A Probabilistic Power-Aware Routing Algorithm for Wireless Ad-Hoc Networks

Ananth Subramanian and Ali H. Sayed, *Fellow IEEE*

Abstract—**In this paper we propose a probabilistic routing algorithm for wireless ad-hoc networks. The focus is on applications that can tolerate end-to-end delays and those for which the final destination for the packets is a small region. The nodes are assumed stationary and establish connections with a master node according to a priority scheme that relates to their distances from the master node. Simulation results illustrate the performance of the proposed algorithm.**

I. INTRODUCTION

Routing algorithms for wireless networks have been discussed in [1]–[4]. In these and other related works, several mechanisms to route packets between nodes in a network have been proposed. In this paper we propose a probabilistic routing algorithm for applications that can tolerate end-to-end delays and those for which the final destination for the packets is a small region, whereby it suffices that the packets reach any of the nodes inside the region.

In the proposed routing mechanism, the space is divided into virtual geographical cells, each containing N nodes with one additional node acting as a master node. A frequency slot is allocated to each node that wishes to communicate with the master node in a cell. We allow for frequency reuse across cells in a manner similar to that in mobile cellular systems. The master nodes perform important tasks like routing and congestion control for high data rate communications, as well as handling some data processing and network information for nodes connected to them. Being a master node is power consuming and hence the nodes are made to take turns as master nodes with equal probability, but with the constraint that there can be only one master node in a cell at any time. Priority to function as a master node is given to the node that has a low interference power from other cells. The nodes communicating in the same frequency slot in other cells cause interference with this cell and this interference is measured in terms of the signal-to-interference ratio (SIR) defined as follows. The SIR for node i at time k on an uplink channel is defined by

$$\gamma_i(k) = \frac{G_{ii}(k)p_i(k)}{\sum\limits_{j\in\mathcal{A}} G_{ij}(k)p_j(k) \;+\; \sigma^2} \tag{1}$$

where, for each time instant k, G_{ij} denotes the channel gain from the $j-$th node to the intended master node of the $i-$th node, p_i is the transmission power from the $i-$th node, and σ^2 is white Gaussian noise power at the receiver of the master node that node i is connected to. Moreover, $\mathcal{A}$ denotes the set of all nodes that are interfering with node i from all cells – see

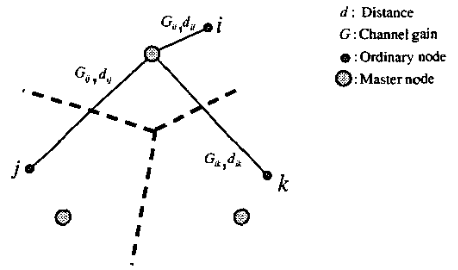

Fig. 1. *A schematic representation with three cells, three master nodes, and active and interfering nodes. The active node is node i and the interfering nodes are nodes j and k.*

Fig. 1. We assume that the transmission power of each node at every instant satisfies $P_{\min} \leq p_i(k) \leq P_{\max}$. We use the model from [5] for the channel gain from the $i-$th node to its master node. In this model, G_{ii} has a lognormal distribution, i.e.,

$$G_{ii} = S_0 d_{ii}^{-\beta} 10^{\alpha/10} \tag{2}$$

where S_0 is a function of the carrier frequency, β is the path loss exponent (PLE), and d_{ii} is the distance of the master node from node i. The value of β depends on the physical environment and varies between 2 and 6 (usually 4), while α is a zero mean Gaussian random variable with variance σ_α^2, which usually ranges between 6 and 12.

The organization of the paper is as follows. In Section II, we describe the routing algorithm and in Section III we propose a strategy to choose the master node. Sections IV and V give certain performance measures of the proposed algorithm in the scenario of a very high density of nodes and Section VI gives some simulation results.

II. ROUTING ALGORITHM

We assume the following rules of connection for nodes in every geographical cell in the network:

1) No two nodes are at the same distance from a master node in a cell. This assumption can be accommodated by assuming slight perturbations in the location of the nodes.

