

Brief paper

A robust power and rate control method for state-delayed wireless networks[☆]

Ananth Subramanian^a, Ali H. Sayed^{b,*}

^a*Institute for Infocomm Research, A-STAR, Singapore*

^b*Electrical Engineering Department, University of California, Los Angeles, CA 90095, USA*

Received 22 July 2003; received in revised form 4 March 2005; accepted 2 May 2005

Abstract

A robust power and rate control algorithm is proposed for distributed wireless networks where the network dynamics is modelled as an uncertain discrete-time state-delayed system.

© 2005 Elsevier Ltd. All rights reserved.

Keywords: Robust estimation; Power control; Rate control; Wireless networks; State-delayed dynamics

1. Introduction

Power consumption is a limiting factor in the performance of wireless networks. This limitation is compounded by the fact that nodes in a wireless network need to cater to certain data rates, which in turn require the SNR level and, consequently, the power level to be above some values. There have been several power control algorithms proposed in the literature (e.g., Zander, 1992; Grandhi, Vijayan, & Goodman, 1994; Foschini & Miljanic, 1993; Chen, Bambos, & Pottie, 1994; Andersin, Rosberg, & Zander, 1996; Shoarinejad, Speyer, & Pottie, 2001; Leung, 1999, 2002; Subramanian & Sayed, 2004a). Most of the available solutions do not approach in a combined manner the tradeoff requirements of power, data rate, and congestion. In recent works (Subramanian, Khajehnouri, & Sayed, 2003; Subramanian &

Sayed, 2005), the authors proposed algorithms that allow for the joint control of rate and power in a network. The algorithms of Subramanian et al. (2003) and Subramanian and Sayed (2005), however, did not account for the presence of feedback delays, which arise from round trip delay propagation in a network. In this paper, we extend these earlier results to the case when there are delayed measurements due to round trip delays. From a system-theoretic perspective, the problem requires that we now deal with state-delayed models. As a result of the analysis, we will end up with a joint rate and power control algorithm that minimizes a bound on the error variance between the desired and actual signal-to-interference ratios (SIR).

Notation: For a column vector z , we write $\|z\|^2$ to denote its squared Euclidean norm. For a positive scalar x , we write \bar{x} to denote its dB value, i.e., $\bar{x} = 10 \log_{10} x$.

2. Power and rate control strategy

Following Subramanian and Sayed (2005), consider a wireless network with nodes organized into local clusters or cells with one node acting as the master node in each cell. Any node that wishes to communicate is allowed to do so only with the master node and using a time slot. Nodes communicating during the same time-slot in other cells

[☆] This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Ioannis Paschalidis under the direction of Editor Ian Petersen. This work was partially supported by NSF grants ECS-0401188 and CCR-0208573. A condensed version of this work was presented at the MTNS meeting, Leuven, Belgium, in July 2004.

* Corresponding author. Tel.: +310 267 2142; fax: +310 206 8495.

E-mail addresses: sananth@i2r.a-star.edu.sg (A. Subramanian), sayed@ee.ucla.edu (A.H. Sayed).

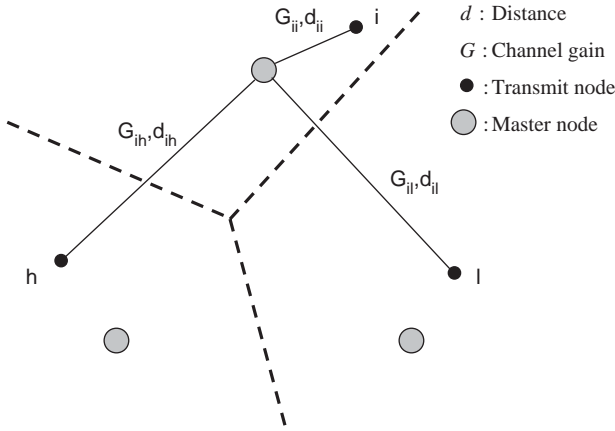


Fig. 1. A schematic representation with three cells, three master nodes, and active and interfering nodes. The active node is node i and the interfering nodes are nodes h and l .

cause interference in this cell. Fig. 1 shows a schematic representation with three cells, three master nodes, and active and interfering nodes.

The signal-to-interference-plus-noise-ratio (SIR) for node i at time k on an uplink channel is defined by

$$\gamma_i(k) = \frac{G_{ii}(k)p_i(k)}{\sum_{j \in \mathcal{A}} G_{ij}(k)p_j(k) + \sigma^2}, \quad (1)$$

where G_{ij} is the channel gain from the j th node to the intended master node of the i th cell, p_j is the transmitted power from the j th node, σ^2 is the power of the white Gaussian noise at the receiver of the master node, and \mathcal{A} is the set of all nodes interfering with node i .

Let $f_i(k)$ denote the flow rate at node i at time k . We shall initially assume that each node in the network employs the following flow-rate control algorithm (Kelly, 2000):

$$f_i(k+1) = f_i(k) + \mu[d(k) - c_1(k)f_i(k) - c_2 f_i(k-\tau)], \quad (2)$$

where $\mu > 0$ is a step-size parameter, and $c_1(k)$ and c_2 are measures of the amount of congestion in the network. Moreover, $d(k)$ controls the rate increase per iteration and τ is non-zero for any controller that incorporates round trip delay time. Subramanian and Sayed (2005) studied the case $c_2 = 0$ and $\tau = 0$.

