sum_{k=1}^{K} c_k Q\left[\gamma_k' - M\left(\sigma_v^2 \mathbf{1} + \boldsymbol{G}_k\right)^T \boldsymbol{w}_k'\right] \le \varepsilon \end{aligned} \tag{46}$$

which can be shown to be convex through the following result.

Lemma 3: If $\gamma_k' \le M(\sigma_v^2 \mathbf{1} + \boldsymbol{G}_k)^T \boldsymbol{w}_k'$, the function $Q[\gamma_k' - M(\sigma_v^2 \mathbf{1} + \boldsymbol{G}_k)^T \boldsymbol{w}_k']$ is concave in $\{\gamma_k', \boldsymbol{w}_k'\}$.

Proof: Refer to Appendix C. ■

By changing the variables $\boldsymbol{W}' = [\boldsymbol{w}_1', \boldsymbol{w}_2', \ldots, \boldsymbol{w}_K']^T$ and $\boldsymbol{\gamma}' = [\gamma_1', \gamma_2', \ldots, \gamma_K']^T$, we can write the objective function in (P4) as

$$\begin{aligned} R(\boldsymbol{W}, \boldsymbol{\gamma}) &= \boldsymbol{r}^T\left[\mathbf{1} - \boldsymbol{P}_f(\boldsymbol{W}, \boldsymbol{\gamma})\right] \\ &= \sum_{k=1}^{K} r_k\left[1 - Q\left(\frac{\gamma_k - M\sigma_v^2 \boldsymbol{w}_k^T \mathbf{1}}{\sigma_v^2 \sqrt{2M\boldsymbol{w}_k^T \boldsymbol{w}_k}}\right)\right] \\ &= \mathbf{1}^T \boldsymbol{r} - \sum_{k=1}^{K} r_k Q\left[\left(\frac{\gamma_k'}{\sigma_v^2} - M\mathbf{1}^T \boldsymbol{w}_k'\right)\right. \\ &\qquad \left. \times \sqrt{\sigma_v^2 + \frac{2{\boldsymbol{w}_k'}^T \operatorname{diag}(\boldsymbol{G}_k) \boldsymbol{w}_k'}{{\boldsymbol{w}_k'}^T \boldsymbol{w}_k'}}\right]. \end{aligned} \tag{47}$$

From the Rayleigh–Ritz theorem [34], we have

$$\min_{1\le n\le N} |H_k(n)|^2 \le \frac{{\boldsymbol{w}'_k}^T \mathrm{diag}(\boldsymbol{G}_k)\boldsymbol{w}'_k}{{\boldsymbol{w}'_k}^T \boldsymbol{w}'_k} \le \max_{1\le n\le N} |H_k(n)|^2 . \tag{48}$$

Now define a new function

$$g_k\left(\gamma'_k, \boldsymbol{w}'_k\right) \triangleq Q\left[\left(\frac{\gamma'_k}{\sigma_v^2} - M\mathbf{1}^T\boldsymbol{w}'_k\right) \times \sqrt{\sigma_v^2 + 2\min_{1\le n\le N}|H_k(n)|^2}\right] \tag{49}$$

for $k = 1, 2, \ldots, K$, the convexity of which is established through the following result.

Lemma 4: If $\gamma'_k \ge \sigma_v^2 M\mathbf{1}^T\boldsymbol{w}'_k$, the function $g_k(\gamma'_k, \boldsymbol{w}'_k)$ is convex in $\{\gamma'_k, \boldsymbol{w}'_k\}$.

Proof: The proof is similar to that of Lemma 3, and thus is omitted. ■

Consequently, the aggregate opportunistic rate can be lower bounded as

$$R(\boldsymbol{W}, \boldsymbol{\gamma}) \ge \sum_{k=1}^{N} r_k\left[1 - g_k\left(\gamma'_k, \boldsymbol{w}'_k\right)\right]. \tag{50}$$

An efficient suboptimal method to solve (P4) is to maximize the lower bound of its objective function, i.e.,

$$\begin{aligned}
\max_{\boldsymbol{W}', \boldsymbol{\gamma}'} \quad & \sum_{k=1}^{K} r_k\left[1 - g_k\left(\gamma'_k, \boldsymbol{w}'_k\right)\right] \qquad \text{(P5)}\\
\text{s.t.} \quad & -\sum_{k=1}^{K} c_k Q\left[\gamma'_k - M\left(\sigma_v^2\mathbf{1} + \boldsymbol{G}_k\right)^T\boldsymbol{w}'_k\right] \le \varepsilon - \mathbf{1}^T\boldsymbol{c}\\
& Q^{-1}(\beta_k)\sqrt{2M{\boldsymbol{w}'_k}^T\boldsymbol{w}'_k} \le \frac{\gamma'_k}{\sigma_v^2} - M\mathbf{1}^T\boldsymbol{w}'_k,\\
& k = 1, 2, \ldots, K\\
& \gamma'_k - M\left(\sigma_v^2\mathbf{1} + \boldsymbol{G}_k\right)^T\boldsymbol{w}'_k \le \sigma_v Q^{-1}(1-\alpha_k),\\
& k = 1, 2, \ldots, K.
\end{aligned} \tag{51}$$

Implied by the practical conditions in (26), this problem is a convex optimization problem and can be efficiently solved.

