

# Two Variants of the FxLMS Algorithm

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## ABSTRACT

We present a time-domain feedback analysis of the FxLMS algorithm, which has been receiving increasing attention in the literature due to its potential application in the active control of noise. In particular, we introduce a generalized FxLMS variant and derive conditions for its  $l_2$ -stability. We also show that the algorithm can in fact be regarded as a member of the class of filtered-error variants. A special case of the generalized algorithm is the so-called MFxLMS recursion, which refers to a recent modification of the standard FxLMS update. While this modification significantly improves the convergence behaviour of FxLMS, it requires of the order of  $3M$  elementary computations per time step. This is in contrast to the  $2M$  operations required by the standard FxLMS. We suggest two new modifications that keep the computational load at the  $2M$  level, and which present improved convergence over the FxLMS algorithm. Simulation results are included to demonstrate the points raised in the paper.

## 1. Introduction

One of the most widely used adaptive algorithms for active noise control is the so-called Filtered-x Least-Mean-Squares (FxLMS) algorithm [1, 2]. It starts with an initial guess  $\mathbf{w}_{-1}$ , for an unknown  $M \times 1$  weight vector  $\mathbf{w}$ , and updates it via an update equation of the form

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu(i) F[\mathbf{u}_i^*] F[d(i) - \mathbf{u}_i \mathbf{w}_{i-1}]. \quad (1)$$

where the  $\{\mathbf{u}_i\}$  are given row vectors and the  $\{d(i)\}$  are noisy measurements of the terms  $\{\mathbf{u}_i \mathbf{w}\}$ , viz.,  $d(i) = \mathbf{u}_i \mathbf{w} + v(i)$ . The factor  $\mu(i)$  is a time-variant step-size parameter.

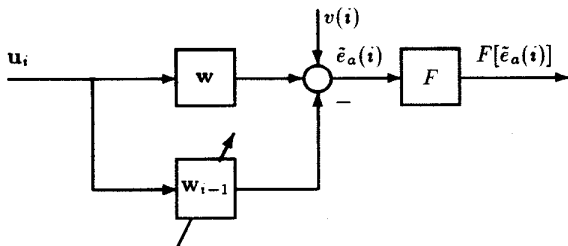


Figure 1: Structure of filtered-error gradient algorithms.

The difference  $[d(i) - \mathbf{u}_i \mathbf{w}_{i-1}]$  will be denoted by  $\tilde{e}_a(i)$  and will be referred to as the *output estimation error*. The following

error measures will also be useful for our later analysis:  $\tilde{\mathbf{w}}_i$  will denote the difference between the true weight  $\mathbf{w}$  and its estimate  $\mathbf{w}_i$ ,  $\tilde{\mathbf{w}}_i = \mathbf{w} - \mathbf{w}_i$ , and  $e_a(i)$  will denote the *a priori estimation error*,  $e_a(i) = \mathbf{u}_i \tilde{\mathbf{w}}_{i-1}$ . As indicated in Figure 1, a filtered version of  $\tilde{e}_a(i)$  is observed (see[2]–[5]), where  $F$  denotes the filter that is assumed to be of finite-impulse response type and of order  $M_F$ , say  $F(q^{-1})[x(i)] = F[x(i)] = \sum_{j=0}^{M_F-1} f_j x(i-j)$ .

Former analyses of the FxLMS algorithm have relied on several approximations. In the simplest case, it has been often assumed that the updated weights change very slowly over the filter length  $M_F$  of the error path, which in effect ignores the contribution of the filter function  $F$ . Another analysis method that has been exploited extensively in the field of echo cancellation is the so-called transfer-functional description [3, 6]. Here many assumptions have to be made (see [7] for details) in order to obtain a linear time-invariant filter function that describes the filter behaviour from the noise sequence to the a priori error sequence. Also, stochastic descriptions have been employed in order to describe the behaviour of the algorithms in the mean sense [1]. Motivated by these considerations, we shall pursue here an analysis within a purely deterministic framework and also suggest two algorithmic variants with improved computational requirements.