Now Shannon's capacity formula suggests a plausible choice for the SIR level in order to achieve a rate value $f_i(k)$, namely, the SIR level should be at least at a value $\gamma'_i(k)$ that is related to $f_i(k)$ via¹

$$f_i(k) = \frac{1}{2} \log_2[1 + \gamma'_i(k)]. \quad (3)$$

Usually, during normal network operation, $\gamma'_i(k) \gg 1$ and, hence, $f_i(k)$ in (3) is proportional to $\log \gamma'_i(k)$. Substituting

¹ We may adjust Eq. (3) to include a buffer zone, namely, we could select $\gamma'_i(k)$ via $f_i(k) = (\rho/2) \log_2[1 + \gamma'_i(k)]$, for some scaling factor $\rho < 1$.

this fact into (2) we find that the desired SIR, in dB scale, would need to vary according to the rule

$$\bar{\gamma}'_i(k+1) = [1 - \mu c_1(k)]\bar{\gamma}'_i(k) - \mu c_2 \bar{\gamma}'_i(k-\tau) + \mu' d(k), \quad (4)$$

where $\mu' = 20\mu/\log_2(10)$ and $\bar{\gamma}_i(k) = 10 \log \gamma_i(k)$.

We shall initially assume that each node in the network adjusts its power according to the power control algorithm (in dB scale):

$$\bar{p}_i(k+1) = \bar{p}_i(k) + \alpha_i[\bar{\gamma}'_i(k) - \bar{\gamma}_i(k)], \quad (5)$$

where α_i is a step-size parameter that is allowed to vary from one node to another, and $\gamma_i(k)$ is the actual SIR that is achieved by $p_i(k)$ as given by (1). Now let

$$\beta_i(k) = \frac{G_{ii}(k)}{\sum_{j \in \mathcal{A}} G_{ij}(k)p_j(k) + \sigma^2}$$

denote the scaling factor that determines how $p_i(k)$ affects the achieved $\gamma_i(k)$ in (1), i.e.,

$$\gamma_i(k) = \beta_i(k)p_i(k)$$

or, equivalently, in dB scale,

$$\bar{\gamma}_i(k) = \bar{\beta}_i(k) + \bar{p}_i(k). \quad (6)$$

It is shown in Subramanian and Sayed (2005) that a random-walk model for $\bar{\beta}_i(k)$ can be derived of the form

$$\bar{\beta}_i(k+1) = \bar{\beta}_i(k) + n_i(k), \quad (7)$$

where $n_i(k) = 10 \log m_i(k) - 10 \log z_i(k)$ is a zero-mean disturbance of some variance σ_n^2 and is independent of $\bar{p}_i(k)$. This model is based on the assumption that nodes in the network do not jointly optimize their power levels in any centralized manner and only do so independently and in a distributed sense. Substituting this model for $\bar{\beta}_i(k)$ into (6), we find that the actual $\bar{\gamma}_i(k)$ varies according to the rule:

$$\bar{\gamma}_i(k+1) = (1 - \alpha_i)\bar{\gamma}_i(k) + \alpha_i \bar{\gamma}'_i(k) + n_i(k). \quad (8)$$

The proposed solution will not use σ_n^2 but rather a bound on it in order to account for uncertainties in the model for $\bar{\beta}_i(k)$. Our objective is to design the power control sequence $\{p_i(k)\}$ such that the actual SIR levels $\{\gamma_i(k)\}$ from (8) will tend to the desired SIR levels $\{\gamma'_i(k)\}$ from (4). To do so, we shall formulate a robust quadratic control problem as follows. First, we drop the node index i for simplicity of notation (it is to be understood that the resulting control mechanism is implemented at each node). Second, we introduce the two-dimensional state vector:

$$x_k = \begin{bmatrix} \bar{\gamma}(k) \\ \bar{\gamma}'(k) \end{bmatrix}.$$

Then combining (4) and (8) we arrive at the time-delayed state-space model:

$$x_{k+1} = \begin{bmatrix} 1 - \alpha & \alpha \\ 0 & 1 - \mu c_1(k) \end{bmatrix} x_k + \begin{bmatrix} 0 & 0 \\ 0 & -\mu c_2 \end{bmatrix} x_{k-\tau} + \begin{bmatrix} n(k) \\ \mu' d(k) \end{bmatrix},$$

or, more compactly,

$$x_{k+1} = A_k x_k + A_d x_{k-\tau} + w_k, \quad (9)$$

where the 2×2 coefficient matrices A_k and A_d are given by

$$A_k = \begin{bmatrix} 1 - \alpha & \alpha \\ 0 & 1 - \mu c_1(k) \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & 0 \\ 0 & -\mu c_2 \end{bmatrix} \quad (10)$$

and where w_k is a 2×1 zero-mean random vector with covariance matrix

$$Q = E w_k w_k^T = \begin{bmatrix} \sigma_n^2 & \\ & \mu'^2 \sigma_d^2 \end{bmatrix} \leq \rho_u I \quad (11)$$

assumed bounded by some known $\rho_u > 0$. In order to drive $\gamma(k)$ towards $\gamma'(k)$ we shall employ a control sequence u_k in (9) as follows:

$$x_{k+1} = A_k x_k + A_d x_{k-\tau} + B u_k + w_k \quad (12)$$

for some given 2×2 matrix B and 2×1 control sequence u_k to be determined. For example, let

$$B u_k = \begin{bmatrix} u_p(k) \\ u_f(k) \end{bmatrix}$$

denote the individual entries of $B u_k$ to be designed. Then the inclusion of the term $B u_k$ in (12) amounts to adding a control signal $u_p(k)$ to the power update (5), and a control signal $u_f(k)$ to the desired SIR update (4)—see Eq. (26) further ahead.

