

Network-Based Wireless Location

Challenges faced in developing techniques for accurate wireless location information

ireless location refers to the geographic coordinates of a mobile subscriber in cellular or wireless local area network (WLAN) environments. Wireless location finding has emerged as an essential public safety feature of cellular systems in response to an order issued by the Federal Communications Commission (FCC) in 1996. The order mandated all wireless service providers to deliver accurate location information of an emergency 911 (E-911) caller to public safety answering points (PSAPs). The FCC mandate aims to solve a serious public safety problem caused by the fact that, at present, a large proportion of all 911 calls originate from mobile phones, the location of which cannot be determined with existing technology. However, many difficulties intrinsic to the wireless environment make meeting the FCC objective challenging; these challenges include channel fading, low signal-to-noise ratios (SNRs), multiuser interference, and multipath conditions. In addition to emergency services, there are many other applications for wireless location technology, including monitoring and tracking for security reasons, location sensitive billing, fraud protection, asset tracking, fleet management, intelligent transportation systems, mobile yellow pages, and even cellular system design and management. This article provides an overview of wireless location challenges and techniques with a special focus on network-based technologies and applications.

WIRELESS NETWORKS

Wireless networks are primarily designed for voice and data communications. The widespread availability of wireless nodes, however, makes it possible to utilize these networks for wireless location purposes as well. It is expected that location-based applications will play an important role in future wireless markets. While location services are now driven by emergency and security requirements imposed on the wireless networks, in the future they will be driven by commercial demands for locationmotivated products. Increasingly, application-level software will incorporate location information into its features to fully utilize such information once it becomes available. For example, asset tracking and management software would incorporate location information into a database for enhanced tracking capabilities. As such, wireless location information will add a new dimension to future applications.

Wireless networking devices constitute the main infrastructure to be utilized for wireless location finding. A location finding system should be able to seamlessly use both cellular and WLANs for location finding by roaming between the networks. The result would be transparent location coverage for both outdoor and indoor environments. Today, the main commercially deployed wireless location finding system is linked to the cellular network in response to requirements by the FCC for emergency 911 calls made through cell phones. These requirements are collectively known as the enhanced 911 (E911) mandate. The details of the FCC requirements for E911 will be discussed.

The purpose of this article is to provide an overview of the basic challenges facing the wireless techniques that are being developed for accurate location information. We start with an overview of the main applications that serve as the major driving force behind the technology.

For ease of reference, Table 1 collects the acronyms that are common in this field and used extensively in subsequent sections.

APPLICATIONS

Figure 1 illustrates some of the available market forecasts for wireless location technology [1], [2]. It is estimated that location-based services (LBSs) will generate annual revenues of the order of US\$15 billion worldwide. In the United States alone, about 170 million mobile subscribers are expected to become covered by the FCC-mandated location accuracy for emergency services. To illustrate the potential of LBS, we will now provide a partial list of applications that will be enhanced using wireless location information [3].

■ *E911*: Currently, a high percentage of E911 calls originate from mobile phones; the percentage is estimated at one third of all 911 calls (170,000 a day) [5], [6]. These wireless E911 calls do not receive the same quality of emergency assistance that fixed-network 911 calls enjoy. This is due to the unknown location of the wireless E911 caller. To face this problem, the FCC issued an order on 12 July 1996 [5], requiring all wireless service providers to report accurate mobile station (MS) location information to the E911 operator at the PSAP. In the FCC order, it was mandated that

[TABLE 1] LIST OF ACRONYMS.

ACRONYMS 2G 3G AOA AP AMPOA BS CDMA E911 FCC GPS LBS ML SS ML SS PDA PSAP rms SINR SINR SINR SINR SINR LDOA TOA TOA LIMTS	DESCRIPTION SECOND GENERATION OF MOBILE SYSTEMS THIRD GENERATION OF MOBILE SYSTEMS ANGLE OF ARRIVAL ACCESS POINT AMPLITUDE OF ARRIVAL BASE STATION CODE DIVISION MULTIPLE ACCESS ENHANCED 911 FEDERAL COMMUNICATIONS COMMISSION GLOBAL POSITIONING SYSTEM LOCATION BASED SERVICES MAXIMUM LIKELIHOOD MOBILE STATION NON-LINE-OF-SIGHT PERSONAL DIGITAL ASSISTANT PUBLIC SAFETY ANSWERING POINT ROOT MEAN SQUARE SIGNAL-TO-INTERFERENCE-NOISE RATIO SIGNAL-TO-NOISE RATIO TIME DIFFERENCE OF ARRIVAL TIME OF ARRIVAL
TOA UMTS WCDMA	TIME OF ARRIVAL UNIVERSAL MOBILE TELECOMMUNICATIONS SYSTEM WIDEBAND CODE DIVISION MULTIPLE ACCESS
WLAN	WIRELESS LOCAL AREA NETWORK

within five years from the effective date of the order, 1 October 1996 (a deadline that is now well passed), wireless service providers must convey to the PSAP the location of the MS within 100 m of its actual location for at least 67% of all wireless E911 calls. (The original FCC requirement was 125 m and was later tightened to 100 m.) This FCC mandate has motivated considerable research efforts towards developing accurate wireless location algorithms for cellular networks and has led to significant enhancements to the wireless location technology (see, e.g., [12]–[25]). According to the latest FCC rules, the new mandate and accuracy requirements will be enforced in 2005. Although the FCC does not have a specific order for indoor environments, a location capability coverage for both indoor and outdoor emergency situations is desirable.

Mobile advertising: Location-specific advertising and marketing will benefit once the location information becomes available. For example, stores will be able to track customer locations and attract them by flashing customized coupons on customers' wireless devices [14]. In addition, a cellular phone or a personal digital assistant (PDA) could act as a handy mobile yellow pages on demand.



[FIG1] Forecast revenues for location-based services [1], [2].



[FIG2] Network-based wireless location finding. (a) Outdoor environment using a cellular network. (b) Indoor environment using a WLAN.

Asset tracking (indoor/outdoor): Wireless location technology can also assist in advanced public safety applications, such as locating and retrieving lost children, patients, or pets. In addition, wireless location technology can be used to track personnel/assets in a hospital or a manufacturing site to provide more efficient management of assets and personnel. One could also consider applications such as smart and interactive tour guides, smart shopping guides that direct shoppers based on their location in a store, and traffic controls in parking structures that guide cars to free parking slots. Department stores, enterprises, hospitals, manufacturing sites, malls, museums, and campuses are some of the potential end users to benefit from the technology.

■ *Fleet management*: Many fleet operators, such as police forces, emergency vehicles, and other services like shuttle and taxi companies, can make use of the wireless location technology to track and operate their vehicles in an efficient manner to minimize response times. In addition, a large number of drivers on roads and highways carry cellular phones while driving. The wireless location technology can help track these phones, thus transforming them into sources of real-time traffic information that can be used to enhance transportation safety.

• *Location-based wireless access security*: New locationbased wireless security schemes can be developed to heighten wireless network security and avoid the interception of digital information. By using location information, only people at specific physical areas could access certain files or databases through a WLAN.

• *Location sensitive billing*: Using the location information of wireless users, wireless service providers can offer variable-rate call plans or services that are based on the caller location.

