A Robust Receiver for Multi-User Uplink DS-CDMA

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Abstract— **This paper presents a robust receiver for uplink direct-sequence code-division multiple access (DS-CDMA) systems. The receiver uses low order auto-regressive models to approximate the multi-path fading channel taps, and a post correlation-based uncertain model for estimation purposes.**

keywords: RAKE receiver, Kalman filter, robust filter, multi-path fading, multiuser detection.

I. INTRODUCTION

Channel estimation in Code Division Multiple Access (CDMA) systems has a significant impact on the overall performance of the receiver [1]-[4]. The main challenge is to estimate and track the channel reliably in the presence of fast-varying multipath signals and large number of interfering users.

There have been many works in the literature (e.g., [5]–[10]) that propose the use of Kalman and extended Kalman filters in communication receivers and specially for joint estimation of the PN code delay and the channel. For example, a robust Kalman filter algorithm was proposed in [10], but the approach does not apply to current CDMA standards where all users transmit the training and information symbols simultaneously. Estimation procedures for CDMA systems need to tackle not only the multi-user interfering scenario where users operate at a Signal to Interference Noise Ratio (SINR) of less than 0 dB, but also cater to current standards (WCDMA and cdma2000) where the pilot channel is sent in parallel with the data and control channels. In this regard, we propose a receiver structure that is based on post-correlation (symbol rate) processing and state-space estimation for channel tracking.

II. CHANNEL MODEL

Consider a base-station receiving signals from N active users within a cell. Without loss of generality, we assume the base-station uses a single antenna receiver, even though the results presented here can be generalized to any receiver dimension. The receiver observes a linear combination of all transmitted data sequences by all active users, each distorted by ISI, under white Gaussian noise. Assuming the maximum number of channel taps to be L, the received signal at time n is

$$y(n) = \sum_{i=1}^{N} \sum_{k=1}^{L} c_{i,k}(n) s_i(n-k) + v(n) \tag{1}$$

where $s_i(n)$ is the transmitted sequence by the ith user and $c_{i,k}(n)$ is the kth tap of the channel from the ith user to the base station at time n. Here time is defined in terms of the chip rate.[1] The multipath channel propagation model is shown in Figure 1.

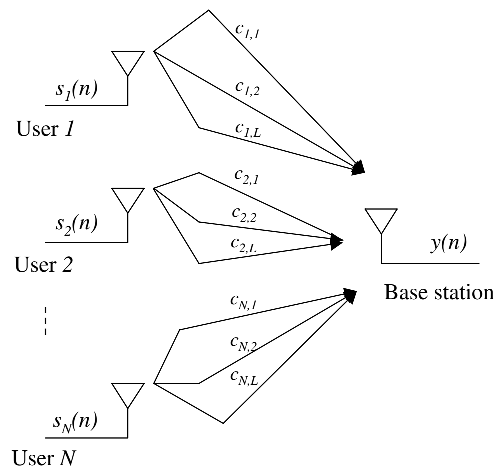

Fig. 1. The channel model

Equation (1) can be rewritten in vector form. Collecting the channel taps for user i into a column vector $c_{i,n}$:

$$c_{i,n} = \text{col}\{c_{i,1}(n) \;\; c_{i,2}(n) \;\; \cdots \;\; c_{i,L}(n)\} \tag{2}$$

and defining a vector of length NL containing all channel taps from all users at time n:

$$h_n = \text{col}\{c_{1,n} \;\; c_{2,n} \;\; \cdots \;\; c_{N,n}\} \tag{3}$$

we can rewrite (1) as

$$y(n) = s_n h_n + v(n) \tag{4}$$

where the NL row data vector s_n contains the transmitted data by all users. The goal is to track the vector h_n, estimate the channel taps, and recover the data in s_n.

This work was partially supported by NSF grants ECS-9820765 and CCR-0208573.

[1] We use n to refer to time instants relative to the chip rate. On the other hand, we shall use m to refer to time instants relative to the symbol rate.