To proceed, we shall assume that we have access to output measurements that are related to the state vector as follows:

$$y_k = C x_k + v_k \quad (13)$$

for some known matrix C and where v_k denotes measurement noise with bounded covariance matrix R ,

$$R = E v_k v_k^T \leq \rho_v I$$

for some known $\rho_v > 0$. Usually, $C = I$ so that the entries of y_k correspond to noisy measurements of the actual and desired SIR levels, $\{\bar{\gamma}(k), \bar{\gamma}'(k)\}$. We now propose a control procedure that takes into account uncertainties that arise due to the lack of perfect knowledge about the network dynamics. For example, the congestion control parameters $c_1(k)$ and c_2 are usually not known exactly and have to be estimated; the estimation process introduces errors in the assumed state-space model. Let us model the uncertainty in $c_1(k)$ as

$$c_1(k) = \bar{c}_1 + g \delta(k) \bar{d}, \quad (14)$$

where $\delta(k)$ is a zero mean random noise with variance σ_δ^2 , g and \bar{d} are known scalars, and \bar{c}_1 is unknown but bounded as

$$c_{1,l} \leq \bar{c}_1 \leq c_{1,u} \quad (15)$$

for some known positive scalars $\{c_{1,l}, c_{1,u}\}$. In other words, we allow for both deterministic and stochastic uncertainties

in $c_1(k)$. In this way, the matrices A_k themselves in (10) are not known exactly but they are modelled as $A_k = \bar{A} + \delta A_k$ where

$$\bar{A} = \begin{bmatrix} 1 - \alpha & \alpha \\ 0 & 1 - \mu \bar{c} \end{bmatrix} \quad (16)$$

and

$$\delta A_k = g \delta(k) D, \quad (17)$$

where

$$D = \begin{pmatrix} 0 & 0 \\ 0 & -\mu \bar{d} \end{pmatrix}. \quad (18)$$

Likewise, let c_2 be bounded as $c_{2,l} \leq c_2 \leq c_{2,u}$. In this way, the matrix A_d in (10) is also not known exactly but is now modelled as belonging to a convex polytope. We shall design the control sequence $\{u_k\}$ as follows. First, we use the robust algorithm of Subramanian and Sayed (2004b) to estimate the state of perturbed state-space models as in (16)–(17). Then, the control sequence $\{u_k\}$ will be designed such that an upper bound on the following stochastic quadratic cost function is minimized (cf. (19)):

$$\mathcal{J} = E \left\{ \sum_{k=0}^{\infty} \|L x_k\|^2 \right\},$$

with $L = [1 \quad -1]$, and where E denotes the expectation operator. This choice of L results in

$$L x_k = \bar{\gamma}(k) - \bar{\gamma}'(k),$$

so that $\|L x_k\|^2$ is a measure of the difference between $\{\bar{\gamma}(k), \bar{\gamma}'(k)\}$. The choice of a quadratic cost function is largely dictated by its convenience and by its interpretation in terms of error variance minimization. Other cost functions may be chosen to address other performance criteria.

Now the resulting control will guarantee the following performance over all models $\{A_k + \delta A_k\}$. Let $\tilde{x}_k = x_k - \hat{x}_k$ denote the state estimation error. Then the construction will determine state estimates $\{\hat{x}_k\}$, and a control sequence $\{u_k\}$ as a function of these state estimates, such that

$$\mathcal{J} < v^2 E \left\{ \sum_{k=0}^{\infty} (\|w_k\|^2 + \|v_k\|^2) \right\} + b \quad (19)$$

for some constant $b > 0$ and for the smallest possible v^2 , and over all zero-mean noise sequences $\{w_k, v_k\}$ satisfying

$$E \left(\sum_{k=0}^{\infty} \|w_k\|^2 \right) < \infty, \quad E \left(\sum_{k=0}^{\infty} \|v_k\|^2 \right) < \infty. \quad (20)$$

The following is the main result assuming $B = I$.

2.1. A robust power and rate control algorithm

Let

$$A_l = \begin{bmatrix} 1 - \alpha & \alpha \\ 0 & 1 - \mu c_{1,l} \end{bmatrix}, \quad A_u = \begin{bmatrix} 1 - \alpha & \alpha \\ 0 & 1 - \mu c_{1,u} \end{bmatrix}$$

and

$$A_{d,l} = \begin{bmatrix} 0 & 0 \\ 0 & 1 - \mu c_{2,l} \end{bmatrix}, \quad A_{d,u} = \begin{bmatrix} 0 & 0 \\ 0 & 1 - \mu c_{2,u} \end{bmatrix}.$$

Given a 1×2 vector $L = [1 \ -1]$, the following is a robust power and rate-flow control strategy:

1. Introduce a 2×2 matrix A_f and a 2×1 vector B_f to be determined. Let

$$\bar{A}_l = \begin{bmatrix} A_l & 0 \\ A_l - A_f - B_f C & A_f \end{bmatrix},$$

$$\bar{A}_u = \begin{bmatrix} A_u & 0 \\ A_u - A_f - B_f C & A_f \end{bmatrix}$$

and define

$$\bar{A}_{d,l} = \begin{bmatrix} A_{d,l} & 0 \\ A_{d,l} & 0 \end{bmatrix}, \quad \bar{A}_{d,u} = \begin{bmatrix} A_{d,u} & 0 \\ A_{d,u} & 0 \end{bmatrix}.$$

