CDMA Location Using Multiple Antennas and Interference Cancellation

Alireza Tarighat, Nima Khajehnouri and Ali H. Sayed

Department of Electrical Engineering
University of California, Los Angeles, CA 90095
Email: {tarighat,nimakh,sayed}@ee.ucla.edu

Abstract—**Location finding for CDMA subscribers using an antenna array at the base station and deploying multiple base stations is studied in this paper. Two methods are considered for radio location: measured time-of-arrival (TOA) and angle-of-arrival (AOA), and algorithms to estimate them are presented. The estimated TOA and AOA from different base stations are combined via data fusion to estimate the user's location by exploiting the topology of the cellular network.**

I. Introduction

The U.S. Federal Communications Commission (FCC) has made E911 a mandatary requirement for wireless communications services [1]. E911 requires all 911 calls from mobile telephones in the U.S. to be located with a certain accuracy in order to route the call to the appropriate emergency service provider.

There are several approaches for implementing a radio location system including those based on TOA, AOA, Time-Difference-Of -Arrival (TDOA), and signal-strength [2]-[5]. This paper presents algorithms for estimating the TOA and AOA in a multiuser CDMA system, and uses interference cancellation techniques for accuracy improvement. To further improve the location estimation, multiple base stations are used to provide independent estimates. The TOA and AOA estimates conducted by different base stations are combined by a central processor via data fusion to estimate the user's location by exploiting the topology of the cellular network.

II. Maximum Likelihood Estimation of TOA

Consider a base station using an M-element antenna array. The received signal at time n over a single-path channel is then an $M \times 1$ vector given by:

$$\mathbf{r}(n) = \mathbf{A}_\theta c(n)s(n-\tau) + \mathbf{v}(n) \tag{1}$$

where $c(n)$ and τ are respectively the unknown channel gain and delay, $s(n)$ is a known sequence transmitted by the user for training purposes, and $\mathbf{v}(n)$ is additive white Gaussian noise. Moreover, $\mathbf{A}_\theta$ is the array response defined by:

$$\mathbf{A}_\theta = \text{col}\{\ 1 \quad e^{j2\pi\frac{d}{\lambda}cos\theta} \quad \ldots \quad e^{j2\pi\frac{(M-1)d}{\lambda}cos\theta}\ \} \tag{2}$$

The maximum likelihood estimates of τ and θ are given by:

$$\{\hat{\tau}, \hat{\theta}\} = \arg\max_{\tau,\theta}[P(\mathbf{r}(1)\cdots\mathbf{r}(K))|\{\tau,\theta\}] \tag{3}$$

*This work was partially supported by NSF grant CCR-0208573.

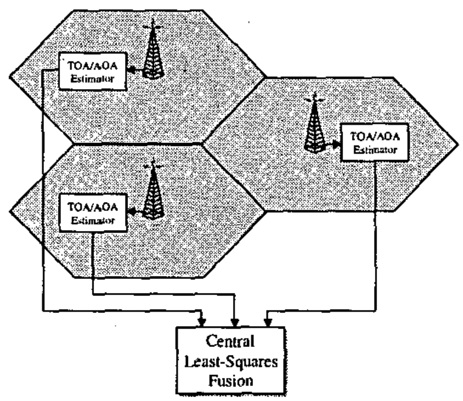

Fig. 1. Top level block diagram of the location finding structure.

in terms of the conditional probability of K vector measurements. It was shown in [4], [5] that this optimization problem can be reduced to maximizing a function of the form

$$J(\tau|\theta) = \frac{1}{P}\sum_{m=1}^{P}\left|\frac{1}{K}\sum_{n=n_0}^{mK}\mathbf{A}_\theta^*\mathbf{r}(n)s(n-\tau)\right|^2 \tag{4}$$

where P is the length of the non-coherent correlator, and $n_0 = (m-1)K+1$. The resulting receiver structure is shown in Fig. 2. The use of an antenna array increases the SNR of the receiver by the beamformer gain and results in higher accuracy in τ compared to a single-antenna receiver.

III. Multi-User Least-Squares AOA Estimation

The channel model (1) can be modified to accommodate a multi-user multi-path scenario. The receiver observes a linear combination of all transmitted data sequences by all active users, each distorted by ISI under white Gaussian noise. Assuming the maximum number of channel taps to be L, and the number of active users to be N, the received signal $\mathbf{r}(n)$ of size $M \times 1$ at time n is now

$$\mathbf{r}(n) = \sum_{i=1}^{N}\sum_{k=1}^{L}\mathbf{h}_{i,k}(n)s_i(n-k) + \mathbf{v}(n) \tag{5}$$

where $s_i(n)$ is the transmitted sequence by the ith user and $\mathbf{h}_{i,k}(n)$ contains the kth channel tap from user i and can be