A. Autocorrelation Model

We use the model in [12] where all channel taps are assumed independent and the variability of the wireless channel over time is reflected in the autocorrelation function of a complex Gaussian process. It is shown in [13] that the theoretical power spectral density associated with either the in-phase or quadrature portion of each channel tap has the well-known U-shaped bandlimited form :

$$S(f) = \begin{cases} \frac{1}{\pi f_d \sqrt{1-(\frac{f}{f_d})^2}} & |f| \leq f_d \\ 0 & \text{elsewhere} \end{cases} \tag{5}$$

where f_d is the maximum Doppler frequency. The corresponding normalized discrete-time autocorrelation of each tap is $R[n] = J_0(2\pi f_m |n|)$ where $J_0(.)$ is the zeroth-order Bessel function of first kind and $f_m = f_d T$ is the Doppler frequency normalized by the sampling rate $1/T$ [13].

B. State-Space Model

The time variation of the vector process $\{h_m\}$ at symbol rate can be approximated by the following AR process of order q [16],

$$h_m = \sum_{l=1}^{q} A(l) h_{m-l} + G_0 w_m \tag{6}$$

where w_n is a zero-mean i.i.d circular complex Gaussian vector process. Assuming Gaussian wide-sense stationary uncorrelated scattering fading (WSSUS), the matrices $A(l), l = 1, \cdots, q$, and G_0 turn out to be diagonal. For the selection of their entries, various criteria of optimality can be adopted, such as requiring the AR model of order q in (6) to provide a "best-fit" to the real channel auto correlation function of (5). In this paper we adopt the AR model presented in [15]. This model has the attractive property that its sampled auto correlation function perfectly matches the desired sampled auto correlation function of (5) up to lag q. This is achieved by solving a set of q Yule-Walker equations [15].

The multichannel AR model (6) can be rewritten in state-space form as :

$$x_m = F x_{m-1} + G w_m \tag{7}$$

where

$$x_m = \text{col}\{h_m \;\; h_{m-1} \;\; \cdots \;\; h_{m-q+1}\}$$

$$F = \begin{bmatrix} A(1) & A(2) & \ldots & A(q) \\ I_{(q-1)NL} & & & 0_{(q-1)NL \times NL} \end{bmatrix}$$

and

$$G = \begin{bmatrix} G_0 \\ 0_{(q-1)NL \times NL} \end{bmatrix} \tag{8}$$

Note that this model is independent of the modulation scheme and depends only on the Doppler frequency and the sampling rate at the receiver. The measurement equation is derived for uplink CDMA in Sec. III.

C. CDMA Uplink Signals

In this paper we use a simplified model for the cdma2000 physical layer where only the pilot and traffic channels are modelled. However, the results can be generalized for any number of traffic channels and any other CDMA standard, e.g., WCDMA. The base-band equivalent of the transmitted signal from mobile user i can be written at chip rate as

$$s_i = (pW_p + j d_i W_d) PN_i \tag{9}$$

where p and d_i are the pilot and data symbols, respectively, W_p and W_d are the orthogonal Walsh-codes for the pilot and data channels, PN_i is the effective PN-sequence used by the ith user:

$$PN_i = PN_{i,L}(PN_I + jPN_Q) \tag{10}$$

where $PN_{i,L}$ is the long PN-code for user i and is different for each user and PN_I and PN_Q are the in-phase and quadrature PN-codes intended to differentiate among cells and are the same within each cell. The pilot portion of the transmitted signal, i.e., $pW_p PN_{i,L}(PN_I + jPN_Q)$ will be used for training purpose in the Kalman estimation step, since it is known at the base station.

III. RECEIVER STRUCTURE

Previous structures [4]–[9] for data recovery use the received signal $y(n)$ directly. For CDMA systems, this will imply using the received CDMA chips to form the measurement equation. This chip-rate approach has some drawbacks :

- The Signal-to-Interference-Noise-Ratio (SINR) at the chip level can be as low as -10 dB in a multi-user environment, which makes it difficult to track channels in such noisy environments.