MOBILE-BASED VERSUS NETWORK-BASED TECHNIQUES

Wireless location technologies fall into two main categories: mobile based and network based. In mobile-based location systems, the MS determines its location from signals received from some base stations (BSs) or from the global positioning system (GPS). In GPS-based estimations, the MS receives and measures the signal parameters from at least four satellites of the current network of 24 GPS satellites. The parameter measured by the MS for each satellite is the time the satellite signal takes to reach the MS. GPS systems have a relatively high degree of accuracy, and they also provide global location information. There is also a hybrid technique that uses both the GPS technology and the cellular infrastructure. In this case, the cellular network is used to aid the GPS receiver embedded in the mobile handset for improved accuracy and/or acquisition time [15].

Still, embedding a GPS receiver into mobile devices leads to increased cost, size, and battery consumption. It also requires the replacement of millions of mobile handsets that are already on the market. In addition, the accuracy of GPS measurements degrades in urban environments as well as inside buildings. For these reasons, some wireless service providers may be unwilling to embrace GPS fully as the sole location technology.

Network-based location technology, on the other hand, relies on some existing networks (either cellular or WLAN) to determine the position of a mobile user by measuring its signal parameters when received at the network BSs. In this technology, the BSs measure the signals transmitted from an MS and relay them to a central site for further processing and data fusion to provide an estimate of the MS location. A significant advantage of network-based techniques is that the MS is not involved in the location-finding process; thus, the technology does not require modifications to existing handsets. However, unlike GPS location systems, many aspects of networkbased location are not yet fully studied.

The rest of this article focuses on network-based wireless location. For location estimation, two operations must be performed at the BSs. The BSs have to measure some signal parameters (such as the time or the angle of arrival) of the received MS signals. Then, the measured signal parameters are combined in a data fusion stage to provide the final estimate for location. Both of these stages are discussed in the following sections. Figure 2 illustrates this two-stage procedure (measurement and data fusion) for an outdoor environment using a cellular network and for an indoor environment using a WLAN. Although the focus of the article is on network-based location systems, most of the network-based location algorithms presented here can be used at the MS as well. Therefore, from now on, network-based wireless location will simply be referred to as *wireless location*.

DATA FUSION METHODS

The data fusion step combines measurements from different BSs to obtain an estimate of the MS location. Let (x_m, y_m) denote the MS location coordinates in a Cartesian coordinate system. Let the coordinates of three BSs (BS₁, BS₂, and BS₃) be denoted by (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) , respectively. For simplicity of presentation, only the x and y coordinates are considered in the derivations and the *z* coordinate is ignored. This corresponds to a case where the BSs and the mobile user are located on a relatively flat plane. Without loss of generality, the origin of the Cartesian coordinate system is set at BS_1 , i.e., $(x_1, y_1) = (0, 0)$. Several data fusion techniques have been introduced in the literature; these techniques depend on what signal parameters are measured at the BSs [3], [4]. (These are several studies in the literature that compare the performance of different fusion algorithms, e.g. [26], [27].) The most common signal parameters are the time, angle, and amplitude of arrival of the MS signal.

TIME OF ARRIVAL DATA FUSION

The time of arrival (TOA) data fusion method is based on combining estimates of the TOA of the MS signal when arriving at three different BSs. Since the wireless signal travels at the speed of light ($c = 3 \times 10^8$ m/s), the distance between the MS and BS_i is given by

$$r_i = (t_i - t^o)c, \tag{1}$$

where t^o is the time instant at which the MS begins transmission and t_i is the TOA of the MS signal at BS_{*i*}. The distances (r_1, r_2, r_3) can be used to estimate (x_m, y_m) by solving the following set of equations (see Figure 3):

$$r_1^2 = x_m^2 + y_m^2 \tag{2}$$

$$r_2^2 = (x_2 - x_m)^2 + (y_2 - y_m)^2 \tag{3}$$

$$r_3^2 = (x_3 - x_m)^2 + (y_3 - y_m)^2.$$
(4)

Without loss of generality, it can be assumed that $r_1 < r_2 < r_3$.

One way to solve this overdetermined nonlinear system of equations is as follows. First, (2) and (3) are solved for the two unknowns (x_m, y_m) to yield two solutions. As shown in Figure 3, (2) and (3) each define a locus on which the MS must lie. Second, the distance between each of the two possible solutions and the circle given by (4) is calculated. The solution that results in the shortest distance from the circle (4) is chosen to be an estimate of the MS location coordinates

[12]. Although this method helps resolve the ambiguity between the two solutions resulting from (2) and (3), it does not combine the third measurement r_3 in an optimal manner. Furthermore, it is not possible in this way to combine TOA measurements from more than three BSs (which would be useful when the measurements { r_i } are subject to inaccuracies and noise).

This issue can be addressed by combining all the available measurements using a least-squares solution as follows (alternative techniques such as maximum likelihood (ML) solution can be found, e.g, in [49], [50]). Subtracting (2) from (3) gives

$$r_2^2 - r_1^2 = x_2^2 - 2x_2x_m + y_2^2 - 2y_2y_m.$$

Similarly, subtracting (2) from (4) gives

$$r_3^2 - r_1^2 = x_3^2 - 2x_3x_m + y_3^2 - 2y_3y_m.$$

Rearranging terms, the above two equations can be written in matrix form as

$$\begin{bmatrix} x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} \begin{bmatrix} x_m \\ y_m \end{bmatrix} = \frac{1}{2} \begin{bmatrix} K_2^2 - r_2^2 + r_1^2 \\ K_3^2 - r_3^2 + r_1^2 \end{bmatrix},$$
 (5)

where

$$K_i^2 = x_i^2 + y_i^2. (6)$$

(7)

Then, (5) can be rewritten as

where

$$\mathbf{H} = \begin{bmatrix} x_2 & y_2 \\ x_3 & y_3 \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} x_m \\ y_m \end{bmatrix}, \ \mathbf{b} = \frac{1}{2} \begin{bmatrix} K_2^2 - r_2^2 + r_1^2 \\ K_3^2 - r_3^2 + r_1^2 \end{bmatrix}.$$

Hx = b,

If more than three TOA measurements are available, it can be verified that (7) still holds with



[FIG3] TOA data fusion using three BSs.

$$\mathbf{H} = \begin{bmatrix} x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ \vdots & \vdots \end{bmatrix}, \qquad \mathbf{b} = \frac{1}{2} \begin{bmatrix} K_2^2 - r_2^2 + r_1^2 \\ K_3^2 - r_3^2 + r_1^2 \\ K_4^2 - r_4^2 + r_1^2 \\ \vdots \end{bmatrix}.$$
(8)

In this case, the least-squares solution of (7) is given by ([3], [7])

$$\hat{\mathbf{x}} = \left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{b}.$$
(9)

It is seen that the TOA data fusion method requires accurate synchronization between the BSs and MS clocks so that the measurements $\{r_i\}$ are adequate approximations for the actual distances. Many of the current wireless system standards only mandate tight timing synchronization among BSs (see, e.g., [30]). The MS clock itself might have a drift that can reach a few microseconds. This drift directly generates an error in the location estimate of the TOA method. In the next subsection we present a data fusion technique that combines time DOA (TDOA) measurements and helps avoid MS clock synchronization errors [3], [31], [34].