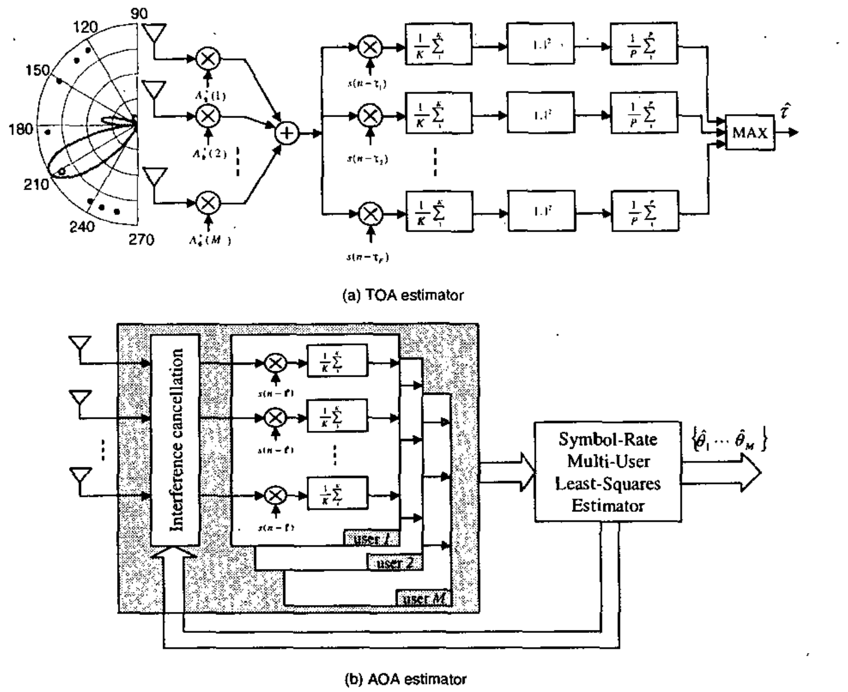

Fig. 2. Receiver structure for TOA and AOA estimation using antenna arrays.

written as

$$\mathbf{h}_{i,k}(n) = \mathbf{A}_{\theta_{i,k}} c_{i,k}(n) \tag{6}$$

where $c_{i,k}(n)$ is the kth tap of the channel from the ith user to the base station, and $\mathbf{A}_{\theta_{i,k}}$ is the array response as a function of the AOA of the kth multipath of the ith user given by (2).

The received SNR at the base station is generally low due to multiuser interference, which makes the channel estimates directly derived from the received samples not accurate. In CDMA systems, the training sequences $s_i(n)$ for different users are designed to have good orthogonality properties. This property can be used to increase the SNR at the receiver by correlating $\mathbf{r}(n)$ with the known sequence transmitted by each user. The correlation results for each specific user and multipath can be used to estimate the AOA for that user and multipath. Assuming constant channel taps during the estimation period, the correlation results , $\mathbf{x}_{i,k}$, for user i and multipath k can be written in vector form :

$$\begin{aligned} \mathbf{x}_{i,k} &= \mathbf{G}_i \mathbf{h}_{i,k} + \mathbf{i}_{i,k} + \mathbf{v}' \\ \mathbf{i}_{i,k} &= \sum_{m=1}^{N} \sum_{\substack{l=1 \\ m \neq i, l \neq k}}^{L} \rho_{i,m,k,l} \mathbf{G}_m \mathbf{h}_{m,l} \end{aligned} \tag{7}$$

where $\rho_{i,m,k,l}$ is the normalized cross-correlation between the known training sequences of users i and m, received on multipaths k and l, respectively, $\mathbf{G}_i$ is the identity matrix of size $M \times M$ scaled by the power of the signal s_i. Equation (7) can be used to form a least-squares estimator for $\mathbf{h}_{i,k}$ and, consequently, the AOA for user i and multipath k, i.e., $\theta_{i,k}$. Collecting R realizations of the correlation results $\mathbf{x}_{i,k}$ into a vector $\mathbf{d}_{i,k}$, for a block LS problem can be formulated as (assuming constant-magnitude training sequences):

$$\mathbf{d}_{i,k} \triangleq \begin{bmatrix} \mathbf{x}_{i,k}(1) \\ \mathbf{x}_{i,k}(2) \\ \vdots \\ \mathbf{x}_{i,k}(R) \end{bmatrix}, \quad \mathbf{H}_i \triangleq \begin{bmatrix} \mathbf{G}_i \\ \mathbf{G}_i \\ \vdots \\ \mathbf{G}_i \end{bmatrix}$$

$$\begin{aligned} \hat{\mathbf{h}}_{LS_{i,k}} &= \arg\min_{\mathbf{h}_{i,k}} \|\mathbf{d}_{i,k} - \mathbf{H}_i \mathbf{h}_{i,k}\|^2 \\ &= (\mathbf{H}_i^* \mathbf{H}_i)^{-1} \mathbf{H}_i^* \mathbf{d}_{i,k} \end{aligned} \tag{8}$$

This LS estimation is repeated for all users and multipaths, $i = 1 \dots N, k = 1 \dots L$.