In [7, 8], a new time-domain method has been suggested for the analysis of filtered-error (FE) gradient-based algorithms. The method describes the update equation in a feedback form with a lossless (or contractive) time-variant feedforward block and a time-variant scalar gain in the feedback loop. The point is that such feedback structures are amenable to stability analysis via tools that are by now standard in system theory. We shall show here that the FxLMS algorithm belongs to the class of FELMS algorithms, which then allows us to apply the conclusions of [7, 8]. More specifically, it is

Algorithm	Complexity ( $M \gg M_F$ )	Memory Capacity	Convergence Behaviour
FxLMS	$2M + M_F$	$3M$	Poor
MFxLMS	$3M + 2M_F$	$3M + M_F$	Good
MFxLMS-1	$2M + 2M_F$	$3M + M_F$	Good
MFxLMS-2	$2M + 3M_F$	$3M + 2M_F$	Reasonable

Table 1: Comparison of complexity and storage capacity.

known that the FxLMS algorithm (1) requires of the order of  $2M$  elementary computations (additions and multiplications) per time step. Its convergence behaviour, however, is

very poor as evidenced by the simulations included at the end of this paper. A modified version of the FxLMS algorithm has been recently suggested in [9]. This so-called MFxLMS variant exhibits a considerably improved convergence behaviour albeit at the cost of increased computations, which are now of the order of  $3M$ . This figure may still be prohibitive in several applications of interest where the value of  $M$  is significantly large. We shall suggest two other variants, (9) together with (11a) and (15a,15b) further ahead. These modifications, while they keep the computational cost at  $2M$ , they nevertheless present an improved convergence behaviour when compared to the standard FxLMS. These comparisons are summarized in Table 1, where the last two lines refer to the two variants proposed herein.

## 2. A Time-Domain Feedback Analysis

For the sake of generality, and for reasons to become clear later, we shall allow for the following more general form of the update equation (instead of (1)):

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu(i)F[\mathbf{u}_i^*]G(i, q^{-1})F[d(i) - \mathbf{u}_i\mathbf{w}_{i-1}], \quad (2)$$

where a time-variant filter  $G(i, q^{-1})$  is included in the update relation. The update equation (2) can be written in terms of the weight-error vector  $\tilde{\mathbf{w}}_i = \mathbf{w} - \mathbf{w}_i$ ,

$$\tilde{\mathbf{w}}_i = \tilde{\mathbf{w}}_{i-1} - \mu(i)F[\mathbf{u}_i^*]G(i, q^{-1})F[\tilde{e}_a(i)]. \quad (3)$$

The above equation allows us to express  $\tilde{\mathbf{w}}_i$  in terms of  $\tilde{\mathbf{w}}_{i-1-l}$ , for some  $l$ .

$$\tilde{\mathbf{w}}_i = \tilde{\mathbf{w}}_{i-1-l} - \sum_{k=0}^l \mu(i-k)F[\mathbf{u}_{i-k}^*]G(i-k, q^{-1})F[\tilde{e}_a(i-k)]$$

or, in a form more suitable for our further investigation,

$$\tilde{\mathbf{w}}_{i-1-l} = \tilde{\mathbf{w}}_{i-1} + \sum_{k=1}^l \mu(i-k)F[\mathbf{u}_{i-k}^*]G(i-k, q^{-1})F[\tilde{e}_a(i-k)].$$

Now using  $\tilde{e}_a(i) = v(i) + \mathbf{u}_i\tilde{\mathbf{w}}_{i-1}$  and the linearity of  $F$ , we note that

$$\begin{aligned} F[\tilde{e}_a(i)] &= F[v(i)] + F[e_a(i)] = F[v(i)] + F[\mathbf{u}_i\tilde{\mathbf{w}}_{i-1}] \\ &= F[v(i)] + F[\mathbf{u}_i]\tilde{\mathbf{w}}_{i-1} \\ &\quad + \sum_{k=1}^{M_F-1} c(i, k)G(i-k, q^{-1})F[\tilde{e}_a(i-k)]. \end{aligned}$$