TDOA DATA FUSION

The TDOA associated with BS_{*i*} is $t_i - t_1$; i.e., it is the difference between the TOAs of the MS signal at BS_{*i*} and BS₁. Now we define the distance differences

$$r_{i1} \stackrel{\Delta}{=} r_i - r_1$$

= $(t_i - t^o)c - (t_1 - t^o)c = (t_i - t_1)c.$ (10)

Note that these differences are not affected by errors in the MS clock time (t^o) as it cancels out when subtracting two TOA measurements. (3) can be rewritten in terms of the TDOA measurement r_{21} as

$$(r_{21}+r_1)^2 = K_2^2 - 2x_2x_m - 2y_2y_m + r_1^2$$

Expanding and rearranging terms gives



[FIG4] Combining AOA measurements.

$$-x_2x_m - y_2y_m = r_{21}r_1 + \frac{1}{2}\left(r_{21}^2 - K_2^2\right).$$

Similarly, (4) leads to

$$-x_3x_m - y_3y_m = r_{31}r_1 + \frac{1}{2}\left(r_{31}^2 - K_3^2\right).$$

Rewriting these equations in matrix form gives

$$\mathbf{H}\mathbf{x} = r_1 \mathbf{c} + \mathbf{d},\tag{11}$$

where

$$\mathbf{c} = \begin{bmatrix} -r_{21} \\ -r_{31} \end{bmatrix}, \qquad \mathbf{d} = \frac{1}{2} \begin{bmatrix} K_2^2 - r_{21}^2 \\ K_3^2 - r_{31}^2 \end{bmatrix}$$

Equation (11) can be used to solve for x in terms of the unknown r_1 to yield

$$\mathbf{x} = r_1 \mathbf{H}^{-1} \mathbf{c} + \mathbf{H}^{-1} \mathbf{d}.$$
 (12)

Substituting this intermediate result into (2) leads to a quadratic equation in r_1 . Solving for r_1 and substituting the positive root back into (12) yields the final solution for x.

If more than three BSs are involved in the MS location, (11) still holds with

$$\mathbf{H} = \begin{bmatrix} x_2 & y_2 \\ x_3 & y_3 \\ \\ x_4 & y_4 \\ \\ \vdots & \vdots \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} -r_{21} \\ -r_{31} \\ \\ -r_{41} \\ \\ \vdots \end{bmatrix}, \ \mathbf{d} = \frac{1}{2} \begin{bmatrix} K_2^2 - r_{21}^2 \\ K_3^2 - r_{31}^2 \\ \\ K_4^2 - r_{41}^2 \\ \\ \vdots \end{bmatrix}$$

which yields the following least-squares intermediate solution

$$\hat{\mathbf{x}} = \left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}(r_{1}\mathbf{c} + \mathbf{d}).$$
(13)

Combining this intermediate result with (2) again, the final estimate for x is obtained. A more accurate solution can be obtained as in [32] if the second-order statistics of the TDOA measurement errors are known.

ANGLE OF ARRIVAL DATA FUSION

At the BS, angle of arrival (AOA) estimates can be obtained using an antenna array. The direction of arrival of the MS signal can be calculated by measuring the phase difference between the antenna array elements or by measuring the power spectral density across the antenna array in what is known as beamforming (see, e.g., [37] and the references therein). By combining the AOA estimates of two BSs, an estimate of the MS position can be obtained (see Figure 4). The number of BSs needed for the location process is less than that of the TOA and TDOA methods. Another advantage of AOA location methods is that they do not require BS or MS clock synchronization. However, one disadvantage of the AOA method

A LARGE PROPORTION OF ALL 911 CALLS ORIGINATE FROM MOBILE PHONES, THE LOCATION OF WHICH SHOULD BE DETERMINED WITH SUFFICIENT ACCURACY.

the angular orientation of the installed antenna arrays. The issue of NLOS is discussed in another section. For the error in the angular orientation of the antenna arrays, some test

is that antenna array structures do not currently exist in second generation (2G) cellular systems. Still, the use of antenna arrays is planned in third generation (3G) cellular systems, such as UMTS and cdma2000 networks [38], [39].

More generally, assume *n* BSs estimate the AOA of the MS signal, and the goal is to combine these measurements to estimate the MS location. As indicated in Figure 4, let α_2 denote the AOA of the MS signal at BS₂. Then

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} r_1 \cos \alpha_1 \\ r_1 \sin \alpha_1 \end{bmatrix}$$

and

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} r_2 \cos \alpha_2 \\ r_2 \sin \alpha_2 \end{bmatrix}$$

Likewise, for any other BS_i ,

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} r_i \cos \alpha_i \\ r_i \sin \alpha_i \end{bmatrix}.$$

Collecting these relations into a single equation yields

$$Hx = b$$
,

where

$$\mathbf{H} = \begin{bmatrix} 1 & 0\\ 0 & 1\\ 1 & 0\\ 0 & 1\\ \vdots & \vdots\\ 1 & 0\\ 0 & 1 \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} x_m\\ y_m \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} r_1 \cos \alpha_1\\ r_1 \sin \alpha_1\\ x_2 + r_2 \cos \alpha_2\\ y_2 + r_2 \sin \alpha_2\\ \vdots\\ x_n + r_n \cos \alpha_n\\ y_n + r_n \sin \alpha_n \end{bmatrix}.$$
(14)

The least-squares solution for x is then

$$\hat{\mathbf{x}} = \left(\mathbf{H}^{\mathrm{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{b}.$$
 (15)

Besides the regular sources of error in AOA measurements, such as noise and interference, AOA measurements can be corrupted by non-line-of-sight (NLOS) effects and errors in antenna arrays, some test measurements can be conducted to calibrate the orientation of the antenna array.

HYBRID DATA FUSION TECHNIQUES

In TOA, TDOA, and AOA methods, two or more BSs are involved in the MS location process. In situations where the MS is much closer to one BS (serving site) than the other BSs, the accuracy of these methods can be degraded due to the relatively low SNR of the received MS signal at one or more BSs. The accuracy is further reduced if some type of power control is used, since this requires that the MS reduce its transmitted power when it approaches a BS. In these cases, an alternate data fusion procedure is used to obtain AOA estimates and combine them with TOA estimates (see, e.g., [40]). In real scenarios, the accuracy of TOA and AOA estimates is usually a function of the environment. For example, in rural areas, AOA measurements can be more accurate than TOA measurements if a large-size antenna array is deployed. On the other hand, TOA measurements are more accurate than AOA measurements if the BS antenna array is surrounded by many scatterers. The following is a simple twostep hybrid least-squares procedure. Assume n BSs estimate the AOA and TOA of the MS. From (9), the least-squares estimate of (x_m, y_m) using TOA measurements is given by

$$\left(\begin{array}{c} x_m \\ y_m \end{array}\right)_{\text{TOA}} = \left(\mathbf{H}_{\text{TOA}}^{\text{T}} \mathbf{H}_{\text{TOA}}\right)^{-1} \mathbf{H}_{\text{TOA}}^{\text{T}} \mathbf{b}_{\text{TOA}}, \quad (16)$$

where

$$H_{\text{TOA}} = \begin{pmatrix} x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \vdots \\ x_n & y_n \end{pmatrix}, \qquad b_{\text{TOA}} = \frac{1}{2} \begin{pmatrix} K_2^2 - r_2^2 + r_1^2 \\ K_3^2 - r_3^2 + r_1^2 \\ \vdots \\ K_n^2 - r_n^2 + r_1^2 \end{pmatrix}$$