2) The probability that a node connects to a master node is inversely proportional to its distance from the master node. In other words, a node closer to the master node has a higher priority of connection than a node farther away.

The details of how the nodes establish a connection with the master node is as follows. A packet is transmitted from a source to its final destination through intermediary master nodes. We assume that each node has a buffer of sufficient size

The authors are with the Department of Electrical Engineering, University of California at Los Angeles, Los Angeles, CA 90095-1594.

E-mail: {msananth,sayed}@ee.ucla.edu.

This work was partially supported by NSF grant CCR-0208573.

to store routed packets. The operation of the nodes in any cell follows a periodic cycle. Each cycle starts with a set-up phase when a node is chosen as a master node. The set-up phase is followed by a transmission phase during which all nodes in a cell that want to communicate with the master node send their packets through available frequency slots. In the set-up phase, every node in a cell expresses its desire to be the master node with a probability that is equal to all nodes. When there is contention, the node with lowest interference power from other cells is chosen as the master node. Once a particular node is chosen as a master node, it lets all other nodes know through a broadcast in that cell that it is the master node for the current cycle. Each node then gauges its distance from the master node to determine the probability with which it could establish a connection. This is done as follows.

Let $d_{\min}$ denote the smallest distance between the master node and its closest neighbor. As soon as the master node is chosen, the master node broadcasts its own coordinates and its variable $d_{\min}$ to all other nodes. Once this is done, every node i calculates the distance d_{ii} that separates it from the master node. Then each node i will try to connect to the master node with probability $(d_{\min}/d_{ii})^{1/\delta}$, where δ is a user defined design parameter that is between 0 and 1. If the nodes are far away from the master node, they connect with lower probability thus conserving power. Note that the smaller the value of δ, the higher the number of nodes that would express a desire to connect with the master node. At the end of this set-up phase, it is decided based on the number of available frequency slots, say Q, which nodes connect with the master node during the transmission phase. If the number of nodes that express a desire to connect with the master node is more than Q, then the master node chooses Q nodes of those with highest probability of connection during the transmission phase.

In the transmission phase, the following routing decisions occur. If the intended final destination node for a packet is in a different cell, then the information is routed through the master node and a centralized base station through specific frequency slots that are separate from those that are available inside each cell. If the final destination node is within the same cell, then the following routing multi-hop algorithm is adopted. All possible routes out of a master node will only lie inside a sector of angle θ in the direction of the final destination, with the current relay node as the vertex. In a particular hop (when a packet is waiting at a relay node to be routed), if the next chosen master node is in any of the possible routes to the final destination, then the relay node routes the packets to the new master node (see Fig 2). Otherwise, the packets wait in the buffer. If the packets move from the relay node to the new master node, then the following occurs. In the next cycle, when the node ceases its functions as master node, it waits till another master node is available in any of the possible routes from the current location to the final destination. Hence, a successful hop in the direction of the final destination may take several cycles. For the analysis of the routing algorithm proposed above, we will assume that the nodes in a cell are uniformly distributed inside a circle of radius r. Only nodes that use the same frequency slot as a particular node i in other cells cause interference with node i. We assume that these interfering nodes are located within radius R from the master node i is connected to (see Figure 3). We will also assume that each geographical cell is surrounded by M other cells within an area A_c and that these cells can cause interference.