B. Sequential Optimization

Here, we present another heuristic approach that divides the optimization of the original problem (P4) into two stages. In the first stage, referred to as *spatial optimization*, we choose the weight coefficients $\boldsymbol{W}$ in order to maximize a performance measure for signal detection. In the second phase, called *spectral optimization*, we fix the values of $\boldsymbol{W}$ obtained from spatial optimization and optimize the thresholds $\boldsymbol{\gamma}$ across all the subbands.

1) Spatial Optimization: A good measure for evaluating the detection performance, called the *modified deflection coefficient* [21], [23], is defined as

$$d_k^2(\boldsymbol{w}_k) = \frac{\left(\mathbb{E}[z_k|\mathcal{H}_{1,k}] - \mathbb{E}[z_k|\mathcal{H}_{0,k}]\right)^2}{\mathrm{Var}(z_k|\mathcal{H}_{1,k})} = \frac{M\left(\boldsymbol{w}_k^T\boldsymbol{G}_k\right)^2}{2\sigma_v^2\boldsymbol{w}_k^T\boldsymbol{\Sigma}_k\boldsymbol{w}_k}. \tag{52}$$

The quantity $d_k^2(\boldsymbol{w}_k)$ can be interpreted as a signal-to-noise ratio. For any given probability of false alarm, a larger value of $d_k^2(\boldsymbol{w}_k)$ will result in a larger probability of detection if z_k is normally distributed under both hypotheses $\mathcal{H}_{0,k}$ and $\mathcal{H}_{1,k}$.

In the spatial optimization, we would like to choose the weight coefficients $\boldsymbol{w}_k$ in order to achieve the maximum modified deflection coefficient for each subband. Note that the feasible set for maximizing $d_k^2(\boldsymbol{w}_k)$ is unbounded. To obtain a unique solution, we confine the weight vector to be on the unit-norm ball and pose

$$\begin{aligned}
\max_{\boldsymbol{w}_k} \quad & d_k^2(\boldsymbol{w}_k) \qquad \text{(P6)}\\
\text{s.t.} \quad & \|\boldsymbol{w}_k\|_2 = 1
\end{aligned}$$

for $k = 1, 2, \ldots, K$.

This problem can be solved as follows. First, apply the linear transform

$$\boldsymbol{w}'_k = \boldsymbol{\Sigma}_k^{1/2}\boldsymbol{w}_k \tag{53}$$

where $\boldsymbol{\Sigma}_k^{1/2}$ is the square root of the matrix $\boldsymbol{\Sigma}$, i.e.,

$$\boldsymbol{\Sigma}_k^{1/2} = \mathrm{diag}\begin{bmatrix} \sqrt{\sigma_v^2 + 2|H_k(1)|^2} \\ \sqrt{\sigma_v^2 + 2|H_k(2)|^2} \\ \vdots \\ \sqrt{\sigma_v^2 + 2|H_k(N)|^2} \end{bmatrix}. \tag{54}$$

In addition, we know that $d_k^2(\boldsymbol{w}_k)$ is upper bounded by

$$\begin{aligned}
d_k^2(\boldsymbol{w}_k) &= \frac{M{\boldsymbol{w}'_k}^T\boldsymbol{\Sigma}_k^{-T/2}\boldsymbol{G}_k\boldsymbol{G}_k^T\boldsymbol{\Sigma}_k^{-1/2}\boldsymbol{w}'_k}{2\sigma_v^2{\boldsymbol{w}'_k}^T\boldsymbol{w}'_k}\\
&\overset{(a)}{\le} \frac{M}{2\sigma_v^2}\lambda_{\max}\left(\boldsymbol{\Sigma}_k^{-T/2}\boldsymbol{G}_k\boldsymbol{G}_k^T\boldsymbol{\Sigma}_k^{-1/2}\right)
\end{aligned} \tag{55}$$

where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a matrix and (a) follows from the Rayleigh–Ritz theorem [34]. Note that the equality in (a) is achieved if

$$\boldsymbol{w}'_k = \boldsymbol{\Sigma}_k^{-T/2}\boldsymbol{G}_k \tag{56}$$

which is the eigenvector corresponding to the maximal eigenvalue of the positive semidefinite matrix $\boldsymbol{\Sigma}_k^{-T/2}\boldsymbol{G}_k\boldsymbol{G}_k^T\boldsymbol{\Sigma}_k^{-1/2}$. Therefore, the optimal solution of (P6) is given by

$$\boldsymbol{w}_k^o = \frac{\boldsymbol{\Sigma}_k^{-1}\boldsymbol{G}_k}{\left\|\boldsymbol{\Sigma}_k^{-1}\boldsymbol{G}_k\right\|_2}. \tag{57}$$