and

$$K_i^2 = x_i^2 + y_i^2$$

Likewise, from (15), the least-squares estimate of (x_m, y_m) using only AOA measurements is given by

$$\widehat{\begin{pmatrix} x_m \\ y_m \end{pmatrix}}_{AOA} = \left(\mathbf{H}_{AOA}^{\mathsf{T}} \mathbf{H}_{AOA} \right)^{-1} \mathbf{H}_{AOA}^{\mathsf{T}} \mathbf{b}_{AOA}$$
(17)

where



The final location estimate could be taken as a linear combination of the two estimates, say as

$$\widehat{\begin{pmatrix} x_m \\ y_m \end{pmatrix}} = \eta \widehat{\begin{pmatrix} x_m \\ y_m \end{pmatrix}}_{\text{TOA}} + (1 - \eta) \widehat{\begin{pmatrix} x_m \\ y_m \end{pmatrix}}_{\text{AOA}}$$
(19)

where the positive parameter η is chosen depending on the relative accuracy of the TOA and AOA measurements.

DATA FUSION WITH NLOS CONDITIONS

An important source of error in TOA-based and AOA-based data fusion is the case where there is no line-of-sight from the mobile station to the BSs. A geometrically constrained data fusion scheme could be used to reduce the effect of such NLOS conditions [41] (see [8] for other ways to exploit the geometry of the problem). Figure 5 shows a representation of a cellular system assuming three BSs. Let the θ_i denote the angles induced by the topology of the BSs. Let also r_{ij} denote the distance between the *i* th and *j* th BSs. Likewise, the α_i denote the AOAs of the MS signal at the BSs. In practice, the distance measurements r_i in (1) are generally corrupted by NLOS offsets arising from the presence of obstacles between the MS and the BS, as well as by measurement noise. Similarly, the AOA



[FIG5] A schematic of a cellular network topology with three BSs.

measurements α_i are corrupted by NLOS effects and by noise. Hence, the available measurements are

$$\bar{\alpha}_i = \alpha_i + v_{\alpha_i}$$

$$\bar{r}_i = r_i + v_{r_i} \tag{20}$$

where v_{α_i} and v_{r_i} represent the corruptions to α_i and r_i . One scheme for recovering (x_m, y_m) from the measurements $\{\bar{r}_i, \bar{\alpha}_i\}$ is based on formulating a constrained optimization problem that reduces the effect of NLOS conditions on location accuracy. The constraints will be a reflection of the topology of the cellular network. Thus, consider the cellular system shown in Figure 5. The constraints are the distances between the BSs, which are given by

$$r_{12}^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\gamma_1 \tag{21}$$

$$r_{23}^2 = r_2^2 + r_3^2 - 2r_2r_3\cos\gamma_2$$
(22)
:

where the angles $\{\gamma_i\}$ are functions of $\{\alpha_i, \theta_i\}$. This formulation is easily extendable to the case of *n* BSs. Then, one could pose the problem of estimating the $\{\alpha_i, r_i\}$ by solving

$$\{\hat{\alpha}_i, \hat{r}_i\}_{i=1}^n = \arg\min_{\{\alpha_i, r_i\}} \sum_{i=1}^n \left(\frac{\bar{\alpha}_i - \alpha_i}{\sigma_{\alpha_i}}\right)^2 + \left(\frac{\bar{r}_i - r_i}{\sigma_{r_i}}\right)^2$$
(23)

subject to

$$r_{12}^2 = r_1^2 + r_2^2 - 2r_1r_2\cos(\pi - (\alpha_1 + \alpha_2))$$

$$r_{23}^2 = r_2^2 + r_3^2 - 2r_2r_3\cos(\pi - (\alpha_3 + (\theta_2 - \alpha_2)))$$

:

where $\sigma_{r_i}^2$ is the variance of the distance error and $\sigma_{\alpha_i}^2$ is the variance of the angle error (both at the *i* th BS). There are some known methods for estimating the variances $\sigma_{r_i}^2$ and $\sigma_{\alpha_i}^2$ (see, e.g., [42]–[44]). These methods generally use the time history of the signals, or the scattering model of the environment, to estimate the noise variance, as in

$$\sigma_{r_i}^2 \approx \frac{1}{K} \sum_{n=0}^{K-1} (\bar{r}_i(n) - \mu_{r_i})^2, \qquad (24)$$

where

$$\mu_{r_i} = \frac{1}{K} \sum_{n=0}^{K-1} \bar{r}_i(n)$$
(25)

for $K \approx 400$ and where $\bar{r}_i(n)$ is the measurement of r_i at experiment *n*. This is also true for $\sigma_{\alpha_i}^2$. Minimizing (23) results in estimates of $\{r_i, \alpha_i\}$. Using the equalized values in (8) or (9) will result in improved location accuracy.

THE WIRELESS ENVIRONMENT

From the previous discussion, it is clear that most wireless location methods depend on combining estimates of the TOA and/or AOA of the received signal at different BSs. Estimating the TOA and amplitude of arrival (AmpOA) of wireless signals has been

studied in many works since it is required in many wireless system designs for online signal decoding purposes. Yet, estimating these same parameters for wireless location purposes is challenging for several reasons.

• Low SINR conditions. Cellular systems tend to suffer from high multiple access interference levels that degrade the SNR of the received signal. Moreover, the ability to detect the MS signal at multiple BSs is limited by the use of power control algorithms, which require the MS to decrease its transmitted power when it approaches the serving BS. This fact, in turn, decreases the received MS signal power level at other BSs. In a typical CDMA IS-95 cellular environment, the received SNR at the serving BS is in the order of -15 dB. However, the received SNR at BSs other than the serving BS can be as low as -40 dB, which poses a challenge for wireless location in such environments.

• *Channel fading and Doppler frequency*. In wireless location applications, the estimation period can be *considerably long* (it might reach several seconds). Thus, in a fading environment, the channel values can change significantly over the location estimation period. In this way, the channel values can no longer be assumed constant during the estimation period.

• Overlapping multipath. In wireless location systems, the accurate estimation of the TOA, AOA, and AmpOA of the *first arriving ray* of the multipath channel is vital. In general, the first arriving (prompt) ray is assumed to correspond to the most direct path between the MS and BS. However, in many wireless propagation scenarios, the prompt ray is succeeded by a multipath component that arrives at the receiver within a short time of the prompt ray. If this delay is smaller than the duration of the pulse-shape used in the wireless system, these two rays overlap, causing errors in the prompt ray TOA and AmpOA estimation. These errors degrade the performance of wireless location algorithms; as such, they demand careful considerations (see, e.g., [9]–[11], [28], and [29]).

In the sections that follow, some algorithms for TOA and AOA estimation are described. These algorithms exploit the nature of the wireless channel and are robust to low SNR and fading conditions [25], [51].