- The training sequence at the chip rate (pilot portion of s_i) is corrupted by unknown traffic channels (data portion of s_i and of other users) as the different traffic channels are transmitted in parallel, as shown by equation (9), in current CDMA standards (cdma2000 and WCDMA).

To overcome these difficulties, we propose a post-correlation structure to derive the measurement equation. In this architecture, the received pilot symbols (rather than the received CDMA chips) are used in the estimation step.

A. Channel Estimation and Tracking

At the base station, the received pilot symbols from all users are estimated using each user's PN-code and the Walsh code of the pilot channel. The pilot symbols are estimated for each channel multipath. The mth estimated pilot symbol for user i, using multipath j, is calculated as:

$$\begin{aligned} \hat{p}_m^{i,j} &= \frac{1}{PG} \sum_{k=\tau^{i,j}+mPG}^{\tau^{i,j}+(m+1)PG-1} y(k)[PN^i(k)W_p(k)]^* \\ &= \frac{1}{PG} \sum_{k=\tau^{i,j}+m\times PG}^{\tau^{i,j}+(m+1)PG-1} y(k)[PN_L^i(k)W_p(k)]^* \times \\ & \quad [PN_I(k) + jPN_Q(k)]^* \end{aligned} \tag{11}$$

where PG is the processing gain used on the pilot channel and $\tau^{i,j}$ is the delay from user i to the base station on multipath j. Note that the estimated pilot symbols in (11) are scaled down by a factor PG in order to have the same magnitude as the transmitted symbols. It is assumed that all PN-codes and channel delays ($\tau^{i,j}$) are known at the base station before estimating the pilot and data symbols. Replacing $y(k)$ in (11) by (4) and assuming the channel taps are constant within each pilot symbol, we have the following (dropping the index m for simplicity of notation):

$$\begin{aligned} \hat{p}_m^{i,j} &= c_{i,j}p^{i,j} + \sum_{l=1,l\neq j}^{L} \rho_{(i,i,j,l)}c_{i,l}(p^{i,l} + jd_{i,l}) + \\ & \sum_{k=1,k\neq i}^{N} \sum_{l=1}^{L} \rho_{(i,k,j,l)}c_{k,l}(p^{k,l} + jd_{k,l}) + v \end{aligned} \tag{12}$$

where $p^{k,l}$ and $d_{k,l}$ are the transmitted pilot and data symbols by user k and received through multipath l at the same time as $p^{i,j}$ is received. The multipath index is necessary for the scenario that the channel maximum delay is larger than the symbol period. Moreover, $\rho_{(i,k,j,l)}$ is the cross correlation between the PN-codes of users i and k, received on multipaths j and l respectively. Collecting the estimated pilot symbols from all users and multipaths ($i = 1, \cdots, N$ and $j = 1, \cdots, L$) into a vector b_m and using the definition (3) we get

$$b_m = R_m h_m + v_m \tag{13}$$

where

$$b_m = [\underbrace{\hat{p}^{1,1}\ \hat{p}^{1,2}\ \cdots\ \hat{p}^{1,L}}_{\text{user } 1}\ \cdots\ \underbrace{\hat{p}^{N,1}\ \hat{p}^{N,2}\ \cdots\ \hat{p}^{N,L}}_{\text{user } N}]^T$$

and

$$R_{NL\times NL} = pI_{NL} + U_{NL\times NL} \tag{14}$$

where I_{NL} is the identity matrix and p is the transmitted pilot symbol which is the same for all users. We have dropped the user and multipath index in $p^{i,j}$ since all users transmit the same sequence of pilot symbols in current CDMA standards. In cdma2000 in particular, the pilot symbols are all 1. In (14), U contains the cross correlation terms with its diagonal elements equal to zero. We complete the derivation of symbol-rate measurement equation by collecting q consecutive realizations of equation (13) into a column vector y_m:

$$y_m = H_m x_m + v_m \tag{15}$$

where

$$y_m = \begin{bmatrix} b_m \\ b_{m-1} \\ \vdots \\ b_{m-q+1} \end{bmatrix},\ x_m = \begin{bmatrix} h_m \\ h_{m-1} \\ \vdots \\ h_{m-q+1} \end{bmatrix}$$

$$H_m = \begin{bmatrix} R_m & & & \mathbf{0} \\ & R_{m-1} & & \\ & & \ddots & \\ \mathbf{0} & & & R_{m-q+1} \end{bmatrix} \tag{16}$$