The above equality then leads to the following relation

$$F[\tilde{e}_a(i)] = \frac{1}{1 - C(i, q^{-1})G(i, q^{-1})} [F[v(i)] + F[\mathbf{u}_i]\tilde{\mathbf{w}}_{i-1}], \quad (4a)$$

where the coefficients of the time-variant filter  $C(i, q^{-1})$  (of order  $M_F$ ) are given by

$$c(i, l) = \mu(i-l)F_l[\mathbf{u}_i]F[\mathbf{u}_{i-l}^*], \quad (4b)$$

for  $l = 1, \dots, M_F-1$ , and where we have also used the following notation for the filtered input vector sequence  $\mathbf{u}_i$ :

$$F_l[\mathbf{u}_i] = \sum_{k=l}^{M_F-1} f_k \mathbf{u}_{i-k} \quad (4c)$$

for  $l = 1, \dots, M_F-1$ . Note that the lower index in (4c) starts at  $l$ . We can now rewrite the weight-error update equation (3) in the form

$$\tilde{\mathbf{w}}_i = \tilde{\mathbf{w}}_{i-1} - \mu(i)F[\mathbf{u}_i^*] \frac{G(i, q^{-1}) [F[v(i)] + F[\mathbf{u}_i]\tilde{\mathbf{w}}_{i-1}]}{1 - C(i, q^{-1})G(i, q^{-1})}, \quad (5)$$

which is of the filtered error type. Hence, the conclusions of [7, 8] are applicable, namely, that  $l_2$ -stability of the generalized FxLMS recursion (2) will be guaranteed if

$$\left\| \mathbf{I} - \mathbf{M}_N \overline{\mathbf{M}}_N^{-\frac{1}{2}} \mathbf{G}_N (\mathbf{I} - \mathbf{C}_N \mathbf{G}_N)^{-1} \overline{\mathbf{M}}_N^{-\frac{1}{2}} \right\|_{2, \text{ind}} < 1,$$

where  $\mathbf{C}_N$  and  $\mathbf{G}_N$  are lower triangular band matrices that describe the linear time-variant filters  $C(i, q^{-1})$  and  $G(i, q^{-1})$ , respectively, and where we have defined

$$\begin{aligned} \bar{\mu}(i) &= \frac{1}{\|F[\mathbf{u}_i]\|_2^2} \\ \mathbf{M}_N &= \text{diag}(\mu(0), \mu(1), \dots, \mu(N)) \\ \overline{\mathbf{M}}_N &= \text{diag}(\bar{\mu}(0), \bar{\mu}(1), \dots, \bar{\mu}(N)). \end{aligned}$$

The original FxLMS algorithm (1) corresponds to  $G = 1$ , and in the case of a constant step-size parameter  $\mu$ , the same analysis shows that a sufficient condition for its  $l_2$ -stability is to require

$$\left\| \mathbf{I} - \frac{\mu}{\bar{\mu}} (\mathbf{I} - \mathbf{C}_N)^{-1} \right\|_{2, \text{ind}} < 1 \quad (6a)$$

with

$$\bar{\mu} \leq \min_{0 \leq i \leq N} \left\{ \frac{1}{\|F[\mathbf{u}_i]\|_2^2} \right\}. \quad (6b)$$

Comparing with the discussion after (1), we see that the above stability condition is applicable to the original FxLMS recursion (1) without either a slow adaptation assumption or a modification as in the MFxLMS algorithm [9].

### 2.1. An Optimal Choice for $G(i, q^{-1})$

It follows from (5) that the update equation for the generalized FxLMS recursion (2) can be written in the form

$$\begin{aligned} \mathbf{w}_i &= \mathbf{w}_{i-1} + \mu(i)F[\mathbf{u}_i^*] \frac{G(i, q^{-1})}{1 - C(i, q^{-1})G(i, q^{-1})} \tilde{e}(i), \\ \tilde{e}(i) &= F[v(i)] + F[\mathbf{u}_i](\mathbf{w} - \mathbf{w}_{i-1}). \end{aligned} \quad (7)$$

If we compare this with the MFxLMS representation in [9], we thus see that the modification carried out in the MFxLMS case amounts to canceling the effect of the extra filtering step  $G/(1 - CG)$ , so that a final update equation that is similar in nature to a standard LMS equation will result. This is achieved by incorporating additional terms into the update relation, namely,  $F[\mathbf{u}_i\mathbf{w}_{i-1}]$  and  $F[\mathbf{u}_i]\mathbf{w}_{i-1}$ . These terms correspond to filtering the input data  $\mathbf{u}_i$  and the signal  $\mathbf{u}_i\mathbf{w}_{i-1}$  by  $F$  as well; thus amounting to an increase in the computational complexity to  $3M$ .