The quantities A_f and B_f are determined as follows (Subramanian & Sayed, 2004b). Given a scalar $0 < \alpha < 1$, we solve the following convex optimization problem over the variables $\{P = \text{diag}\{P_1, P_2\}, R, A_f, B_f\}$ (all of dimension 2×2):

$$\min \text{Tr}[\rho_u(P_1 + P_2) + \rho_v B_f^T P_2 B_f] \quad (21)$$

subject to the conditions

$$\left(\begin{array}{cc|cc} \bar{H} & -\bar{A}_m^T P \bar{A}_{d,m} & \bar{A}_m^T P & 0 \\ -\bar{A}_{d,m}^T P \bar{A}_m & R - \bar{A}_{d,m}^T P \bar{A}_{d,m} & 0 & 0 \\ \hline P \bar{A}_m & 0 & P & 0 \\ 0 & 0 & 0 & I \end{array} \right) > \alpha I \quad (22)$$

for $m = l, u$, and with $P > I, R > I$,

$$\bar{H} \triangleq P - R - \sigma_\delta^2 \bar{D}^T \bar{G}^T P \bar{G} \bar{D}$$

and

$$\bar{G} = \begin{pmatrix} gI & 0 \\ gI & 0 \end{pmatrix}, \quad \bar{D} = \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix}. \quad (23)$$

The A_f and B_f found in this manner are such that they minimize a bound on the covariance of \tilde{x}_k in the absence of control (Subramanian & Sayed, 2004b). In addition, the construction below ensures asymptotic stability in the presence of a control signal (the argument is similar to that in Appendix D of Subramanian & Sayed, 2005).

2. Using the just found $\{A_f, B_f\}$, define

$$\check{A}_l = \begin{bmatrix} A_l - K_c & K_c \\ A_l - A_f - B_f C & A_f \end{bmatrix},$$

$$\check{A}_u = \begin{bmatrix} A_u - K_c & K_c \\ A_u - A_f - B_f C & A_f \end{bmatrix},$$

$$\check{B} = \begin{bmatrix} I & 0 \\ I & -B_f \end{bmatrix}$$

for some 2×2 matrix K_c to be determined. Determine K_c, X, Y , and the smallest positive v^2 that guarantee

$$\left(\begin{array}{ccc} \check{H}_m & \check{A}_m^T X \bar{A}_{d,m} & -\check{A}_m^T X \check{B} \\ -\bar{A}_{d,m}^T X \check{A}_m & Y - \bar{A}_{d,m}^T X \bar{A}_{d,m} & -\bar{A}_{d,m}^T X \check{B} \\ -\check{B}^T X \check{A}_m & -\check{B}^T X \bar{A}_{d,m} & v^2 I - \check{B}^T X \check{B} \end{array} \right) > 0, \quad (24)$$

where

$$\check{H}_m = X - Y - \check{A}_m^T X \check{A}_m - \check{L}^T \check{L} - \sigma_\delta^2 \bar{D}^T \bar{G}^T X \bar{G} \bar{D}$$

for $m = l, u$ and

$$\check{L} = [L \ 0].$$

Then set

$$u_k = -K_c \hat{x}_k,$$

$$\hat{x}_{k+1} = A_f \hat{x}_k + B_f y_k + u_k. \quad (25)$$

3. Partition u_k as

$$u_k = \begin{bmatrix} u_p(k) \\ u_f(k) \end{bmatrix}$$

and update the rate flow and the power at the relevant node as follows (compare with (2) and (5)). Let $\kappa = (\log_2(10))/20$. Then

$$\bar{\gamma}'_i(k) = f_i(k)/\kappa,$$

$$\bar{p}_i(k+1) = \bar{p}_i(k) + \alpha_i [\bar{\gamma}'_i(k) - \bar{\gamma}_i(k)] + u_p(k),$$

$$f_i(k+1) = f_i(k) + \mu [d(k) - c_1(k) f_i(k) - c_2 f_i(k - \tau)] + \kappa u_f(k). \quad (26)$$

3. Simulations

To illustrate the performance of the proposed algorithm, we simulate the model adopted in Subramanian et al. (2003) and Subramanian and Sayed (2005) and summarized as follows. The space is divided into virtual geographical cells, each containing many nodes with one node acting as a master node. A frequency slot is allocated to each node that wishes to communicate with the master node in a cell. We allow for frequency reuse across cells in a manner similar to that in mobile cellular systems. The nodes communicating in the same frequency slot in other cells cause interference with this cell and this interference is measured in terms of the SIR. The channel G_{ii} is assumed to have a lognormal distribution, i.e.,

$$G_{ii} = S_0 d_{ii}^{-\beta} 10^{\alpha/10}, \quad (27)$$

where S_0 is a function of the carrier frequency, β is the path loss exponent (PLE), and d_{ii} is the distance of the master node from the node. The value of β depends on the physical environment and changes between 2 and 6 (usually 4), while α is a zero mean Gaussian random variable with variance σ_α^2 ,