TOA ESTIMATION

The aim of a TOA estimation scheme is to estimate an unknown delay, τ^{o} , of a known sequence {*s*(*n*)}. (At the serving site, the MS signal can be decoded with reasonably high accuracy; thus, it can be assumed to be known almost perfectly.) The signal is

assumed initially to be transmitted over a single path fading channel. A total of *K* measurements r(n) are collected, which are related to s(n) via

$$r(n) = A h(n) s(n - \tau^{0}) + v(n), \quad n = \{1, \dots, K\},$$
(26)

where v(n) is additive white Gaussian noise with variance σ_v^2 , {h(n)} is the fading channel gain, and A is an unknown amplitude (real value) that accounts for the gain of the

static channel if fading were not present. The autocorrelation function of h(n) is defined as

$$R_h(i) = Eh(n)h^*(n-i).$$
 (27)

Without loss of generality, we will assume that the sequence h(n) has unit variance, i.e., $R_h(0) = 1$. The ML estimates of $\{\tau^o, h(n)\}$ are defined by

$$\{\hat{\tau}, \hat{h}(n)\} = \arg\max_{\tau, h(n)} \left[P(r(1) \cdots r(K) | \tau, h(n)) \right],$$
 (28)

where the likelihood function $P(r(1) \cdots r(K) | \tau, h(n))$ is of the form

$$C_1 \exp\left\{-C_2 \frac{1}{K} \sum_{n=1}^{K} \|r(n) - Ah(n)s(n-\tau)\|^2\right\}$$
(29)

for some positive constants C_1 and C_2 that are independent of the unknowns { τ , h(n)}. Thus, the ML estimates of { τ , h(n)} are given by [25]

$$\{\hat{\tau}, \hat{h}(n)\} = \arg \max_{\tau, h(n)} J_{ML}(\tau, h(n)),$$
 (30)

where the cost function $J_{\rm ML}$ is given by

$$I_{\text{ML}}(\tau, h(n)) = \frac{2A}{K} \sum_{n=1}^{K} \text{Re}[r(n)h^*(n)s^*(n-\tau)] - \frac{A^2}{K} \sum_{n=1}^{K} |h(n)|^2 |s(n-\tau)|^2.$$

This construction requires an infinite dimensional search over $\{\tau, h(n)\}$ and is not feasible in practice even when τ and h(n) are evaluated over a dense grid. To arrive at a feasible algorithm, we assume the channel variations are sufficiently slow, namely, that h(n) is piecewise constant over intervals of N samples. The value of N depends on the environmental conditions; an optimal choice for N is discussed later in this

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article. Under the slow channel variation assumption, it can be argued that the maximization problem (30) over τ and h(n) can be reduced to maximizing the following cost function instead over τ [25]:

$$J(\tau) = \frac{1}{M} \sum_{m=1}^{M} \left| \frac{1}{N} \sum_{n=(m-1)N+1}^{mN} r(n) s^*(n-\tau) \right|^2, \quad (31)$$

where the integers *N* and *M* satisfy NM = K. A practical scheme for maximizing (31) over τ is shown in Figure 6 and was derived in [25]. The received sequence $\{r(n)\}$ is correlated with a delayed replica of the transmitted sequence, $\{s(n - \tau)\}$, for different values of τ . The resulting sequence is averaged coherently



[FIG6] A scheme for TOA estimation over fading channels.

over an interval of *N* samples and further averaged noncoherently for *M* samples after squaring to build the power delay profile, $J(\tau)$. The branch that results in the largest value for τ provides the desired estimate $\hat{\tau}$. Moreover, the SNR at the output of the searcher is [25]

SNR =
$$\frac{A^2}{\sigma_v^2} \left(R_h(0) + \sum_{i=1}^{N-1} \frac{2(N-i)R_h(i)}{N} \right).$$

The optimal value of the coherent averaging period (N_{opt}) is obtained by maximizing the SNR with respect to N, which leads to the following expression for finding N_{opt} :

$$\sum_{i=1}^{N_{\text{opt}}-1} i R_h(i) = 0.$$
 (32)

For a Rayleigh fading channel, $R_h(i)$ is given by

$$R_h(|i|) = J_o\left(2\pi f_D T_s i\right),$$

where $J_o(\cdot)$ is the first-order Bessel function, T_s is the sampling period of the received sequence $\{r(n)\}$, and f_D is the maximum Doppler frequency. Therefore, (32) shows that the coherent averaging interval N should be adapted according to the channel autocorrelation function.

TOA ESTIMATION WITH ANTENNA ARRAY

Further improvement in the TOA estimation can be accomplished by deploying an antenna array at the BS [51]. Thus assume that the BS uses an N_a -element antenna array. Then, in contrast to (26), the received signal at time *n* is now an $N_a \times 1$ vector:

$$\mathbf{r}(n) = \mathbf{a}h(n)s(n - \tau^{o}) + \mathbf{v}(n), \tag{33}$$

(Note that we are assuming a scattered MS model. Since in a typical cellular system the mobile station is usually far from the BS, the reflected rays from the scatterers around the MS reach the BS at close angles. These reflected rays cause a fading effect in h(n) with almost the same AOA. Moreover, it is assumed that any bias in the direction or the angle of the arrays can be ignored from the derivations through some calibration procedures.) where now v(n) is an additive white Gaussian noise vector with covariance matrix $\sigma_n^2 \mathbf{I}$, and a is the array response defined by

$$\mathbf{a} = \operatorname{col}\left\{1, e^{j2\pi \frac{d}{\lambda}\sin\alpha}, \dots, e^{j2\pi \frac{(N_d-1)d}{\lambda}\sin\alpha}\right\}, \qquad (34)$$

where α is the AOA of the signal measured with respect to the array, *d* is the antenna spacing, and λ is the wavelength corresponding to the carrier frequency. Note that the array response defined by (34) is valid only for a uniform linear array (ULA). This array response could be modified if other array structures, such as circular array, are used.

The ML estimates of τ and α are given by

$$\{\hat{\tau}, \hat{\alpha}\} = \arg\max_{\tau, \alpha} \left[P(\mathbf{r}(1) \cdots \mathbf{r}(K)) | \tau, \alpha \right], \tag{35}$$

where the likelihood function is now proportional to

$$\exp\left\{-C_{2}\frac{1}{K}\sum_{n=1}^{K}\|\mathbf{r}(n)-\mathbf{a}h(n)s(n-\tau)\|^{2}\right\},$$
 (36)

in which $\|.\|$ is the Euclidean norm of the vector and C_2 is some positive constant. Therefore, the ML estimates of $\{\tau, \alpha\}$ can be found by solving

$$\{\hat{\tau}, \hat{\alpha}\} = \arg \max_{\tau, \alpha} J_{\rm ML}(\tau, \alpha), \qquad (37)$$

where

$$J_{\mathrm{ML}}(\tau,\alpha) = \frac{1}{K} \sum_{n=1}^{K} \|\mathbf{r}(n) - \mathbf{a}h(n)s(n-\tau)\|^2.$$

This cost function can be simplified by noting that $\|\mathbf{a}\|^2 = N_a$ and that the entries of a have unit norm, so that

$$J_{\rm ML}(\tau, \alpha) = \frac{1}{K} \sum_{n=1}^{K} |\mathbf{a}^* \mathbf{r}(n) - h(n)s(n-\tau)|^2.$$
(38)

Rather than perform a two-dimensional (2-D) search, the estimator can perform the search for τ and α separately as follows.

Assume first that α is known. Then the term $a^*r(n)$ in (38) can be interpreted as the output of a beamformer (antenna combiner) steered to direction α , as shown in Figure 7. The optimization problem for τ , given α , then reduces to the single-antenna case of Figure 6 using a beamformer at direction α .