2) Spectral Optimization: Substituting the weight vectors $\{\boldsymbol{w}_k^o\}_{k=1}^K$ obtained in the first subproblem (P6) into (34) and (35), the probabilities of false alarm and detection become functions of only the threshold γ_k. Following the procedure in (P1), we can solve the following subproblem for the threshold vector $\boldsymbol{\gamma}$:

$$\begin{aligned}
\min_{\boldsymbol{\gamma}} \quad & \sum_{k=1}^{K} r_k P_f^{(k)}\left(\boldsymbol{w}_k^o, \gamma_k\right) \qquad \text{(P7)}\\
\text{s.t.} \quad & \sum_{k=1}^{K} c_k P_m^{(k)}\left(\boldsymbol{w}_k^o, \gamma_k\right) \le \varepsilon && (58)\\
& \gamma'_{\min,k} \le \gamma_k \le \gamma'_{\max,k}, k = 1, 2, \ldots, K && (59)
\end{aligned}$$

TABLE I
POWER DELAY PROFILE IN EXAMPLE 1

Delays (ns)	0	20	50	120
Avg. Power Gain (dB)	-2	-8	-19	-21

where

$$\gamma'_{\min,k} = M\sigma_v^2 \mathbf{1}^T \boldsymbol{w}_k^o + \sigma_v^2 Q^{-1}(\beta_k)\sqrt{2M\boldsymbol{w}_k^{oT}\boldsymbol{w}_k^o} \tag{60}$$

and

$$\gamma'_{\max,k} = M\left(\sigma_v^2\mathbf{1} + \boldsymbol{G}_k\right)^T \boldsymbol{w}_k^o + \sigma_v Q^{-1}(1-\alpha_k)\sqrt{2M\boldsymbol{w}_k^{oT}\boldsymbol{\Sigma}_k\boldsymbol{w}_k^o}. \tag{61}$$

As before, the problem is convex and can be solved efficiently.

As an alternative example, the spatial–spectral joint detection problem can be reformulated to minimize the interference subject to some constraint on the aggregate opportunistic throughput δ, i.e.,

$$\begin{aligned} \min_{\boldsymbol{W},\boldsymbol{\gamma}} \quad & \boldsymbol{c}^T \boldsymbol{P}_m(\boldsymbol{W},\boldsymbol{\gamma}) \qquad \text{(P8)} \\ \text{s.t.} \quad & R(\boldsymbol{W},\boldsymbol{\gamma}) \geq \delta \\ & \boldsymbol{P}_m(\boldsymbol{W},\boldsymbol{\gamma}) \preceq \boldsymbol{\alpha} \\ & \boldsymbol{P}_f(\boldsymbol{W},\boldsymbol{\gamma}) \preceq \boldsymbol{\beta}. \end{aligned}$$

Near-optimal solutions can be obtained using the same techniques as in solving (P4).

V. SIMULATION RESULTS

In this section, we numerically evaluate the proposed spectrum sensing schemes. Consider a 48-MHz primary system where the wideband channel is equally divided into eight subbands. For each subband k $(1 \leq k \leq 8)$, we assume an achievable throughput rate r_k if used by CRs and a cost coefficient c_k indicating the penalty if the primary signal is interfered with by secondary users. It is expected that the opportunistic spectrum utilization is at least 50%, i.e., $\beta_k = 0.5$, and the probability that the primary user is interfered with is at most $\alpha_k = 0.2$. For simplicity it is assumed that the noise power level is $\sigma_v^2 = 1$, and the length of each detection interval is $M = 100$.

A. Example 1: Multiband Joint Detection for Individual CRs

This example studies multiband joint detection in a single CR. The proposed spectrum sensing algorithms are examined by comparing with an approach that searches for a uniform threshold to maximize the aggregate opportunistic throughput. We consider a power delay profile, as given in Table I, that specifies the frequency selective channel between the primary transmitter and the secondary receiver. The channel gain, opportunistic rate, and interference penalty on each subband are given in Table II.

We would like to maximize the aggregate opportunistic throughput over the eight subbands subject to the constraints on the interference to the primary users, as formulated in (P1). Fig. 4 plots the maximum aggregate opportunistic rates against the aggregate interference to the primary communication system. It can be seen that the multiband joint detection algorithm with optimized thresholds can achieve a much higher opportunistic rate than that achieved by the uniform threshold method. That is, the proposed multiband joint detection makes better use of the wide frequency band by balancing the conflict between improving spectral utilization and reducing the interference. In addition, it is observed that the aggregate opportunistic rate increases as we relax the constraint on the aggregate interference ε.

TABLE II
OTHER PARAMETERS USED IN EXAMPLE 1

$\boldsymbol{G}$	.61	.49	.35	.25	.23	.35	.52	.59
$\boldsymbol{r}$ (kbps)	612	524	623	139	451	409	909	401
$\boldsymbol{c}$	1.91	8.17	4.23	3.86	7.16	6.05	0.82	1.30

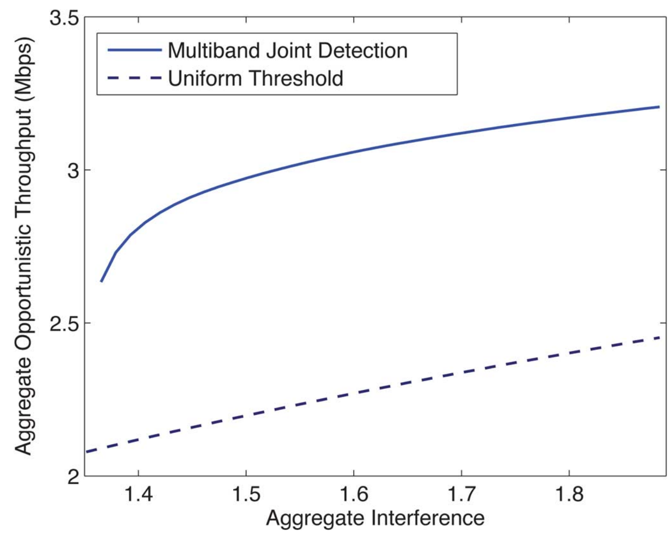

Fig. 4. The aggregate opportunistic throughput versus the constraint on the aggregate interference to the primary communication system.