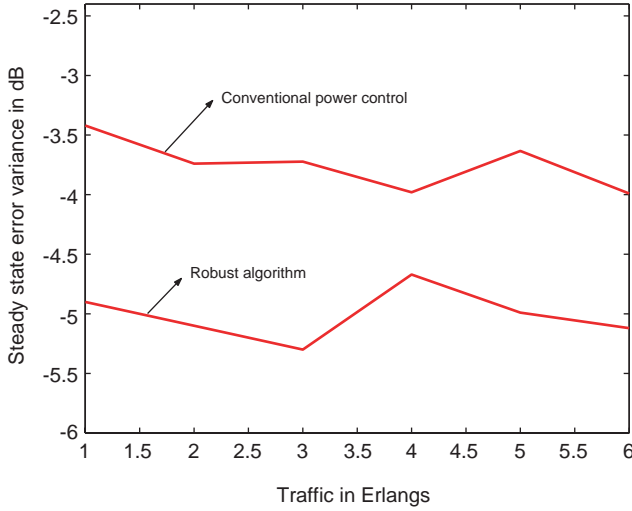


Fig. 2. Steady-state variance $E[\gamma(k) - \gamma'(k)]^2$ in SIR tracking using a conventional power control algorithm (5) and the proposed joint rate and control algorithm (26). [Erlangs is a unit that measures the ratio of the arrival rate of nodes to the departure rate of nodes after transmission.]

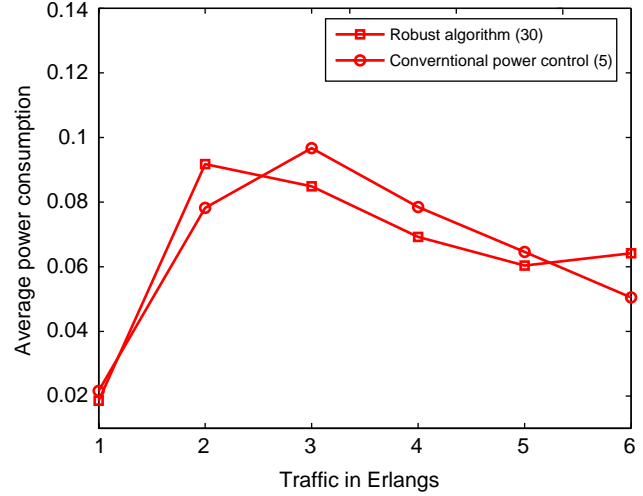


Fig. 4. Power consumption using a standard power control algorithm (5) and the joint rate and power control algorithm (26).

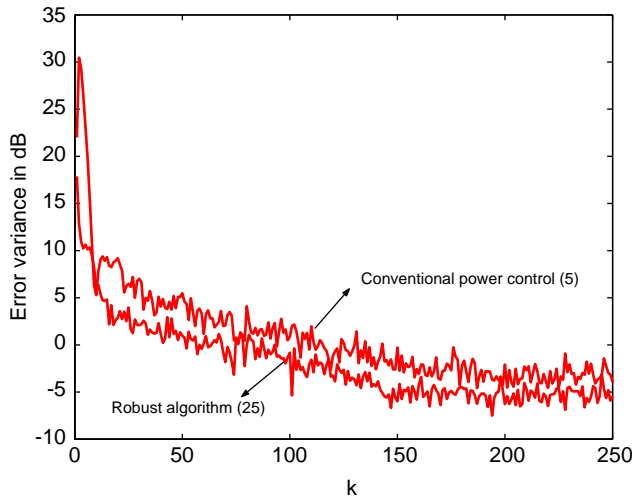


Fig. 3. Error variance curves as a function of time in SIR tracking using the conventional power control algorithm (5) and the proposed robust algorithm (26). The curves are obtained by averaging over 350 experiments.

which usually ranges between 6 and 12. We assume that the transmission power of each node at every instant satisfies $P_{\min} \leq p_i(k) \leq P_{\max}$.

Fig. 2 illustrates the performance of the algorithm (26) in comparison to the algorithm (5) from Foschini and Miljanic (1993) using a fixed $\alpha = 0.2$ for all nodes. Fig. 3 shows the error variance curves as a function of time for the power control strategy with and without the additive control term. It is seen that the robust algorithm (26) leads to smaller error variance in steady state. Fig. 4 shows that there is no significant difference in power consumption between the conventional power control algorithm and the proposed

robust algorithm. Fig. 4 shows the mean power $E[p_i(k)]$ consumed by a node averaged over 350 experiments. We assume $P_{\min} = 0$ and $P_{\max} = 1$.

Appendix A. Properties of the robust filter

Consider the filter (25) without the control signal u_k , namely

$$\hat{x}_{k+1} = A_f \hat{x}_k + B_f y_k,$$

with A_f and B_f found by solving (21). Define

$$\eta_k \triangleq \begin{pmatrix} x_k \\ \tilde{x}_k \end{pmatrix}. \quad (28)$$

Then combining (9) and (28) gives

$$\begin{aligned} \eta_{k+1} = & \left\{ \begin{pmatrix} \bar{A} & 0 \\ \bar{A} - A_f - B_f C & A_f \end{pmatrix} + \begin{pmatrix} g \\ g \end{pmatrix} \delta(k)(D \ 0) \right\} \eta_k \\ & + \begin{pmatrix} A_d & 0 \\ A_d & 0 \end{pmatrix} \eta_{k-\tau} \\ & + \begin{pmatrix} I & 0 \\ I & -B_f \end{pmatrix} \begin{pmatrix} w_k \\ v_k \end{pmatrix}, \end{aligned} \quad (29)$$

where \bar{A} and A_d belong to the following convex polyhedral domains:

$$\begin{aligned} \bar{A} &= \varepsilon A_l + (1 - \varepsilon) A_u, \quad 0 \leq \varepsilon \leq 1, \\ A_d &= \lambda A_{d,l} + (1 - \lambda) A_{d,u}, \quad 0 \leq \lambda \leq 1. \end{aligned} \quad (30)$$

The following properties are special cases of Theorems 1 and 3 in Subramanian and Sayed (2004b).