To get further insight into the nature of the modification induced by the MFxLMS algorithm, let us analyze it from the point of view of the generalized FxLMS recursion (7). We see that in order to cancel the effect of the additional

filtering operation  $G/(1 - CG)$ , we need to choose a  $G$ , say  $G_o(i, q^{-1})$ , such that

$$\frac{G_o(i, q^{-1})}{1 - C(i, q^{-1})G_o(i, q^{-1})} = 1.$$

This leads to the expression

$$G_o(i, q^{-1}) = \frac{1}{1 + C(i, q^{-1})}, \quad (8a)$$

or in terms of the matrix representations  $\mathbf{C}_N$  and  $\mathbf{G}_{o,N}$ ,

$$\mathbf{G}_{o,N} = [\mathbf{I}_N + \mathbf{C}_N]^{-1}. \quad (8b)$$

Using  $G_o(i, q^{-1})$  in the generalized recursion (7) will obviously lead to the MFxLMS algorithm. This also means that the MFxLMS recursion can be equivalently rewritten in the form (2) with the above  $G_o(i, q^{-1})$ , namely,

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu(i)F[\mathbf{u}_i^*] \frac{1}{1 + C(i, q^{-1})} F[d(i) - \mathbf{u}_i \mathbf{w}_{i-1}]. \quad (9)$$

We now propose two new modifications with lower computational requirements.

## 2.2. The MFxLMS-1 Algorithm

The first modification replaces the time-variant coefficients  $c(i, l)$  by constant approximations. This is especially useful when statistical information is available about the data. In particular, assume the input sequence is stationary with autocorrelation function  $r_l = E[u(k)u^*(k - l)]$ . If the process is ergodic and the order  $M$  of the input vector  $\mathbf{u}_i$  is sufficiently large, the terms  $\mathbf{u}_{i-l}\mathbf{u}_i^*$  can be approximated by  $\mathbf{u}_{i-l}\mathbf{u}_i^* \approx M\mathbf{r}_l$ . We shall also assume that the time-variant step size  $\mu(i)$  in (9) is chosen to be

$$\mu(i) = \frac{\alpha}{\|F[\mathbf{u}_i]\|_2^2}, \quad (10)$$

which is known as the projection step-size. The term  $\|F[\mathbf{u}_i]\|_2^2$  can be approximated by

$$\|F[\mathbf{u}_i]\|_2^2 \approx M \sum_{i=0}^{M_F-1} \sum_{j=0}^{M_F-1} r_{i-j} f_i f_j,$$

and the filter coefficients  $c(i, l)$  can also be approximated by:

$$\begin{aligned} c(i, l) &= \mu(i-l)F_l[\mathbf{u}_i]F[\mathbf{u}_{i-l}^*] \\ &\approx \alpha \frac{\sum_{i=l}^{M_F-1} \sum_{j=0}^{M_F-1} r_{i-j} f_i f_j}{\sum_{i=0}^{M_F-1} \sum_{j=0}^{M_F-1} r_{i-j} f_i f_j} \\ &= \alpha \bar{c}(l), \end{aligned} \quad (11a)$$

where we have defined the averaged coefficient

$$\bar{c}(l) = \frac{\sum_{i=l}^{M_F-1} \sum_{j=0}^{M_F-1} r_{i-j} f_i f_j}{\sum_{i=0}^{M_F-1} \sum_{j=0}^{M_F-1} r_{i-j} f_i f_j}. \quad (11b)$$

These approximate coefficients depend only on the error-filter  $F$  and on the autocorrelation coefficients  $\{r_l\}$ . They can therefore be computed in advance, assuming knowledge of  $F$ .