An alternative example is depicted in Fig. 5, showing the numerical results of minimizing the aggregate interference subject to the constraints on the aggregate opportunistic throughput as formulated in (P3). It can be observed that the multiband joint detection strategy outperforms the one using uniform thresholds in terms of the induced interference to the primary users for any given opportunistic throughput target. For illustration purposes, the optimized thresholds and the associated probabilities of missed detection and false alarm are given in Fig. 6 for (P1) and (P3).

B. Example 2: Cooperative Wideband Sensing among CRs

In this example, we consider the case in which two CRs cooperatively sense the eight subbands by exchanging the summary statistics of their sensed data. We compare the two proposed spatial–spectral joint detection schemes with the multiband joint detection algorithms performed individually without cooperation. The two CRs experience different channel models as specified in Table III. Other parameters are given in Table IV.

TABLE III
POWER DELAY PROFILES IN EXAMPLE 2

CR1 – Delays (ns)	0	30	80	120		
CR1 – Avg. Power Gain (dB)	-2.6	-10.9	-19.5	-22.6		
CR2 – Delays (ns)	0	20	70	110	160	210
CR2 – Avg. Power Gain (dB)	-2.4	-8.3	-16.4	-22.1	-25	-20.2

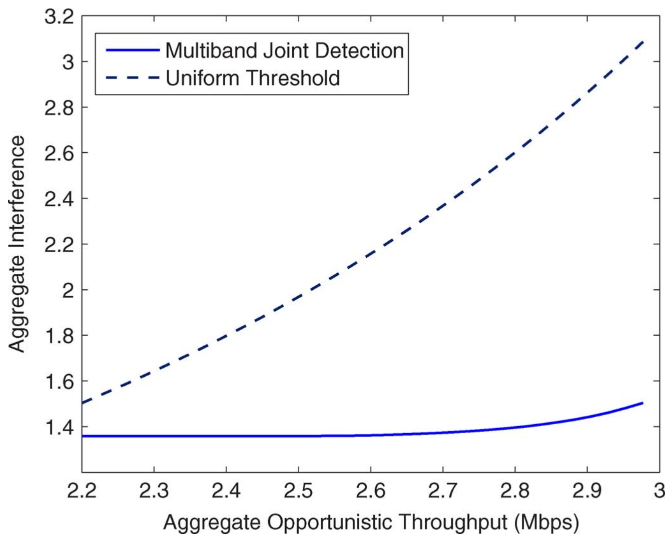

Fig. 5. The aggregate interference to the primary communication system versus the constraint on the aggregate opportunistic throughput.

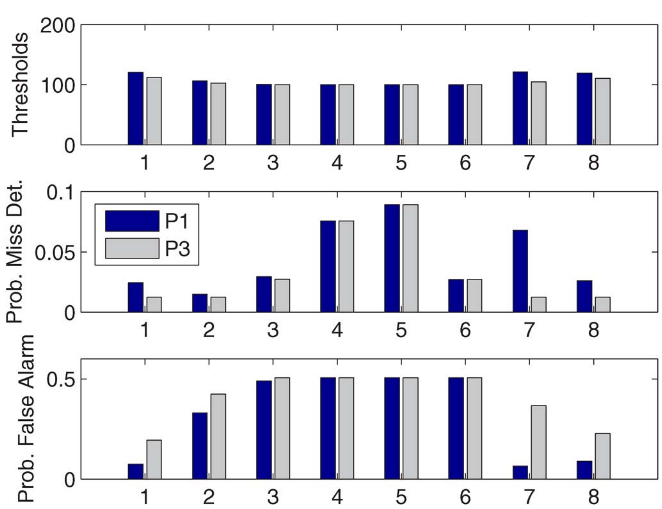

Fig. 6. The optimized thresholds and the associated probabilities of missed detection and false alarm on individual subbands: (P1) $\varepsilon = 1.474$ and (P3) $\delta = 2.354$ Mbps.