Theorem A.1 (Asymptotic stability). *Given a positive scalar $\alpha < 1$, let the matrices $\{A_f, B_f, P > I, R > I\}$ be chosen to satisfy (22). Then the process $\{\eta_k\}$ in (29) is asymptotically*

stable in the absence of the noises $\{w_k, v_k\}$ and for all uncertainties in $\{c_1(k), c_2\}$.

Theorem A.2 (Exponential stability). *The vector process $\phi_k = [\eta_k, \eta_{k-1}, \dots, \eta_{k-\tau}]^T$ is exponentially stable in the presence of measurement and process noises $\{w_k, v_k\}$ for all uncertainties in $\{c_1(k), c_2\}$. Specifically, there exists $\theta > 1$ such that*

$$E\|\phi_k\|^2 < \frac{1}{\lambda_{\min}(\Gamma)} \left\{ \frac{V(\phi_0)}{\theta^k} + U \frac{\theta}{\theta-1} \left(1 - \frac{1}{\theta^{k+1}} \right) \right\}, \quad (31)$$

where

$$U \triangleq \rho_u \text{Tr}(P_1 + P_2) + \rho_v \text{Tr}(B_f^T P_2 B_f) \quad (32)$$

and

$$\Gamma \triangleq \begin{pmatrix} P & & & \\ & R & & \\ & & \ddots & \\ & & & R \end{pmatrix} > I. \quad (33)$$

Appendix B. Robust performance

We now verify that the proposed algorithm (21)–(26) ensures a robust performance level of v^2 , as in (19). Define

$$o_k \triangleq \begin{pmatrix} w_k \\ v_k \end{pmatrix} \quad (34)$$

and let

$$V(\phi_k) = \eta_k^T X \eta_k + \sum_{i=k-\tau}^{k-1} \eta_i^T Y \eta_i \quad (35)$$

for some $X > 0$ and $Y > 0$ to be determined in order to satisfy the inequality

$$EV(\phi_{k+1}) - EV(\phi_k) - v^2 E \times (\|w_k\|^2 + \|v_k\|^2) + E \tilde{z}_k^T \tilde{z}_k < 0, \quad (36)$$

where $\tilde{z}_k = \check{L} \eta_k = \bar{y}(k) - \bar{y}'(k)$. We will show that, for a given A_f and B_f , if X and Y are determined such that the above inequality is satisfied, then (19) is guaranteed. Indeed, if we sum inequality (36) over k , and if we use the fact that the system is asymptotically stable (which is shown subsequently), we would get

$$E \left\{ \sum_{k=0}^{\infty} |\bar{y}(k) - \bar{y}'(k)|^2 \right\} < EV(\eta_0) + v^2 E \left\{ \sum_{k=0}^{\infty} \|w_k\|^2 + \|v_k\|^2 \right\} \quad (37)$$

as desired. Now assume a control structure of the form (26), i.e.,

$$\hat{x}_{k+1} = A_f \hat{x}_k + B_f y_k + u_k, \quad u_k = -K_c \hat{x}_k \quad (38)$$

for some given $\{A_f, B_f\}$ and unknown K_c . Combining this equation with (12), i.e.,

$$x_{k+1} = (A_k + \delta A_k)x_k + A_d x_{k-\tau} + u_k + w_k,$$

we can see that η_k satisfies the state-space model:

$$\eta_{k+1} = (\check{A} + \delta \check{A}_k) \eta_k + \bar{A}_d \eta_{k-\tau} + \check{B} o_k, \quad (39)$$

where

$$\check{A} = \begin{pmatrix} \bar{A} - K_c & K_c \\ \bar{A} - A_f - B_f C & A_f \end{pmatrix}, \quad \bar{A}_d = \begin{pmatrix} A_d & 0 \\ A_d & 0 \end{pmatrix}, \quad (40)$$

$$\check{B} = \begin{pmatrix} I & 0 \\ I & -B_f \end{pmatrix},$$

with

$$\delta \check{A}_k = \begin{pmatrix} g \\ g \end{pmatrix} \delta(k) (D \quad 0) \quad (41)$$

and where \bar{A} and A_d take values in the convex polyhedral domains (30) and \check{A} and \bar{A}_d belong to the following convex polyhedral domains:

$$\check{A} = \varepsilon \check{A}_l + (1 - \varepsilon) \check{A}_u, \quad (42)$$

$$\bar{A}_d = \lambda \bar{A}_{d,l} + (1 - \lambda) \bar{A}_{d,u}.$$

Using (39) and expanding (36) gives

$$E \{ \eta_k^T \check{A}^T X \check{A} \eta_k - \eta_k^T X \eta_k + \sigma_\delta^2 \eta_k^T \bar{D}^T \bar{G}^T X \bar{G} \bar{D} \eta_k + \eta_k^T \check{A}^T X \check{B} o_k + o_k^T \check{B}^T X \check{A} \eta_k + \eta_k^T Y \eta_k + \eta_k^T \check{A}^T X \bar{A}_d \eta_{k-\tau} + \eta_{k-\tau}^T \bar{A}_d^T X \check{A} \eta_k + \eta_{k-\tau}^T \bar{A}_d^T X \bar{A}_d \eta_{k-\tau} - \eta_{k-\tau}^T Y \eta_{k-\tau} + \eta_{k-\tau}^T \bar{A}_d^T X \check{B} o_k + o_k^T \check{B}^T X \bar{A}_d \eta_{k-\tau} - v^2 o_k^T o_k + o_k^T \check{B}^T X \check{B} o_k + \eta_k^T \check{L}^T \check{L} \eta_k \} < 0. \quad (43)$$