This approximate solution also has another advantage, beyond the simplification in the computations. It provides us with an approximate stability result for the FxLMS algorithm. Since the filter coefficients  $c(i, l)$  are equivalent in the FxLMS algorithm as well as in the optimal filter  $G_o(i, q^{-1})$ , we can use the averaged coefficients to conclude stability bounds. More specifically, condition (6a) can be approximated by

$$\max_{\Omega} \left| 1 - \frac{\alpha}{1 - \alpha \bar{C}(e^{j\Omega})} \right| < 1 \quad (12a)$$

which is now given in terms of the normalized step-size  $\alpha$  and in terms of the constant filter  $\bar{C}(q^{-1})$  whose coefficients are the  $\bar{c}(l)$ . This condition also suggests a choice for  $\alpha$  in order to speed up the convergence of the resulting algorithm: it can be chosen as the solution of

$$\min_{\alpha} \max_{\Omega} \left| 1 - \frac{\alpha}{1 - \alpha \bar{C}(e^{j\Omega})} \right|. \quad (12b)$$

## 2.3. The MFxLMS-2 Algorithm

We have shown in the earlier sections that the MFxLMS algorithm can be written in two equivalent forms. The first one is:

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu(i)F[\mathbf{u}_i^*] \bar{\epsilon}(i), \quad (13a)$$

$$\bar{\epsilon}(i) = F[v(i)] + F[\mathbf{u}_i](\mathbf{w} - \mathbf{w}_{i-1}), \quad (13b)$$

and the second one is (9):

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu(i)F[\mathbf{u}_i^*] \frac{1}{1 + C(i, q^{-1})} F[\bar{\epsilon}_a(i)]. \quad (13c)$$

The difference between both representations is that the second one operates directly on the available signal  $F[\bar{\epsilon}_a(i)]$  by filtering it through  $1/(1+C)$ , while the first one modifies the update equation and uses the filtering operations  $F[\mathbf{u}_i \mathbf{w}_{i-1}]$  and  $F[\mathbf{u}_i] \mathbf{w}_{i-1}$ . The net result, however, is the same since we already know that both representations are equivalent and, in particular, that

$$\bar{\epsilon}(i) = \frac{1}{1 + C(i, q^{-1})} F[\bar{\epsilon}_a(i)] = G_o(i, q^{-1})F[\bar{\epsilon}_a(i)]. \quad (14)$$

If we know  $\bar{\epsilon}(i)$  then it can be used in the update form (13a). The MFxLMS algorithm computes it by introducing the terms  $F[\mathbf{u}_i \mathbf{w}_{i-1}]$  and  $F[\mathbf{u}_i] \mathbf{w}_{i-1}$ , as explained above. The generalized FxLMS algorithm computes it by filtering through  $1/(1+C)$ , which in turn requires the evaluation of the filters  $C(i, q^{-1})$  themselves.

This suggests the following modification. We have in (14) a relation between  $\bar{\epsilon}(i)$  and  $1/(1+C)$ . The only known quantity in (14) is  $F[\bar{\epsilon}_a(i)]$ , and we can rewrite the expression in the form

$$\bar{\epsilon}(i) = F[\bar{\epsilon}_a(i)] - C(i, q^{-1})[\bar{\epsilon}(i)].$$

Since  $\bar{\epsilon}(i)$  is also unknown, we need an estimate, say  $\hat{\epsilon}(i)$ . The above equation reads then

$$\hat{\epsilon}(i) = F[\bar{\epsilon}_a(i)] - \hat{C}(i, q^{-1})[\hat{\epsilon}(i)].$$

An approximate solution would be to use a gradient-type algorithm in order to estimate the coefficients of  $C(i, q^{-1})$ ,

that is  $\hat{C}(i, q^{-1})$  and also  $\hat{\epsilon}(i)$ . If the error energy  $|\hat{\epsilon}(i)|^2$  (or as often used in literature, its mean-value  $E[|\hat{\epsilon}(i)|^2]$ ) is minimized, the following gradient-type update would result:

$$\hat{\epsilon}(i) = F[\bar{\epsilon}_a(i)] - \sum_{k=1}^{M_F-1} \hat{c}(i-1, k)\hat{\epsilon}(i-k), \quad (15a)$$

$$\hat{c}(i, l) = \hat{c}(i-1, l) + \frac{\hat{\epsilon}(i)\hat{\epsilon}(i-l)}{1 + \sum_{k=1}^{M_F-1} |\hat{\epsilon}(i-k)|^2}. \quad (15b)$$