The numerical results of solving (P4), which maximizes the aggregate opportunistic throughput subject to the constraints on the aggregate interference, are illustrated in Fig. 7. It is observed that the spectrum sensing algorithms with cooperation result in higher opportunistic rates than the sensing algorithms without cooperation. In Fig. 8, we examine the problem (P8), which minimizes the aggregate interference under the constraints on

TABLE IV
OTHER PARAMETERS USED IN EXAMPLE 2

$\boldsymbol{G}(1)$	.38	.29	.23	.26	.35	.39	.33	.27
$\boldsymbol{G}(2)$	.51	.40	.31	.19	.21	.27	.43	.50
$\boldsymbol{r}$ (kbps)	806	755	356	327	68	720	15	972
$\boldsymbol{c}$	5.95	3.91	0.71	4.21	0.44	2.03	0.58	2.85

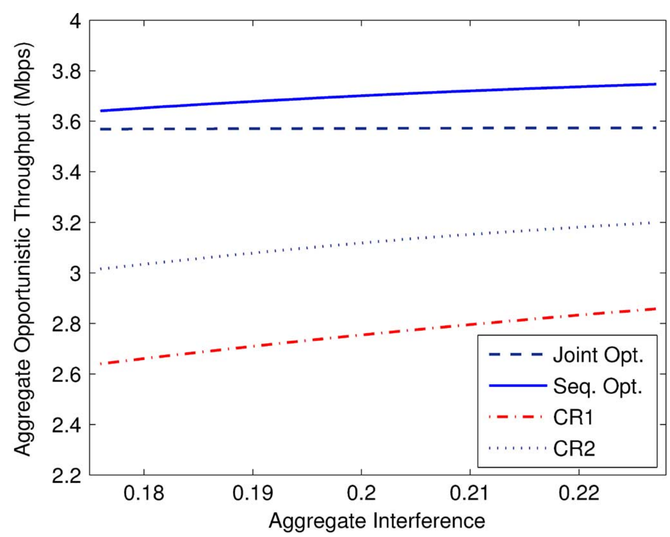

Fig. 7. The aggregate opportunistic throughput versus the constraint on the aggregate interference to the primary communication system.

the minimum aggregate opportunistic throughput. Similarly, the algorithms with cooperation perform much better than those without cooperation, and the joint optimization outperforms the sequential optimization.

Generally speaking, these numerical results show that multiband joint detection can improve the spectral efficiency considerably by making better use of the spectral diversity, and the spatial–spectral joint detection strategies can further enhance the system performance by exploiting the spatial diversity.

VI. CONCLUSION

In this paper, we have proposed multiband joint detection for wideband spectrum sensing in CR networks. The basic strategy is to take into account the detection of primary users jointly across a bank of narrowband subbands rather than considering only one single band at a time. We have formulated the joint detection problem into a class of optimization problems to improve

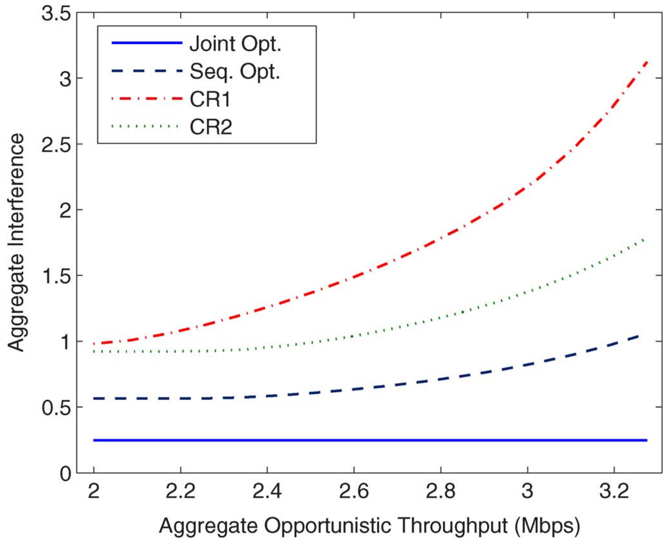

Fig. 8. The aggregate interference to the primary communication system versus the constraint on the aggregate opportunistic throughput.

the spectral efficiency and reduce the interference. By exploiting the hidden convexity in the seemingly nonconvex problem formulations, we have obtained the optimal solution under practical conditions. In addition, we have presented a spatial–spectral joint detection strategy for cooperative wideband spectrum sensing, in which spatially distributed CRs can collaborate with each other to improve the sensing reliability by exchanging the individual sensing statistics. We have provided efficient suboptimal solutions for the problems that jointly optimize the cooperation among spatially distributed secondary users and the decision thresholds over multiple bands. The proposed spectrum sensing algorithms have been examined numerically and have been shown to perform well.

Appendix A
Proof of Lemma 1

Proof: Taking the second derivative of $P_f^{(k)}(\gamma_k)$ from (10) gives

$$\frac{d^2 P_f^{(k)}(\gamma_k)}{d\gamma_k^2} = \frac{-1}{2\sigma_v^2\sqrt{2\pi M}}\frac{d}{d\gamma_k}\exp\left[-\frac{\left(\gamma_k - M\sigma_v^2\right)^2}{4M\sigma_v^4}\right] = \frac{\gamma_k - M\sigma_v^2}{8M\sigma_v^6\sqrt{2\pi M}}\exp\left[-\frac{\left(\gamma_k - M\sigma_v^2\right)^2}{4M\sigma_v^4}\right]. \quad (62)$$

Since $P_f^{(k)}(\gamma_k) \leq 1/2$, we have $\gamma_k \geq M\sigma_v^2$. Consequently, the second derivative of $P_f^{(k)}(\gamma_k)$ is larger than or equal to zero, which implies that $P_f^{(k)}(\gamma_k)$ is convex in γ_k. ■