AOA ESTIMATION WITH ANTENNA ARRAY

Now assume τ in (38) is known and evaluate the correlation

$$\mathbf{z}_{m} = \frac{1}{N} \left(\sum_{n=(m-1)N+1}^{mN} \mathbf{r}(n) s^{*}(n-\tau^{o}) \right), \quad m = 1, \dots, M$$
(39)

over intervals of length N, during which h(n) is essentially invariant. Then, using (33),

$$z_{m} = ah(mN) \frac{1}{N} \left(\sum_{n=(m-1)N+1}^{mN} |s(n-\tau^{o})|^{2} \right) + \frac{1}{N} \left(\sum_{n=(m-1)N+1}^{mN} v(n) s^{*}(n-\tau^{o}) \right).$$
(40)

In other words,

$$\mathbf{z}_m = p(m)h(mN)\mathbf{a} + \mathbf{u}_m,\tag{41}$$

where p(m) denotes the constant known power term

$$p(m) = \frac{1}{N} \left(\sum_{n=(m-1)N+1}^{mN} |s(n-\tau^{o})|^2 \right)$$

and u_m refers to the noise term in (40). Collecting (41) into vector form yields





[FIG7] TOA estimation over fading channels using an antenna array.

The channel gains $\{h(N), h(2N), \ldots, h(MN)\}$ can be estimated roughly from (41) by noting that the top entry of a is unity, so that

$$\hat{h}(mN) = \mathbf{z}_m(1)/p(m).$$

The LS estimate can then be obtained as

$$\hat{\mathbf{a}} = (\mathbf{A}^* \mathbf{A})^{-1} \mathbf{A}^* \mathbf{z}.$$
 (43)

The AOA information can be extracted from the estimated array response \hat{a} based only on the phase rotation between the entries of \hat{a} (see Figure 8).

MULTIPATH, MULTIUSER ENVIRONMENT

As mentioned previously, wireless propagation suffers from multipath conditions, in which case the prompt ray may be succeeded by multipath components that arrive at the receiver with short delays (e.g., [52]). In this section, we describe one way to perform TOA and AOA estimation under multipath



[FIG8] AOA estimation using an antenna array over a single path fading channel.

conditions. First, we modify the channel model (26) to accommodate a more general multiuser, multipath environment. Assuming the maximum number of channel taps to be L and the number of mobile users to be N_u , the received signal r(n) of size $N_a \times 1$ is now given by

$$\mathbf{r}(n) = \sum_{k=1}^{N_u} \sum_{l=1}^{L} \mathbf{a}_{k,l} h_{k,l}(n) s_k(n - \tau_{k,l}^o) + \mathbf{v}(n), \qquad (44)$$

where $s_k(n)$ is the transmitted sequence by the *k*th user and v(n) is an $N_a \times 1$ additive white Gaussian noise vector. Moreover, $h_{k,l}(n)$ and $\tau_{k,l}$ are the channel gain and delay, respectively, for user *i*, and $a_{k,l}$ is the array response corresponding to the *l* th channel tap from user *k* to the BS, namely

$$\mathbf{a}_{k,l} = \operatorname{col}\left\{1, e^{j2\pi \frac{d}{\lambda} \sin \alpha_{k,l}}, \dots, e^{j2\pi \frac{(N_d-1)d}{\lambda} \sin \alpha_{k,l}}\right\}, \quad (45)$$

where $\alpha_{k,l}$ is the AOA for the *l* th tap and *k*th user.

As in the single path case discussed earlier [see (39)], we define the correlation vectors:

$$\mathbf{z}_{k,l,m} = \frac{1}{N} \sum_{n=(m-1)N+1}^{mN} \mathbf{r}(n) s_k^* \left(n - \tau_{k,l}^o \right),$$

where N is the coherent correlation length. Then [compare with (41)]

$$\mathbf{z}_{k,l,m} = p_{k,l}(m)h_{k,l}(mN)\mathbf{a}_{k,l} + \mathbf{i}_{k,l,m} + \mathbf{u}_{k,l,m}, \qquad (46)$$

where

$$p_{k,l}(m) = \frac{1}{N} \left(\sum_{n=(m-1)N+1}^{mN} |s_k \left(n - \tau_{k,l}^o \right)|^2 \right)$$

and

$$\mathbf{u}_{k,l,m} = \frac{1}{N} \left(\sum_{n=(m-1)N+1}^{mN} \mathbf{v}(n) s_k^* \left(n - \tau_{k,l}^o \right) \right)$$
(47)

$$\mathbf{i}_{k,l,m} = \sum_{k'=1}^{N_u} \sum_{\substack{l'=1\\k' \neq k, l' \neq l}}^{L} \rho_{k,l,k',l'}(m) h_{k',l'}(mN) \mathbf{a}_{k',l'}$$
(48)

where

$$\rho_{k,l,k',l'}(m) = \frac{1}{N} \sum_{n=(m-1)N+1}^{mN} s_{k'} \left(n - \tau_{k',l'}^o\right) s_k^* \left(n - \tau_{k,l}^o\right)$$
(49)

represents the correlation between the sequences of user k and all other users. Collecting M such realizations into a vector $\mathbf{z}_{k,l}$ yields

$$\begin{bmatrix}
z_{k,l,1} \\
z_{k,l,2} \\
\vdots \\
z_{k,l,M}
\end{bmatrix} = \begin{bmatrix}
p_{k,l}(1)h_{k,l}(N)I_{N_a \times N_a} \\
p_{k,l}(2)h_{k,l}(2N)I_{N_a \times N_a} \\
\vdots \\
p_{k,l}(M)h_{k,l}(MN)I_{N_a \times N_a}
\end{bmatrix} a_{k,l} \\
+ \begin{bmatrix}
i_{k,l,2} \\
\vdots \\
i_{k,l,M}
\end{bmatrix} + \begin{bmatrix}
u_{k,l,1} \\
u_{k,l,2} \\
\vdots \\
u_{k,l,M}
\end{bmatrix}.$$
(50)

The least-squares estimation of $a_{k,l}$ can be obtained as

$$\hat{\mathbf{a}}_{k,l} = \left(\mathbf{A}_{k,l}^* \mathbf{A}_{k,l}\right)^{-1} \mathbf{A}_{k,l}^* \mathbf{z}_{k,l}.$$
(51)

The AOA information is finally extracted from the estimated array response $\hat{\mathbf{a}}_{k,l}$ based only on the phase rotation between the entries of $\hat{\mathbf{a}}_{k,l}$ according to (45). The above least-squares estimation is repeated for all users and multipaths, $k = 1, \ldots, N_u$, $l = 1, \ldots, L$.

As the number of users and multipaths increases, multiple access interference (MAI) and intersymbol interference (ISI) in (46) become stronger. For practical scenarios with a large number of active users in a cell, the accuracy of the least-squares estimation of AOA is limited by MAI and ISI. One solution for reducing the effect of MAI in (46) is to increase the coherent correlation length N, as well as the realization length M. However, the correlation or estimation length should be short enough such that the channel taps can be assumed constant during the estimation process.