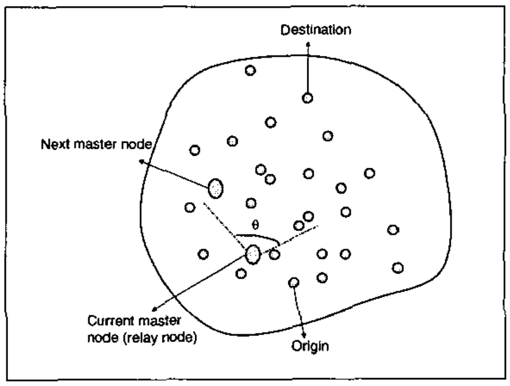

Fig. 2. *A schematic showing the origin and final destination of a packet with intermediary relay node and master node.*

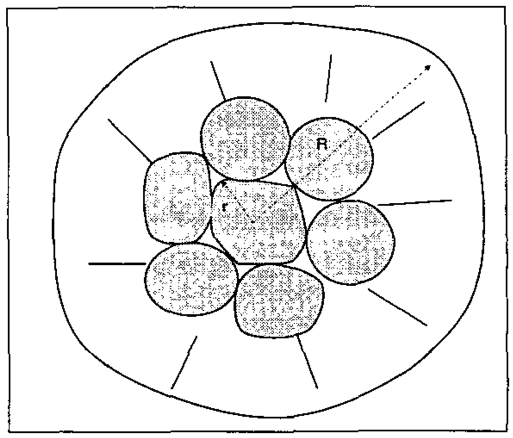

Fig. 3. *Nodes beyond a radius R do not cause interference with the nodes inside a cell of radius r.*

III. Choice of Master Node

We now show how we choose the master node in every cycle. Before we proceed, we establish the following result.

Theorem 1 [Average Power per Node] Under log-normal channel conditions, the average power required at a node i in order to achieve an SIR level of γ is

$$E(p_i(k)) = \frac{\gamma E(J_i(k))}{\frac{S_0'((2r)^{1-\beta}-(d_o)^{1-\beta})}{(2r-d_o)(1-\beta)}} \tag{3}$$

where

$$S_0' = e^{\frac{(\sigma_\alpha \ln 10/10)^2}{2}} S_0 \tag{4}$$

and $E(J_i(k))$ is the expected interference at the master node to which node i is connected. Moreover, σ^2 is the noise variance, and d_o is the smallest possible distance between any two nodes in the network.

Proof : Assuming a uniform distribution for d_{ii} between d_o and $2r$, we have

$$f_{d_{ii}}(d) = 1/(2r - d_o) \qquad d_o \le d \le 2r$$

where $f_{d_{ii}}(d)$ is the probability density function of the distance of a node from the master node in a cell. Since

$$E(G_{ii}|d_{ii}) = S_0' d_{ii}^{-\beta}$$

we have

$$\begin{aligned} E(G_{ii}) &= \int_{d_o}^{2r} E(G_{ii}|d_{ii} = d) f_{d_{ii}}(d) dr \\ &= \frac{S_0'((2r)^{1-\beta} - (d_o)^{1-\beta})}{(2r - d_o)(1-\beta)} \end{aligned}$$

Let an SIR level of γ be attained at all master nodes in steady state throughout the network. Then, from (1), we have

$$J_i(k) = p_i(k) G_{ii}(k)/\gamma \tag{5}$$

Taking expectations of both sides of the above equation, we have

$$\frac{S_0'((2r)^{1-\beta} - (d_{\min})^{1-\beta})}{(2r - d_{\min})(1-\beta)} E(p_i(k)) = \gamma E[J_i(k)] \tag{6}$$

or

$$E(p_i(k)) = \frac{\gamma E[J_i(k)]}{\frac{S_0'((2r)^{1-\beta} - (d_o)^{1-\beta})}{(2r-d_o)(1-\beta)}} \tag{7}$$

$\diamond$

It can be seen from the above result that if the master node is chosen in such a way that the interference is the least, then the average power consumed by a node will be smaller. Hence, in order to reduce the power consumption in the network, we choose the master node that has the least amount of interference.