Appendix B
Proof of Lemma 2

Proof: This can be proved using a similar technique to that used in proving Lemma 1. By taking the second derivative of (11), we can show that $P_d^{(k)}(\gamma_k)$ is concave, and hence that $P_m^{(k)}(\gamma_k) = 1 - P_d^{(k)}(\gamma_k)$ is a convex function. ■

Appendix C
Proof of Lemma 3

Proof: To prove the lemma, we take the Hessian of $Q[\gamma_k' - M(\sigma_v^2\mathbf{1} + \boldsymbol{G}_k)^T\boldsymbol{w}_k']$ over $[\gamma_k', \boldsymbol{w}_k']$ and obtain

$$\nabla^2 Q\left[\gamma_k' - M\left(\sigma_v^2\mathbf{1} + \boldsymbol{G}_k\right)^T\boldsymbol{w}_k'\right] = \left[\gamma_k' - M\left(\sigma_v^2\mathbf{1} + \boldsymbol{G}_k\right)^T\boldsymbol{w}_k'\right]\frac{e^{-\left[\gamma_k' - M\left(\sigma_v^2\mathbf{1}+\boldsymbol{G}_k\right)^T\boldsymbol{w}_k'\right]^2/2}}{\sqrt{2\pi}} \times \begin{bmatrix} 1 \\ -M\left(\sigma_v^2 + |H_k(1)|^2\right) \\ \vdots \\ -M\left(\sigma_v^2 + |H_k(N)|^2\right)\end{bmatrix} \times \begin{bmatrix} 1 \\ -M\left(\sigma_v^2 + |H_k(1)|^2\right) \\ \vdots \\ -M\left(\sigma_v^2 + |H_k(N)|^2\right)\end{bmatrix}^T.$$

Since $\gamma_k' \leq M(\sigma_v^2\mathbf{1} + \boldsymbol{G}_k)^T\boldsymbol{w}_k'$, the $N+1$ by $N+1$ matrix $\nabla^2 Q[\gamma_k' - M(\sigma_v^2\mathbf{1}+\boldsymbol{G}_k)^T\boldsymbol{w}_k']$ is a negative semidefinite matrix, denoted by $\nabla^2 Q[\gamma_k' - M(\sigma_v^2\mathbf{1} + \boldsymbol{G}_k)^T\boldsymbol{w}_k'] \preceq 0$. Consequently, $Q[\gamma_k' - M(\sigma_v^2\mathbf{1} + \boldsymbol{G}_k)^T\boldsymbol{w}_k']$ is concave in $\{\gamma_k', \boldsymbol{w}_k'\}$. ■

Acknowledgment

The authors would like to thank S. Shellhammer at Qualcomm for his helpful discussions.

References

[1] J. Mitola, III and G. Q. Maguire, "Cognitive radio: Making software radios more personal," *IEEE PersonalCommun.*, vol. 6, pp. 13–18, 1999.
[2] Federal Communications Commission, Et docket no. 03-322, Dec. 2003.
[3] The XG Architectural Framework Request for Comments v1.0, Defense Advance Research Projects Agency (DARPA), 2003 [Online]. Available: http://www.darpa.mil/ato/programs/xg/rfc_af.pdf
[4] D. Cabric, S. M. Mishra, and R. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Proc. 38th Asilomar Conf. Signals, Systems Computers*, Pacific Grove, CA, Nov. 2004, pp. 772–776.
[5] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Upper Saddle River, NJ: Prentice-Hall, 1998.
[6] S. J. Shellhammer, S. Shankar N, R. Tandra, and J. Tomcik, "Performance of power detector sensors of DTV signals in IEEE 802.22 WRANs," presented at the ACM 1st Int. Workshop Technology Policy for Accessing Spectrum (TAPAS), Boston, MA, Aug. 2006.
[7] H. V. Poor, *An Introduction to Signal Detection and Estimation*. New York: Springer-Verlag, 1994.
[8] S. Enserink and D. Cochran, "A cyclostationary feature detector," in *Proc. 28th Asilomar Conf. Signals, Systems, Computers*, Pacific Grove, CA, Oct. 1994, pp. 806–810.
[9] A. Sahai, N. Hoven, and R. Tandra, "Some fundamental limits on cognitive radio," in *Proc. 42nd Allerton Conf. Communication, Control, Computing*, Monticello, IL, Oct. 2004, pp. 131–136.
[10] D. Cabric, A. Tkachenko, and R. W. Brodersen, "Experimental study of spectrum sensing based on energy detection and network cooperation," presented at the ACM 1st Int. Workshop Technology Policy for Accessing Spectrum (TAPAS), Boston, MA, Aug. 2006.
[11] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[12] R. S. Blum, S. A. Kassam, and H. V. Poor, “Distributed detection with multiple sensors: Part II—Advanced topics,” *Proc. IEEE*, vol. 85, no. 1, pp. 64–79, Jan. 1997.

[13] P. K. Varshney, *Distributed Detection and Data Fusion*. New York: Springer-Verlag, 1997.

[14] V. Aalo and R. Viswanathan, “Asymptotic performance of a distributed detection system in correlated Gaussian noise,” *IEEE Trans. Signal Process.*, vol. 40, no. 1, pp. 211–213, Jan. 1992.