Equations (15) and (8) form the state and measurement equations :

$$\begin{cases} x_m &= Fx_{m-1} + Gw_m \\ y_m &= H_m x_m + v_m \end{cases} \tag{17}$$

where m is the pilot symbol index. Note that H_m is a block diagonal matrix with its diagonal elements equal to p (or 1 in the case of cdma2000) and the non-diagonal elements representing the cross correlation terms, as shown in (14). The exact values of the non-diagonal elements depend on many parameters, e.g., the PN-codes, the relative delays of different multipaths and the transmitted data symbols by all users. Therefore, the non-diagonal elements are not practically available at the base station. However, we know that the non-diagonal elements have lower power than the diagonal elements by a factor of PG due to the properties of PN-codes. A robust filter used for estimating and tracking the channel taps is given in the next section. The estimated channel taps are then used in a Rake receiver for data estimation. The resulting receiver structure is shown in figure 2.

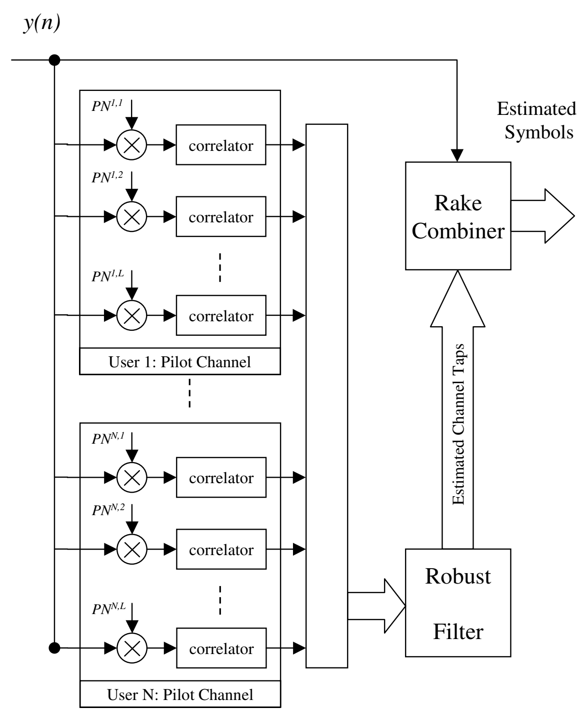

Fig. 2. Proposed receiver structure

B. Robust Kalman filter

Due to modelling errors in model (17), we rewrite the dynamics of the channel and measured signal at the receiver at symbol rate as follows :

$$x_{m+1} = (F + \delta F_m)x_m + Gw_m, \quad m \geq 0 \tag{18}$$

$$y_m = (H + \delta H_m)x_m + v_m \tag{19}$$

where the perturbations in $\{F_m, H_m\}$ are modeled as

$$[\ \delta F_m \quad \delta H_m\ ] = M_m \Delta_m [\ E_{f,m} \quad E_{h,m}\ ] \tag{20}$$

for some known matrices $\{E_{f,m}, E_{h,m}\}$, scalar M_m and for an arbitrary contraction Δ_m, $\|\Delta_m\| \le 1$. Observe that for generality we are allowing the quantities $\{M_m, E_{f,m}, E_{h,m}\}$ to vary with time. To estimate the channel taps in the presence of uncertainties in the state matrices, we use the robust algorithm developed in [14] and listed in Table 1. The filter performance is sensitive to the value of the constant α which can be chosen by means of the following procedure. The covariance matrix of the extended state vector $\bar{x_m} = [x_m \ \tilde{x_m}]$, where $\tilde{x_m} = x_m - \hat{x_m}$, satisfies the Lyapunov equation