IV. Routing Delay

Consider again a cell with N nodes in addition to the master node. An illustration of routes through which packets are routed from the sender node to the destination node is shown in Fig. 4. It is known that as N increases, the distribution of the nodes in the cell satisfies a Poisson distribution [9]. In other words, as $N \to \infty$, the number of nodes in a given area, say of $\bar{H}$ units, within a cell of area H, is Poisson distributed with mean $\epsilon\bar{H}$, where $\epsilon = N/H$ is the density of nodes. That is to say, the probability that $\bar{H}$ contains j nodes is $(\epsilon\bar{H})^j(1/j!)\exp(-\epsilon\bar{H})$. We now derive an expression for the probability density function of the distance travelled per hop in any route from source to destination within a cell.

Consider a relay node X shown in Fig. 5 that wishes to send packets to a destination node D. Let Y be an intermediary

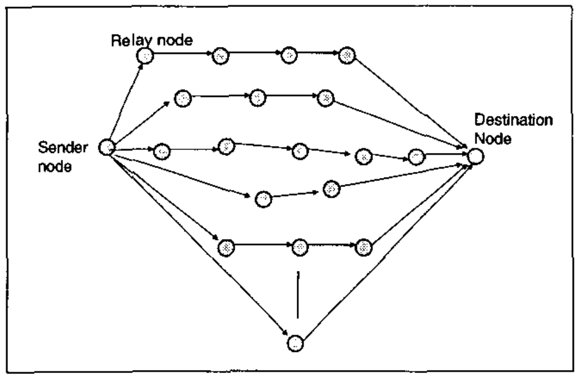

Fig. 4. *Relaying of packets through multiple routes.*

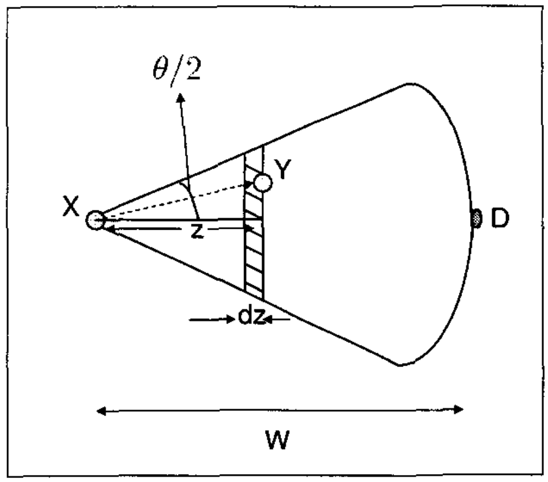

Fig. 5. *A packet is routed to its final destination node D through an intermediary master node Y.*

node. Packets will travel from X to Y only if Y becomes a master node. Let $\bar{E}$ denote the event that a node Y in a strip of width dz and distance z from X becomes the master node. Then, since the nodes are distributed uniformly, the probability that $\bar{E}$ occurs is $2z\tan(\theta/2)dz/H$, where H units is the area of the cell. In a transmission cycle, the probability that node X connects to Y is $\{d_{\min}/d_{ii}\}^{1/\delta} \ge \{d_o/W\}^{1/\delta}$, where d_o is the minimum distance between any two nodes in the cell and W is the distance between X and D. Note that, in a cycle, the event Y being the master node rules out the possibility of any other master node in the sector shown in Figure 5. If z denotes the distance travelled by a packet from X to Y in the direction of D, then the probability density function of z satisfies

$$f(z)dz \ge \{d_o/W\}^{1/\delta} \times \{2z\tan(\theta/2)dz/H\}$$

Hence, the expected value of z for a small θ is

$$\begin{aligned}E(z) &= \int_{d_o}^{W} z f(z) dz \\ &\geq \int_{d_o}^{W} \{d_o/W\}^{1/\delta} \times \{2z^2 \tan(\theta/2) dz/H\} \\ &= 2\{d_o/W\}^{1/\delta} \times \{\tan(\theta/2)/H\}\{W^3/3 - d_o^3/3\}\end{aligned}$$

The above inequality says that on an average a packet moves at least $\phi \times W$, where

$$\phi = 2\{d_o/W\}^{1/\delta} \times \{\tan(\theta/2)/(HW)\}\{W^3/3 - d_o^3/3\}$$

Note that if $d_o \ll W$ and $\delta = 1$, we get

$$\begin{aligned}\phi &\approx (2/3)\{d_o\} \times \{\tan(\theta/2)/(H)\}\{W\} \\ &> (2/3)\tan(\theta/2) d_o^2/H \triangleq \psi\end{aligned}$$

Note that the distance to destination from the next relay node would be utmost $(1-\psi)W$, which establishes the following result.