[15] A. Ghasemi and E. Sousa, “Collaborative spectrum sensing for opportunistic access in fading environments,” in *Proc. IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, Baltimore, MD, Nov. 2005, pp. 131–136.

[16] E. Vistotsky, S. Kuffner, and R. Peterson, “On collaborative detection of TV transmissions in support of dynamic spectrum sharing,” in *Proc. IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, Baltimore, MD, Nov. 2005, pp. 338–345.

[17] G. Ghurumuruhan and Y. Li, “Agility improvement through cooperative diversity in cognitive radio,” in *Proc. IEEE Global Commun. Conf. (IEEE GLOBECOM)*, St. Louis, MO, Nov. 2005, pp. 2505–2509.

[18] J. Lunden, V. Koivunen, A. Huttunen, and H. V. Poor, “Spectrum sensing in cognitive radios based on multiple cyclic frequencies,” in *Proc. 2nd Int. Conf. Cognitive Radio Oriented Wireless Networks Communications (CrownCom)*, Orlando, FL, Jul. 2007, pp. 502–506.

[19] M. Gandetto and C. Regazzoni, “Spectrum sensing: A distributed approach for cognitive terminals,” *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 546–557, Apr. 2008.

[20] C. R. C. M. da Silva, B. Choi, and K. Kim, “Distributed spectrum sensing for cognitive radio systems,” in *Proc. Information Theory Applications Workshop*, La Jolla, CA, Jan. 2007, pp. 120–123.

[21] Z. Quan, S. Cui, and A. H. Sayed, “Optimal linear cooperation for spectrum sensing in cognitive radio networks,” *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 28–40, Feb. 2008.

[22] Y.-C. Liang, Y. Zeng, E. Peh, and A. T. Hoang, “Sensing-throughput tradeoff for cognitive radio networks,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1326–1337, Apr. 2008.

[23] Z. Quan, S. Cui, and A. H. Sayed, “An optimal strategy for cooperative spectrum sensing in cognitive radio networks,” in *Proc. IEEE GLOBECOM*, Washington DC, Nov. 2007, pp. 2947–2951.

[24] A. Sahai and D. Cabric, “A tutorial on spectrum sensing: Fundamental limits and practical challenges,” in *Proc. IEEE Symp. New Frontiers Dynamic Spectrum Access Networks (DySPAN)*, Baltimore, MD, Nov. 2005.

[25] Z. Tian and G. B. Giannakis, “A wavelet approach to wideband spectrum sensing for cognitive radios,” in *Proc. 1st Int. Conf. Cognitive Radio Oriented Wireless Networks Communications (CrownCom)*, Mykonos Island, Greece, Jun. 2006, pp. 1–5.

[26] Y. Hur, J. Park, W. Woo, K. Lim, C.-H. Lee, H. S. Kim, and J. Laskar, “A wideband analog multi-resolution spectrum sensing (MRSS) technique for cognitive radio (CR) systems,” in *Proc. IEEE Int. Symp. Circuits Systems (ISCAS)*, Island of Kos, Greece, May 2006, pp. 4090–4093.

[27] A. Goldsmith, *Wireless Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2006.

[28] C. Cordeiro, K. Challapali, and D. Birru, “IEEE 802.22: An introduction to the first wireless standard based on cognitive radios,” *J. Commun.*, vol. 1, no. 1, pp. 38–47, Apr. 2006.

[29] Initial Evaluation of the Performance of Prototype TV-Band White Space Devices, FCC/OET 07-TR-1006, Jul. 2007.

[30] B. V. Gendenko and A. N. Kolmogorov, *Limit Distributions for Sums of Independent Random Variables*. Reading, MA: Addison-Wesley, 1954.

[31] Z. Quan, S. Cui, A. H. Sayed, and H. V. Poor, “Wideband spectrum sensing in cognitive radio networks,” in *Proc. IEEE Conf. Commun.*, Beijing, China, May 2008, pp. 901–906.

[32] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2003.

[33] Z. Quan, S. Cui, A. H. Sayed, and H. V. Poor, “Spatial-spectral joint detection for wideband spectrum sensing in cognitive radio networks,” in *Proc. IEEE Int. Conf. Acoustic, Speech, Signal Processing (ICASSP)*, Las Vegas, NV, Apr. 2008, pp. 2793–2796.

[34] A. H. Sayed, *Fundamentals of Adaptive Filtering*. New York: Wiley, 2003.

[35] Z. Quan, S. Cui, H. V. Poor, and A. H. Sayed, “Collaborative wideband sensing for cognitive radios,” *IEEE Signal Process. Mag.*, vol. 25, no. 6, pp. 60–73, Dec. 2008.

Zhi Quan (S’07) received the B.E. degree in communication engineering from the Beijing University of Posts and Telecommunications, China, and the M.S. degree in electrical engineering from Oklahoma State University, Stillwater, OK. He is currently working towards the Ph.D. degree in electrical engineering at the University of California, Los Angeles (UCLA).

He was a visiting researcher with Princeton University, Princeton, NJ, during summer 2007, and was an interim engineering intern with Qualcomm during summer 2008. His current research interests include statistical signal processing, wireless communication and networking, cognitive radios, and multimedia.