With \check{A} and \bar{A}_d taking values in the convex domains (42), condition (43) is satisfied if we require

$$\begin{pmatrix} \check{H}_m & -\check{A}_m^T X \bar{A}_{d,m} & -\check{A}_m^T X \check{B} \\ -\bar{A}_{d,m}^T X \check{A}_m & Y - \bar{A}_{d,m}^T X \bar{A}_{d,m} & -\bar{A}_{d,m}^T X \check{B} \\ -\check{B}^T X \check{A}_m & -\check{B}^T X \bar{A}_{d,m} & v^2 I - \check{B}^T X \check{B} \end{pmatrix} > 0 \quad (44)$$

for $m=l, u$ and for some $K_c, v^2, X > 0$ and $Y > 0$, as desired. Inequality (44) also implies that the system is asymptotically stable. To see this, first note from (20) that $E\|w_k\|^2 \rightarrow 0$ and $E\|v_k\|^2 \rightarrow 0$. Assuming zero mean noises, it follows that $w_k \rightarrow 0$ and $v_k \rightarrow 0$ w.p.1. Therefore, it is sufficient for our purposes to show that η_k is asymptotically stable in the absence of noise. Thus let $o_k = 0$ and observe that

$$E[V(\phi_{k+1}) | \phi_k, \dots, \phi_0] - V(\phi_k) = -(\eta_k^T \quad \eta_{k-\tau}^T) \begin{pmatrix} \check{H} & -\check{A}^T X \bar{A}_d \\ -\bar{A}_d^T X \check{A} & Y - \bar{A}_d^T X \bar{A}_d \end{pmatrix} \times \begin{pmatrix} \eta_k \\ \eta_{k-\tau} \end{pmatrix}, \quad (45)$$

where

$$\tilde{H} = X - Y - \tilde{A}^T X \tilde{A} - \sigma_\delta^2 \tilde{D}^T \tilde{G}^T X \tilde{G} \tilde{D}.$$

The centre matrix in (45) is positive definite whenever (44) holds. Therefore, condition (44) guarantees

$$E[V(\phi_{k+1})|\phi_k, \dots, \phi_0] - V(\phi_k) < 0, \quad (46)$$

which in turn implies asymptotic stability of the process ϕ_k (Subramanian & Sayed, 2004b; Kushner, 1967).

Appendix C. Optimization

We now show how to determine K_c , X and the smallest v^2 in step 2 of the robust algorithm in order to guarantee (44). We shall restrict X to a block diagonal structure as

$$X = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix}, \quad X_1 > 0, \quad X_2 > 0 \quad (47)$$

and define

$$Q = K_c X_2, \quad Y = \begin{pmatrix} Y_1 & Y_3 \\ Y_3^T & Y_2 \end{pmatrix}. \quad (48)$$

Now, through a Schur complementation argument, condition (37) is satisfied if, for any given A_f and B_f , there exist positive definite matrices $\{X_1, X_2\}$ and a matrix Q that satisfy

$$\begin{pmatrix} \hat{S} & -Y_3 & 0 & 0 & 0 & 0 & A^T X_1 + Q^T & \hat{J} & L^T \\ -Y_3^T & X_2 - Y_2 & 0 & 0 & 0 & 0 & Q^T & A_f^T X_2 & 0 \\ 0 & 0 & S' & Y_3 & 0 & 0 & A_d^T X_1 & A_d^T X_2 & 0 \\ 0 & 0 & Y_3^T & Y_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma^2 I & 0 & B_1^T X_1 & B_1^T X_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma^2 I & 0 & -B_f^T X_2 & 0 \\ X_1 A + Q & Q & X_1 A_d & 0 & X_1 B_1 & 0 & X_1 & 0 & 0 \\ \hat{J}^T & X_2^T A_f & X_2 A_d & 0 & X_2 B_1 & -X_2^T B_f & 0 & X_2 & 0 \\ L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{pmatrix} > 0, \quad (49)$$

where

$$\hat{S} \triangleq X_1 - \sigma_\delta^2 g D^T (X_1 + X_2) D g - Y_1,$$

$$\hat{J} \triangleq -C^T B_f^T X_2 - A_f^T X_1 + A^T X_2$$

and

$$S' \triangleq Y_1 - A_d^T (X_1 + X_2) A_d. \quad (50)$$

Finding the $\{X_1, X_2, Y_1, Y_2, Y_3, Q\}$ that solve the above inequality for the smallest v^2 is a convex optimization problem. Once $\{X_2, Q\}$ have been determined, K_c is obtained from $K_c = Q X_2^{-1}$.

References

Andersin, M., Rosberg, Z., & Zander, J. (1996). Gradual removals in cellular PCS with constrained power control and noise. *Wireless Networks*, 2(1), 27–43.