### 3. Simulation Results

In the following simulations we demonstrate several of the points raised in our previous discussions. In all experiments we have chosen a Gaussian white random sequence with variance one as the input signal, and the additive noise was set at  $-60dB$  below the input power. We provide plots of learning curves for the relative system mismatch, defined as

$$S_{rel}(i) = \frac{\|\bar{\mathbf{w}}_i\|_2^2}{\|\bar{\mathbf{w}}_{-1}\|_2^2}.$$

The curves are averaged over 50 Monte Carlo runs in order to approximate  $E[S_{rel}(i)]$ . The results in the figures are also indicated in dB. In all experiments we employed the projection normalization (10). Figure 2 exhibits learning curves of the several algorithms when run with their optimal normalized step-size  $\alpha_o$  in order to have fastest convergence. The order of the system to be identified was taken as  $M = 20$  and the error filter path was defined as

$$F(q^{-1}) = 1 + q^{-1} + q^{-2} + q^{-3}.$$

indicating a low pass behaviour as it is common in acoustic ducts. The coefficients of the corresponding averaged filter  $\bar{C}(q^{-1})$  are given by

$$\bar{c}_1 = 0.75 \quad \bar{c}_2 = 0.5 \quad \bar{c}_3 = 0.25,$$

that is,

$$\bar{C}(q^{-1}) = 0.75q^{-1} + 0.5q^{-2} + 0.25q^{-3}.$$

We continue to use the projection step-size parameter (10.) If we use the above averaged coefficients as approximations, we obtain an approximate stability range for the MFxLMS-1 algorithm at  $0 < \alpha < 0.5$  (recall (12a)); the optimal convergence speed is attained at  $\alpha_o = 0.45$  (recall (12b)). In the simulations that were carried out, the results were very close to these values with a stability bound at 0.57 and fastest convergence at 0.5. In particular, the optimal step-size from [2] for this case is 0.8333, which is already in the unstable region.

As Figure 2 shows, the average filter solution that corresponds to the proposed version MFxLMS-1 leads to a learning curve (indicated by the letter (c)) which is close to the optimal one (i.e., the one that corresponds to the MFxLMS recursion and is indicated by the letter (d) in the figure). We obtained fastest convergence for  $\alpha_o = 1.2$  and stability bound at 2 precisely as in the MFxLMS case.

The figure also indicates the result of the second modification MFxLMS-2 (curve (b)), which is appropriate when the statistics of the input sequence is not known apriori. While curve

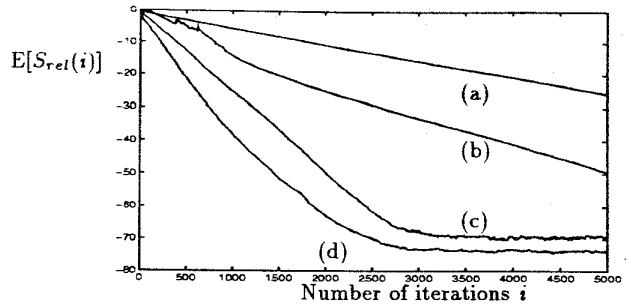


Figure 2: Learning curves for FxLMS algorithm with  $\alpha = 0.5$  (a) and modifications: MFxLMS-2 (b)  $\alpha = 1.15$  and MFxLMS-1 (c)  $\alpha = 1.2$  in comparison to MFxLMS (d)  $\alpha = 1.2$ .

(b) is less appealing than the curves (c) and (d), it nevertheless improves on the convergence of the standard FxLMS recursion, which is indicated by curve (a). The optimal convergence speed for the MFxLMS-2 algorithm was found for  $\alpha_o = 1.15$  and stability bound at 1.3. A fifth learning curve for the LMS algorithm, with  $u_i$  and  $v(i)$  prefiltered by  $F$ , is not explicitly shown in the figure since it essentially coincides with the MFxLMS algorithm (curve (d)).

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