$$\mathcal{M}_{m+1} = (\mathcal{F}+\delta\mathcal{F}_m)\mathcal{M}_m(\mathcal{F}+\delta\mathcal{F}_m)^* + \mathcal{G}\begin{pmatrix} Q & 0 \\ 0 & R \end{pmatrix}\mathcal{G}^* \quad (21)$$

where

$$\mathcal{F} = \begin{pmatrix} F & 0 \\ F - F_p - F_p P H^T \hat{R}^{-1} H & F_p \end{pmatrix} \quad (22)$$

$$\mathcal{G} = \begin{pmatrix} G & 0 \\ G & -F_p P H^T \hat{R}^{-1} \end{pmatrix} \quad (23)$$

$$\delta\mathcal{F}_m = \begin{pmatrix} M\Delta_m E_f & 0 \\ M\Delta_m E_f & 0 \end{pmatrix} \quad (24)$$

where $F_p = \hat{F}(I - PH^T R_e^{-1} H)$ and P is the steady state solution of

$$P_{m+1} = F_m P_m F_m^* - \overline{K}_m \overline{R}_{e,m}^{-1} \overline{K}_m^* + \widehat{G}_m \widehat{Q}_m \widehat{G}_m^* \quad (25)$$

Under the detectability of $\{F, \bar{H}\}$ and the stabilizability of $\{F, GQ^{1/2}\}$, it can be shown that the estimation error of the robust filter satisfies

$$\lim_{i\to\infty} E\tilde{x_m}\tilde{x_m}^* \le \mathcal{P}_{22} \quad (26)$$

where $\mathcal{P}_{22}$ is the (2,2) block entry with the smallest trace among all (2,2) block entries of positive-definite matrices $\mathcal{P}$ satisfying

$$\mathcal{P} - (\mathcal{F}+\delta\mathcal{F}_m)\mathcal{P}(\mathcal{F}+\delta\mathcal{F}_m)^* + \mathcal{G}\begin{pmatrix} Q & 0 \\ 0 & R \end{pmatrix}\mathcal{G}^* > 0 \quad (27)$$

for all contractive matrices Δ_m. The above inequality holds iff there exists a scalar $\omega > 0$ such that the following linear matrix inequality is feasible with $\Sigma > \mathcal{P} + \mathcal{G}\bar{\mathcal{Q}}\mathcal{G}^*$.

$$\begin{pmatrix} \Sigma & \mathcal{F} & M \\ \mathcal{F}^* & \mathcal{P}^{-1} - \omega \bar{E}^T \bar{E} & 0 \\ M & 0 & \omega I \end{pmatrix} > 0 \quad (28)$$

where

$$\bar{\mathcal{Q}} = \begin{pmatrix} Q & 0 \\ 0 & R \end{pmatrix} \quad (29)$$

$$\bar{E} = \begin{pmatrix} E & 0 \\ E & 0 \end{pmatrix} \quad (30)$$

Now, we take a grid $[\lambda_1, \lambda_2, \ldots, \lambda_n]$ in the interval $(\lambda_l, \lambda_l + \delta\lambda)$ where $\delta\lambda$ is a given positive scalar(chosen by the user). We evaluate the minimum $\mathcal{P}$ in (28) for each of the λ_m and choose the λ_m that results in a minimum bound on the variance of the state estimation error. From the value of λ_m that minimizes the error variance, α is easily known. The computational complexity of the proposed robust algorithm is $\mathcal{O}(K^3)$ where K is the dimension of x_m.

Assumed uncertain model: Eqs. (18)–(20). Also, $\Pi_0 > 0$, $R_m > 0$, $Q_m > 0$ are given weighting matrices.

Initial conditions: $\hat{x}_0 = 0$, $P_0 = \Pi_0$, and $\widehat{R}_0 = R_0$.