Chen, S. C., Bambos, N., & Pottie, G. J. (1994). Admission control schemes for wireless communication networks with adjustable transmitted powers. *Proceedings of the IEEE INFOCOM*, Toronto, Canada, June 1994, pp. 21–28.

Foschini, G. J., & Miljanic, Z. (1993). A simple distributed autonomous power control algorithm and its convergence. *IEEE Transactions on Vehicular Technology*, 42(4), 641–646.

Grandhi, S. A., Vijayan, R., & Goodman, D. J. (1994). Distributed power control in cellular radio systems. *IEEE Transactions on Communications*, 42(2), 226–228.

Kelly, F. P. (2000). Models for a self managed internet. *Philosophical Transactions of the Royal Society*, A358, 2335–2348.

Kushner, H. (1967). *Stochastic stability and control*. New York: Academic Press.

Leung, K. (1999). A Kalman-filter method for power control in broadband wireless networks. *Proceedings of IEEE INFOCOM*, vol. 2, New York, March 1999, pp. 948–956.

Leung, K. (2002). Power control by interference predictions for wireless packet networks. *IEEE Transactions on Wireless Communications*, 1(2), 256–265.

Shoarinejad, K., Speyer, J. L., & Pottie, G. J. (2001). A distributed scheme for integrated predictive dynamic channel and power allocation in cellular radio networks. *Proceedings of the GLOBECOM*, San Antonio, TX, pp. 3623–3627.

Subramanian, A., Khajehnouri, N., & Sayed, A. H. (2003). A minimum variance power control algorithm for wireless networks. *Proceedings of the VTC*, Orlando, FL, October 2003, pp. 1468–1472.

Subramanian, A., & Sayed, A. H. (2004a). A power and rate control algorithm for wireless networks with state-delayed dynamics. *Proceedings of the MTNS*, Leuven, Belgium.

Subramanian, A., & Sayed, A. H. (2004b). Multi-objective filter design for uncertain stochastic time-delay systems. *IEEE Transactions on Automatic Control*, 49(1), 149–154.

Subramanian, A., & Sayed, A. H. (2005). Joint rate and power control algorithms for wireless networks. *IEEE Transactions on Signal Processing*, to appear.

Zander, J. (1992). Distributed cochannel interference control in cellular radio systems. *IEEE Transactions on Vehicular Technology*, 41, 305–311.



Ananth Subramanian received the Ph.D. degree in electrical engineering in 2004 from UCLA, Los Angeles, CA. He is currently Associate Scientist at Institute for Infocomm Research, A-STAR, Singapore. He obtained his Bachelors in Electrical and Electronics Engineering and Masters in Mathematics in 1997 from BITS, Pilani. He obtained his Masters in Electrical Engineering from the Indian Institute of Science in 1999 where he was awarded the Gold Medal for being the best overall graduate student. He has held

the position of Teaching Fellow at UCLA and was awarded the Henry Samueli Excellence in Teaching Award in 2002. He was also awarded the UCLA Chancellors Fellowship in 2003. He is currently a member of IEEE. His areas of interests include statistical signal processing, wireless sensor and ad-hoc networks, signal processing for communications, and signal processing for sensor and wireless networks.



Ali H. Sayed received the Ph.D. degree in electrical engineering in 1992 from Stanford University, Stanford, CA. He is Professor and Chairman of electrical engineering at the University of California, Los Angeles. He is also the Principal Investigator of the UCLA Adaptive Systems Laboratory (www.ee.ucla.edu/asl). He is the author of the textbook 'Fundamentals of Adaptive Filtering' (New York: Wiley, 2003), is co-author of the research monograph 'Indefinite Quadratic

Estimation and Control' (Philadelphia, PA: SIAM, 1999) and of the graduate-level textbook 'Linear Estimation' (Englewood Cliffs, NJ: Prentice-Hall, 2000). He is also co-editor of the volume 'Fast Reliable Algorithms for Matrices with Structure' (Philadelphia, PA: SIAM, 1999).

He has served on the editorial boards of the 'SIAM Journal on Matrix Analysis and its Applications', the 'International Journal of Adaptive Control and Signal Processing', and the European 'Signal Processing Journal'. He has contributed several articles to engineering and mathematical encyclopedias and handbooks and has served on the program committees of several international meetings. He has also consulted with industry in the areas of adaptive filtering, adaptive equalization, and echo cancellation. His research interests span several areas, including adaptive and statistical signal processing, filtering and estimation theories, signal processing for communications, interplays between signal processing and control methodologies, system theory, and fast algorithms for large-scale problems. He has published widely in these areas. Dr. Sayed is recipient of the 1996 IEEE Donald G. Fink Award, a 2002 Best Paper Award from the IEEE Signal Processing Society in the area of Signal Processing Theory and Methods, and co-author of two Best Student Paper awards at international meetings. He is also recipient of the 2003 Kuwait Prize in Basic Sciences and of the 2005 Terman Award. He is a Fellow of IEEE and a member of the technical committees on signal processing theory and methods (SPTM) and on signal processing for communications (SPCOM), both of the IEEE Signal Processing Society. He is a member of the editorial board of the IEEE SIGNAL PROCESSING MAGAZINE. He has also served twice as Associate Editor of the IEEE TRANSACTIONS ON SIGNAL PROCESSING, of which he is now serving as Editor-in-Chief.