We may use joint least-squares estimation followed by multiuser interference cancellation to provide an accurate AOA estimation in the presence of interfering users. This joint technique takes advantage of the following two facts:

• A BS (in normal operating mode) detects and decodes the signals from all users simultaneously. Therefore, the BS knows the training sequences used by the users, i.e., $s_k(n)$, $k = 1, ..., N_u$.

The BS performs the channel and array response estimation for all users and multipaths. Therefore, the estimation of the channel taps ($\hat{a}_{k,l}$ and $h_{k,l}$, $k = 1, ..., N_u$, l = 1, ..., L) are available at the BS.

The above information is used in a secondary stage to further enhance the accuracy of the estimation process by canceling the MAI in (46). The interfering signal is regenerated using the known $s_k(n)$, the estimated channel gain $\hat{h}_{k,l}$, and the array response $\mathbf{a}_{k,l}$. It is then subtracted from the previous correlation results $\mathbf{z}_{k,l}$, and a new estimate is obtained. The following steps are performed in the proposed architecture:

1) Use (46)–(51) to calculate $\{\hat{\alpha}_{k,l}, \hat{h}_{k,l}(mN)\}$.

2) Use the estimated channel taps to regenerate (estimate) the MAI term $i_{k,l,m}$ in (48), i.e.,

$$\hat{\mathbf{i}}_{k,l,m} = \sum_{k'=1}^{N_u} \sum_{\substack{l'=1\\k' \neq k, l' \neq l}}^{L} \rho_{k,l,k',l'}(m) \hat{h}_{k',l'}(mN) \hat{\mathbf{a}}_{k',l'}.$$
 (52)

3) Subtract the estimated interference (52) from the correlation vector in (46). The new $\mathbf{z}_{k,l,m}$ becomes

$$\bar{\mathbf{z}}_{k,l,m} = \mathbf{z}_{k,l,m} - \hat{\mathbf{i}}_{k,l,m}$$

with the interference term $i_{k,l,m}$ reduced.

4) Use the $\bar{z}_{k,l,m}$ in (51) to find an improved $\{\hat{\alpha}_{k,l}, \hat{h}_{k,l}(mN)\}$.

5) Repeat steps 2–4 for all users and multipaths as necessary. The above procedure can be repeated until an AOA estimate within the desirable range is achieved.

WLANS

Similar technical challenges also arise for wireless location determination in indoor environments, where WLAN is currently the most widely deployed wireless network. WLAN standards such as IEEE802.11b, and more recently IEEE802.11g, have been widely adopted in offices, homes, hospitals, restaurants, and schools. WLAN connectivity has also become a standard feature for laptop computers and PDAs as well as the new generation of smart cellular phones. As such, there is an increasing interest in utilizing these networks for location purposes to help provide good coverage for indoor scenarios.

THE INDOOR ENVIRONMENT

LOCATION-BASED APPLICATIONS WILL

PLAY AN IMPORTANT ROLE IN FUTURE

WIRELESS MARKETS.

Yet, the indoor channel environment is challenging for a number of reasons.

• *Channel fading.* The channel variation as a function of position due to the scatterer-rich nature of indoor environments can be significant. In a scattererrich environment, the channel

can be considered correlated only over a distance of $\lambda/2$, where λ is the wavelength corresponding to the carrier frequency. At 2.4 GHz, which is the band of operation for IEEE802.11b and IEEE802.11g, $\lambda/2$ is less than 10 cm. In other words, a movement of about 10 cm in an indoor environment can result in significant change in the channel gain. *Path loss and shadow fading.* The distance between the access point (AP) and mobile users causes path loss in the signal strength. The path loss in a typical office area is proportional to $d^{-3.5}$, where *d* is the distance between the mobile user and the AP. In addition to the path loss, the shadow fading caused by walls further contributes to attenuation of the signal strength.

Interference. The 2.4-GHz band is an unlicensed band where devices such as Bluetooth, cordless phones, and even microwave ovens operate. The interference from these other active devices can limit the achievable location accuracy.

AMPOA ESTIMATION

Some of the older proposed location-aware systems for indoor environments [53]-[57] require specialized hardware such as ultrasound transmitters, camera, and infrared transmitters. But since the IEEE 802.11b and IEEE 802.11g MAC layer software provides the signal strength and the SNR, a software-level location technique could be developed for WLAN networks based on the AmpOA at different access points [58]–[66]. Specifically, when an IEEE802.11 wireless network operates in the infrastructure mode, there are several APs and many end users within the network. RF-based systems that use the signal strength for location purposes can monitor the received signal strength from different APs and use the obtained statistics to build a conditional probability distribution network to estimate the location of the mobile client. These schemes usually work in two phases: the first phase is the offline training and data gathering phase, and the second phase is the location determination phase using the online signal strength measurements. In the training phase, signal strength measurements are used to build an a priori probability distribution of the received signal strength at the mobile user from all APs. Assume there are N APs in the system and the radio map is created based on measurements from M user locations. The radio map model is described by [55]-[58]

 $p(A_i|x_j, y_j) \stackrel{\Delta}{=}$ the PDF of the received signal strength,

where

 A_i = received signal strength from the *i* th AP (x_j, y_j) = coordinates of the *j*th measurement point $i = 1, 2, ..., N, \quad j = 1, 2, ..., M.$

After constructing a Bayesian network, the online determination phase uses ML estimation to locate the mobile user. Thus, assume that the mobile user measures the received signal strength from all APs, as in

 $A'_i \stackrel{\Delta}{=}$ measured signal from the *i*th AP.



[FIG9] An illustration of the UCLA WLAN Location Simulator interface. The estimated location of a mobile user using different algorithms would be plotted for different realizations. Moreover, the estimated accuracy of the different methods would be shown by circles surrounding the mobile location.



[FIG10] An illustration of the UCLA Cellular Location Simulator interface. The estimated location of a mobile user using different algorithms would be plotted for different realizations. Moreover, the estimated accuracy of the different methods and the FCC accuracy requirements would be shown by circles surrounding the mobile locations. The design engine allows the user to place blocking objects in the simulator environment and to select the trajectory of the mobile user such that it experiences different situations, fadings, and shadowings. The various parameters that control the environment can be adjusted as well.

Then, using Bayes' rule, the probability of having the mobile user at location (x_j, y_j) given the received signal strengths from all APs is given by

$$\mathbf{A}' \stackrel{\Delta}{=} [A'_1, \dots, A'_N]$$
$$p(x_j, y_j | \mathbf{A}') = \frac{p(\mathbf{A}' | x_j, y_j) p(x_j, y_j)}{p(\mathbf{A}')}$$
$$= \frac{p(x_j, y_j) \prod_{i=1}^N p(A'_i | x_j, y_j)}{p(\mathbf{A}')}$$

where $\prod_{i=1}^{N} p(A'_i|x_j, y_j)$ is the approximation for the conditional probability density function of the received signal strength when the location of the mobile user is given. Thus, the location of the mobile user can be estimated as

$$(\hat{x}_m, \hat{y}_m) = \arg \max_{x_j, y_j} p(x_j, y_j | \mathbf{A}')$$

$$j = 1, 2, \dots, M.$$
(53)

TOA/AOA ESTIMATION

The TOA and AOA estimation techniques and the data fusion schemes presented in the previous sections can be used for indoor environments as well. However, the accuracy desired for indoor applications is higher than that required for outdoor environments. While an accuracy of 50 m is acceptable for many outdoor applications, for indoor applications an accuracy of few meters is desired. Therefore, the performance of the estimation algorithms should be boosted to meet the accuracy requirements. The following facts will improve the accuracy of location finding algorithms for indoor applications:

■ *Higher clocking rates*. The clocking rates of WLAN systems are higher than the ones used in cellular systems; this is due to the fact that the WLAN physical layers are intended for higher data rates and occupy a wider bandwidth than the physical layer of cellular systems. The higher clocking rate (and, equivalently, the higher sampling rate at the receiver) translates into higher accuracy in TOA measurements and into more accurate location estimates. Additionally, the bandwidth per channel used in 3G cellular networks is about 4 MHz, as opposed to the 11-MHz bandwidth in IEEE802.11b and 16-MHz bandwidth in IEEE802.11g. The higher bandwidth and clocking rate effectively provide a higher resolution in estimating the TOA of the signals.