Theorem 2 [Average Distance per Hop] For a cell with a high density of N nodes, a packet moving from a source to its destination node would cover on average a distance of

$$L_h \geq \{2/3\}(d_o/W)^{1/\delta} \times (\tan(\theta/2)/H)(W^3 - d_o^3) \quad (8)$$

in the first hop from a relay node, where d_o is the smallest distance between any two nodes in the network, W is the distance between the current relay and destination nodes, θ is the angle of the sector in the direction of the destination node and is assumed small, and H is the area of the cell. Moreover, if $d_o \ll W$ and $\delta = 1$, then the distance to destination from the next relay node would be utmost $(1-\psi)W$ where $\psi = (2/3)\tan(\theta/2)d_o^2/H$.

$\diamondsuit$

V. Cell Coverage

Let q_h denote the probability of a successful transmission of a packet in a cycle to the final destination node in the last hop. Note that q_h is $\mathcal{O}(1/N)$ since the probability that the destination node becomes master node is $\mathcal{O}(1/N)$ in a cell containing N nodes. Then

$$\text{Prob}(\text{delay in last hop of a packet} \geq k) = (1-q_h)^{k-1} \quad (9)$$

This equation reveals that there is no almost sure bound on the delay that a particular packet may experience.

If the final destination for a packet is not a node but a destination area S, we can then perform the following analysis. With the location of the master node taking random positions in the cell with uniform distribution, one can correlate this process with a coverage process in which the cell is covered with circles of radius o centered at the positions of the master nodes – see Fig. 6. Let again the density of the Poisson distributed nodes be ϵ. Let S denote the selected destination area

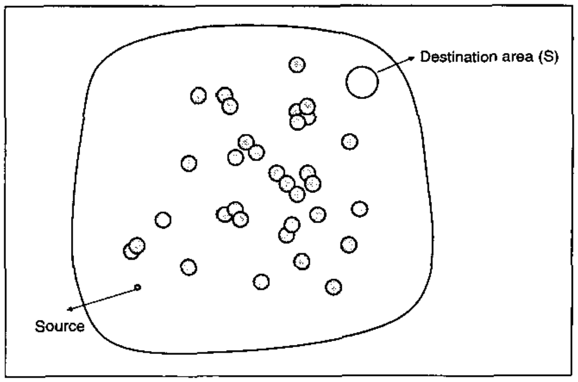

Fig. 6. *A schematic representation of the selected areas in a cell as master nodes change every cycle. One circular area corresponds to one master node chosen in one cycle.*

within the cell. Then the probability that the coverage process (formed by the circles with consecutive master nodes as centers) never intersects S is $\exp(-\epsilon S)$ [9]. As $\epsilon \to \infty$ (i.e., as the density of nodes increases), the area S is almost surely covered or, in other words, a packet almost surely reaches a master node in S. Also, if we let V denote the area that is never covered, then from [9], we know that if $o \leq 1/2$ and $\epsilon \geq 1$, then

$$P(V > 0) < 3\min\{1, (1+\pi o^2\epsilon^2)e^{-\pi o^2 \epsilon}\} \quad (10)$$

Corollary 1 [Cell Coverage] As the density of nodes ϵ increases, the entire cell is almost surely covered. $\diamondsuit$

The above corollary says that any non-zero area around the final destination node (if not the actual destination node) can be reached by a packet almost surely when the density of nodes is arbitrarily large.