Step 1a. Using $\{\widehat{R}_m, H_m, P_m\}$ compute $P_{m|m}$:

$$\begin{aligned} P_{m|m} &= (P_m^{-1} + H_m^* \widehat{R}_m^{-1} H_m)^{-1} \\ &= P_m - P_m H_m^* (\widehat{R}_m + H_m P_m H_m^*)^{-1} H_m P_m \end{aligned}$$

Step 1b. If $M_m = 0$, then set $\hat{\lambda}_m = 0$. Otherwise, set $\hat{\lambda}_m = (1+\alpha)\lambda_{l,m}$

$$\lambda_{l,m} \triangleq \|M_m^* R_{m+1}^{-1} M_m\|$$

Step 2. Replace $\{Q_m, R_{m+1}, P_{m|m}, G_m, F_m\}$ by:

$$\begin{aligned} \widehat{Q}_m^{-1} &= Q_m^{-1} \\ &\quad + \hat{\lambda}_m E_{g,m}^* \left[I + \hat{\lambda}_m E_{f,m} P_{m|m} E_{f,m}^* \right]^{-1} E_{g,m} \\ \widehat{R}_{m+1} &= R_{m+1} - \hat{\lambda}_m^{-1} M_m M_m^* \\ \widehat{G}_m &= G_m - \hat{\lambda}_m F_m \widehat{P}_{m|m} E_{f,m}^* E_{g,m} \\ \widehat{F}_m &= (F_m - \hat{\lambda}_m \widehat{G}_m \widehat{Q}_m E_{g,m}^* E_{f,m}) \times \\ &\quad (I - \hat{\lambda}_m \widehat{P}_{m|m} E_{f,m}^* E_{f,m}) \\ \widehat{P}_{m|m} &= P_{m|m} - P_{m|m} E_{f,m}^* (\hat{\lambda}_m^{-1} I \\ &\quad + E_{f,m} P_{m|m} E_{f,m}^*)^{-1} E_{f,m} P_{m|m} \end{aligned}$$

If $\hat{\lambda}_m = 0$, then simply set $\widehat{Q}_m = Q_m$, $\widehat{R}_{m+1} = R_{m+1}$, $\widehat{P}_{m|m} = P_{m|m}$, $\widehat{G}_m = G_m$, and $\widehat{F}_m = F_m$.

Step 3. Now update $\{\hat{x}_m, P_m\}$ to $\{\hat{x}_{m+1}, P_{m+1}\}$ as follows:

$$\begin{aligned} \hat{x}_{m+1} &= \widehat{F}_m \hat{x}_m + \widehat{F}_m P_m H_m^* R_{e,m}^{-1} e_m \\ e_m &= y_m - H_m \hat{x}_m \\ P_{m+1} &= F_m P_m F_m^* - \overline{K}_m \overline{R}_{e,m}^{-1} \overline{K}_m^* + \widehat{G}_m \widehat{Q}_m \widehat{G}_m^* \\ \overline{K}_m &= F_m P_m \overline{H}_m^*, \quad \overline{R}_{e,m} = I + \overline{H}_m P_m \overline{H}_m^* \\ \overline{H}_m^T &= \left[\ H_m^T \widehat{R}_m^{-T/2} \quad \sqrt{\hat{\lambda}_m} E_{f,m}^T \ \right] \end{aligned}$$

Table 1: Listing of a robust filtering algorithm.

IV. SIMULATION RESULTS

An uplink cdma2000 environment is simulated to evaluate the performance of the architecture presented in the paper. The proposed architecture is compared to an ideal Rake combiner (knowing the exact channel taps) and a conventional Rake combiner, which uses an averaging technique for channel estimation. The performance of the link is simulated for different number of active users (up to 10). Every user sends the pilot and data channels simultaneously using orthogonal Walsh codes. Simulated for high data rate applications, the processing gain on data channels is 64, which corresponds to a data rate of 19.2 Kbps for each user per channel. This is due to the fact that the chip