• *Higher SNR*. WLAN networks operate at higher SNR than cellular networks. The higher SNR results in more accurate estimates for TOA and AOA.

• Oversampling at the receiver. The received signal can be oversampled to further increase the resolution of TOA estimation. Since the received SNR in WLAN networks is relatively high, an accurate TOA estimation after oversampling is possible. ■ *Slow-varying channel*. The channel variation in indoor environments is slower than in outdoor environments. This is due to the range of speeds present in indoor and outdoor environments. The slow-varying channel allows for longer coherent averaging periods at the receiver (the parameter *N* used in the derivations), which results in a higher effective SNR for TOA and AOA estimation.

■ *Power up scheme*. Due to the local nature of WLAN networks, the mobile user can be requested by the network to raise the level of transmitted power momentarily. This instant increase in the level of transmitted power will not degrade the network performance as significantly as it would for cellular networks.

For further information on indoor location algorithms, see [15] and [61]. Moreover, [62] describes localization algorithms in a cooperative sensor network setting.

SIMULATION ENVIRONMENT

To test several of the techniques and algorithms described in the previous section, a software simulator called the Wireless Location Simulator has been developed at the UCLA Adaptive Systems Laboratory [67]. A high degree of testability and flexibility, along with a user-friendly interface, are designed into the simulator. Selected snapshots of the Wireless Location Simulator are shown in Figures 9–12. The simulator consists of an interface and a location-finding engine. The location engine performs the following tasks:

Data fusion techniques: Different data fusion techniques are implemented using TOA, AOA, or a combination of both.

• *Channel modeling*: A multipath, multiuser channel environment is created that models path loss, shadowing, Rayleigh fading, and Doppler frequency effects.

Parameter estimation: TOA and AOA estimation algorithms are implemented as part of the location finding engine. Different variations of the algorithms are implemented for performance and comparison purposes.

Most of the algorithms used in the location-finding engine are generic in the sense that they could be used for both indoor and outdoor applications with minimum alteration. The location-finding engine and the capabilities listed below make the simulator usable for both networks.

• Configuring the physical layer for different wireless networks. Adjusting the network physical layer parameters enables the simulator to be used for different wireless networks. Among the programmable parameters are the spreading factor, packet size, training length, constellation type, modulation technique, carrier frequency, level of transmitted signal power, and the number of antennas at the transmitter or receiver.

• Configuring the mobile user conditions. The wireless channel models depend on the Doppler frequencies present in the environment. The Doppler frequency depends on the mobile speed and the carrier frequency of the system. The simulator accepts different speed and carrier frequencies as input and generates a Rayleigh fading channel with the U- shape Doppler spectrum. Moreover, the simulator allows the user to define the trajectory of the mobile, and then tracks the user on the defined trajectory.

• Configuring the environmental parameters and network geographical structure. One of the factors affecting the performance of the wireless location system is the environment type (e.g., bad urban, urban, suburban, or rural.) For example, in a bad urban area with many blocking objects and buildings, NLOS effects play an important role in the estimation accuracy. The simulator enables the user to place an arbitrary number of buildings of different sizes and shapes in the simulation environment. The simulator modifies the channel models to capture the effect of the buildings (shadow fading and NLOS effects).



[FIG11] Snapshot of the UCLA Location Simulator. The location estimation accuracy and the FCC requirements are shown by two circles in this lateral view generated by the 3-D simulation visualizer.



[FIG12] Snapshot of the UCLA Location Simulator. The location estimation accuracy and the FCC requirements are shown by two circles in this top view generated by the 3-D simulation visualizer.

Performance evaluation and monitoring tools. The simulator contains different tools to illustrate the results and accuracy of the location procedure, such as

1) comparing the estimated trajectory with the true trajectory

2) statistics of the location error, including the 67% and 95% outage values (the 67% and 95% outage values are of interest due to the FCC

with an interactive 3-D environment that shows the

movement of the mobile user on the predefined trajecto-

ry. It also shows the accuracy of the algorithm and com-

requirements)

3) a three-dimensional (3-D) simulator that creates a 3-D virtual reality environment. The 3-D viewer has different adjustable camera views from different angles. It provides the user

pares it with the FCC standard.

A LOCATION FINDING SYSTEM SHOULD BE ABLE TO SEAMLESSLY USE BOTH CELLULAR AND WLANS FOR LOCATION FINDING BY ROAMING BETWEEN THE NETWORKS.

SOME SIMULATION RESULTS

Some simulation results are shown in Figure 13(a) and (b) for a CDMA cellular network with the following parameters: CDMA chip rate of 4 MHz, a processing gain of 64, a 3-tap Rayleigh fading channel with path loss exponent of two, an antenna array of size four at the BS, and a Doppler frequency corresponding to a maximum speed of 30 mph. The location

> algorithms are simulated in a multi-user environment with a different number of active users. Figure 13(a) and (b) shows the resulting accuracy outage curves. An outage curve measures the probabilities that the loca-

tion estimation errors will be below certain values.

For example, in Figure 13(a), it is seen that for a receiver employing both TOA and AOA measurements, the location error is below 100 m 90% of the time. The FCC-mandated requirement for a network-based solution is for the location



[FIG13] Outage curves for location accuracy in outdoor environment (IC denotes interference cancellation method). (a) Assuming one user. (b) Assuming six users.

error to be below 100 m only 67% of the time. For a mobilebased solution, on the other hand, the FCC requirement mandates an outage probability of 67% for 50 m. Both of these requirements are met by the TOA/AOA fusion method presented earlier. Figure 13(b) repeats the simulation for the case of six total users.

CONCLUSION

Network-based wireless location poses several interesting problems from a signal processing perspective. The estimation algorithms must provide accurate parameter estimates under challenging conditions such as fast fading channels, low SNR, multipath effects, and multiuser interference. Small errors in estimation can lead to large errors in location. For example, in 3G CDMA, the chip duration is roughly 0.25 μ s. An error in TOA estimation of the order of $T_c/2$ can translate into 37.5 m in location error. Likewise, for a cell with a two-mile radius, an error of 1° in the AOA measurement can result in a location error of the order of 55 m. For this reason, the location estimation algorithms and the data fusion methods must exploit any available information about the environment (e.g., fading conditions, Doppler frequency, and network topology) to attain high accuracy. In addition, the resulting location searchers need to exhibit a certain degree of adaptation to changing conditions (e.g., mobile speed) so as to maintain reliable performance.

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