Mr. Quan was the recipient of the UCLA Chancellor’s Dissertation Fellowship (2008–2009).

Shuguang Cui (S’99–M’05) received the B.Eng. degree in radio engineering (with the highest distinction) from Beijing University of Posts and Telecommunications, Beijing, China, in 1997, the M.Eng. degree in electrical engineering from McMaster University, Hamilton, ON, Canada, in 2000, and the Ph.D. degree in electrical engineering from Stanford University, Stanford, CA, in 2005.

From 1997 to 1998, he was a System Engineer at Hewlett-Packard, Beijing, China. In summer 2003, he worked at National Semiconductor, Santa Clara, CA, on the ZigBee project. From 2005 to 2007, he worked as an Assistant Professor at the Department of Electrical and Computer Engineering, University of Arizona, Tucson. He is now working as an Assistant Professor in Electrical and Computer Engineering at the Texas A&M University, College Station. His current research interests include cross-layer energy minimization for low-power sensor networks, hardware and system synergies for high-performance wireless radios, statistical signal processing, and general communication theories.

Dr. Cui was a recipient of the NSERC graduate fellowship from the National Science and Engineering Research Council of Canada and the Canadian Wireless Telecommunications Association (CWTA) graduate scholarship. He has been serving as the TPC co-chair for the 2007 IEEE Communication Theory Workshop and the ICC’08 Communication Theory Symposium. He is currently serving as an Associate Editor for the IEEE COMMUNICATION LETTERS and IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY.

Ali H. Sayed (S’90–M’92–SM’99–F’01) received the Ph.D. degree from Stanford University, Stanford, CA, in 1992.

He is Professor and Chairman of Electrical Engineering at the University of California, Los Angeles (UCLA) and Principal Investigator of the Adaptive Systems Laboratory. He has published widely, with over 300 articles and five books, in the areas of statistical signal processing, estimation theory, adaptive filtering, signal processing for communications and wireless networking, and fast algorithms for large structured problems. He is coauthor of the textbook *Linear Estimation* (Prentice-Hall, 2000), of the research monograph *Indefinite Quadratic Estimation and Control* (SIAM, 1999), and co-editor of *Fast Algorithms for Matrices with Structure* (SIAM, 1999). He is also the author of the textbooks *Fundamentals of Adaptive Filtering* (Wiley, 2003) and *Adaptive Filters* (Wiley, 2008). He has contributed several encyclopedia and handbook articles.

Dr. Sayed has served on the editorial boards of the *IEEE Signal Processing Magazine*, the *European Signal Processing Journal*, the *International Journal on Adaptive Control and Signal Processing*, and the *SIAM Journal on Matrix Analysis and Applications*. He also served as the Editor-in-Chief of the IEEE TRANSACTIONS ON SIGNAL PROCESSING from 2003 to 2005 and the *EURASIP Journal on Advances in Signal Processing* from 2006 to 2007. He is a member of the Signal Processing for Communications and the Signal Processing Theory and Methods technical committees of the IEEE Signal Processing Society. He has served on the Publications (2003–2005), Awards (2005), and Conference (2007–present) Boards of the IEEE Signal Processing Society. He served on the Board of Governors (2007–2008) of the same Society and is now serving as Vice-President of Publications (2009–present). His work has received several recognitions, including the 1996 IEEE Donald G. Fink Award, the 2002 Best Paper Award from the IEEE Signal Processing Society, the 2003 Kuwait Prize in Basic Sciences, the 2005 Terman Award, the 2005 Young Author Best Paper Award from the IEEE Signal Processing Society, and two Best Student Paper Awards at international meetings (1999 and 2001). He has served as a 2005 Distinguished Lecturer of the IEEE Signal Processing Society and as General Chairman of ICASSP 2008.

H. Vincent Poor (S'72–M'77, SM'82–F'87) received the Ph.D. degree in electrical engineering and computer science from Princeton University, Princeton, NJ, in 1977.

From 1977 until 1990, he was on the faculty of the University of Illinois at Urbana-Champaign. Since 1990, he has been on the faculty at Princeton University, where he is the Michael Henry Strater University Professor of Electrical Engineering and Dean of the School of Engineering and Applied Science. His research interests are in the areas of stochastic analysis, statistical signal processing and their applications in wireless networks, and related fields. Among his publications in these areas are the recent books *MIMO Wireless Communications* (Cambridge University Press, 2007) and *Quickest Detection* (Cambridge University Press, 2009).

Dr. Poor is a member of the National Academy of Engineering, a Fellow of the American Academy of Arts and Sciences, and a former Guggenheim Fellow. He is also a Fellow of the Institute of Mathematical Statistics, the Optical Society of America (OSA), and other organizations. In 1990, he served as President of the IEEE Information Theory Society, and in 2004–2007, he served as the Editor-in-Chief of the IEEE TRANSACTIONS ON INFORMATION THEORY. He was the recipient of the 2005 IEEE Education Medal. Recent recognition of his work includes the 2007 IEEE Marconi Prize Paper Award, the 2007 Technical Achievement Award of the IEEE Signal Processing Society, and the 2008 Aaron D. Wyner Award of the IEEE Information Theory Society.