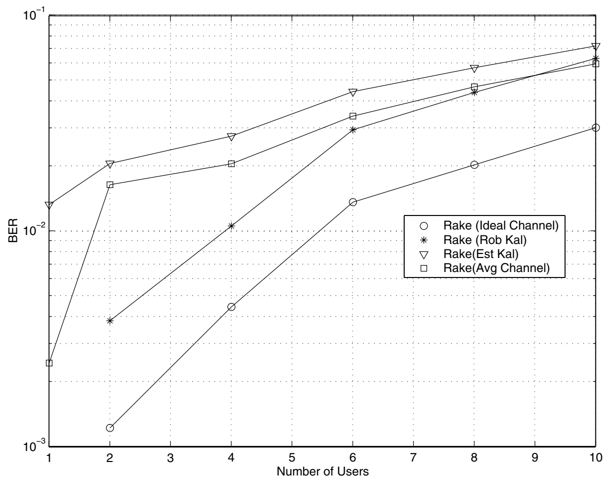

Fig. 3. BER vs. the number of users for different algorithms, SNR= 0dB

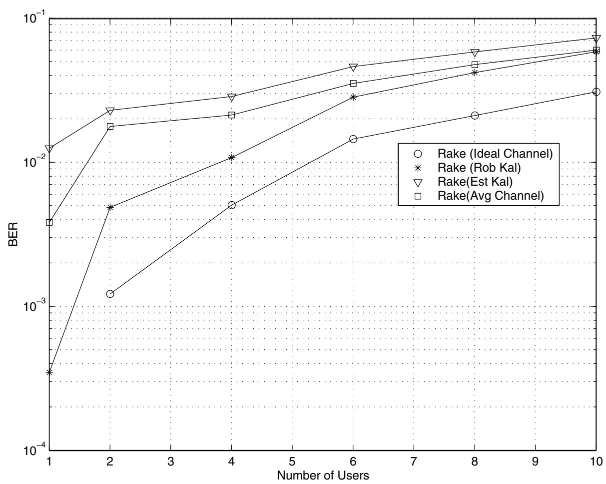

Fig. 4. BER vs. the number of users for different algorithms, SNR=-3dB

frequency in cdma2000-1X is 1.2288 MHz. On the base station side, the correlation length for the pilot channel (PG) is 256, and the averaging period used by the conventional Rake combiner is 10 symbols. Results are simulated for received chip rate SNR of -6dB to 0 dB . The channel model used in the simulation has 2 independent taps for each user and is approximated by an AR model of order 3. The maximum doppler frequency in the channel corresponds to a velocity of about 80 mph. The results for different number of users and different tracking techniques are shown in Figures 3-5. The proposed robust tracking technique outperforms the averaging algorithm used in CDMA systems. The difference is more significant for small number of active users, which is the desired scenario in high data rate links. The performance improvement is also significant when the actual channel taps have larger order unknown variations.

V. CONCLUSION

The issue of symbol-rate post-correlation model-based channel tracking and symbol detection for uplink CDMA systems has been addressed. In order to deal with the uncertainties in the model that arise due to the unknown correlation between different PN sequences and errors in doppler estimation, a robust filter has been proposed in tandem with a RAKE receiver.

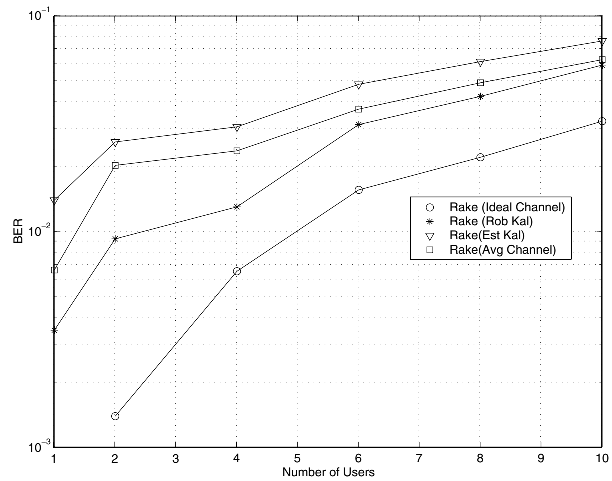

Fig. 5. BER vs. the number of users for different algorithms, SNR=-6dB

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