VI. Simulations

In order to demonstrate the performance of the proposed routing protocol, we simulate a cell in the network with N nodes independently and uniformly distributed within an area. Figure 7 shows the frequency chart of the delay in number of cycles when 10,000 experiments are conducted. Figure 8 gives the average distance covered in a hop for a total distance between the node and destination node.

VII. Conclusions

In this paper we proposed a probabilistic routing algorithm for wireless ad-hoc networks. The focus is on applications that can tolerate end-to-end delays and those for which the final destination for the packets is a small region, where it suffices that the packets reach any of the nodes inside the destination area.

References

[1] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad-hoc wireless networks," *IEEE/ACM Transactions on Networking*, vol. 10, no. 4, pp. 477–486, Aug. 2002.

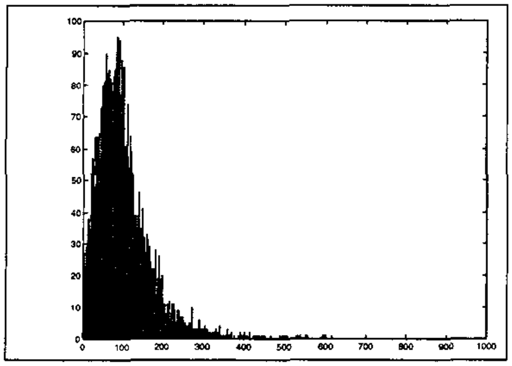

Fig. 7. *Frequency chart of average delay in cycles for a successful transmission of a packet from the sender node to destination node for* $N = 100$ *nodes* ($\theta = 45^o$).

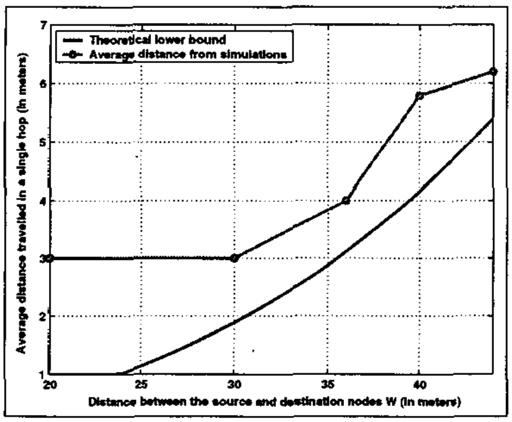

Fig. 8. *Average distance covered in the first hop vs. total distance between the node and destination node* ($\theta = 45^o$).

[2] S. Park and M. Srivatsava, "Power aware routing in sensor networks using dynamic source routing," *ACM MONET Special Issue on Energy Conserving Protocols in Wireless Networks*, 1999.
[3] C. Intanagonwiwat, R. Govindan and D. Estrin, Directed Diffusion: A scalable and robust communication paradigm for sensor networks, *Proc. MobiCom*, Boston, MA, pp. 56–67, Aug. 2000.
[4] S. Singh, M. Woo, and C. Raghavendra, "Power aware routing in mobile ad-hoc networks," *Proc. 4th Annual ACM/IEEE MobiCom*, Oct 1998.
[5] M. Gudmundson, "Correlation model for shadow fading in mobile radio systems," *Electronic Letters*, vol. 27, no. 23, pp 2145–2146, Nov. 1991.
[6] D. Baker, A. Ephremides, and J. Flynn, "The deign and simulation of a mobile radio network with distributed control," *IEEE Journal on Selected Areas in Communications*, vol. SAC-2, pp. 226–237, Jan. 1984.
[7] C. Lin and M. Gerla, "Adaptive clustering for mobile wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 15, pp. 1265–1275, Sep. 1997.
[8] W. B. Heinzelman, A. P. Chandrakasan, and H. Balakrishnan, "An application specific protocol architechture for wireless microsensor networks," *IEEE Transactions on Wireless Communications*, vol. 1, no. 4, pp. 660–670, Oct. 2002.
[9] P. Hall, *Introduction to the Theory of Coverage Processes*, Wiley, NY, 1998.