As the number of users and multipaths increases, Multiple-Access-Interference (MAI) and Inter-Symbol-Interference (ISI) in equation (7) become stronger. In practical scenarios with a large number of active users in a cell, the accuracy of the LS estimation of AOA is limited by MAI and ISI. One solution for reducing the effect of MAI in (7) is to increase the coherent correlation length K. This is due to the fact that the cross correlation terms ρ in equation (7) are in the order of $\frac{1}{K}$ for well designed training sequences. However, the correlation or estimation length cannot be increased indefinitely in order to achieve a higher estimation accuracy. The limit on the correlation length is set by the *Channel Coherence Time*.

We propose a joint least-squares estimation followed by a multiuser interference cancellation technique to provide an accurate AOA estimation even in the presence of a large number of interfering users. This joint technique takes advantage of the following two facts:

- A base station decodes the signals from all users simultaneously(the base station knows the training sequences used by the users).
- The LS estimation of the channel taps for all users ($\hat{\mathbf{h}}_{LS_{i,k}}, i = 1 \dots N, k = 1 \dots L$) are available at the base station.

The above information is then used in a secondary stage to further increase the accuracy of the estimation by cancelling the MAI in (7). The following are the steps performed in the proposed architecture:

1) Use equations (7) and (8) to calculate the LS estimation of the channel taps for all users and multipaths.
2) Use the estimated channel taps to regenerate (estimate) the MAI in equation (7), i.e.,

$$\hat{\mathbf{i}}_{i,k} = \sum_{m=1}^{N} \sum_{\substack{l=1 \\ m \neq i, l \neq k}}^{L} \rho_{i,k,m,l} \mathbf{G}_m \hat{\mathbf{h}}_{LS_{m,l}} \qquad (9)$$

3) Subtract the estimated interference in step (2) from the correlation results in equation (7).
4) Use the new $\mathbf{x}_{i,k}$ in the LS estimation to calculate a new estimate for channel taps referred to as MU-LS estimation.
5) Repeat steps 2-4 for all users and multipaths.

IV. Combining Location Estimates From Different Base Stations

Now assume n BSs estimate the AOA and TOA of the MS. We shall define the cellular system features as follows. See Fig. 3.

- $\begin{pmatrix} x_m \\ y_m \end{pmatrix}$: mobile location.
- $\begin{pmatrix} x_i \\ y_i \end{pmatrix}$: ith base station location.
- r_i: the distance from the MS to the ith BS.
- α_i: the angle of arrival from the MS to the ith BS.
- τ_i: the time difference of arrival of the ith BS.

The location of the mobile station can be determined from knowledge of these features. For instance, we know that $r_i = \tau_i \times C$, where C is the speed of light (3×10^8m/s) and, moreover,

$$r_i^2 = (x_i - x_m)^2 + (y_i - y_m)^2$$

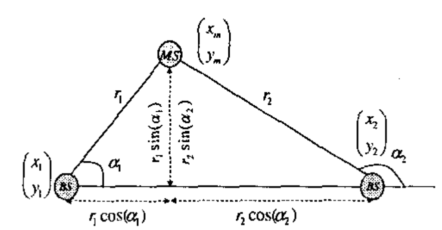

Fig. 3. Using the topology of the cellular network to improve location accuracy.

If we choose the first BS as the origin of the coordinate system (i.e., if we set $x_1 = y_1 = 0$), then the location of the MS can be estimated via the least-squares solution:

$$\begin{pmatrix} x_m \\ y_m \end{pmatrix} = (\mathbf{H}^{\mathbf{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathbf{T}}\mathbf{b} \qquad (10)$$

where

$$\mathbf{H} = \begin{pmatrix} x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \vdots \\ x_n & y_n \end{pmatrix}, \ \mathbf{b} = \frac{1}{2}\begin{pmatrix} K_2^2 - r_2^2 + r_1^2 \\ K_3^2 - r_3^2 + r_1^2 \\ \vdots \\ K_n^2 - r_n^2 + r_1^2 \end{pmatrix}$$

$$K_i^2 = x_i^2 + y_i^2$$

We can improve the measurement accuracy by incorporating the angle measurements. These measurements provide additional equations of the from:

$$\begin{pmatrix} x_m \\ y_m \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix} + \begin{pmatrix} r_i\cos(\alpha_i) \\ r_i\sin(\alpha_i) \end{pmatrix}$$

which can be incorporated into the least-square solution (10) by redefining H and b as

$$\mathbf{H} = \begin{pmatrix} x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \vdots \\ x_n & y_n \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} \frac{1}{2}(K_2^2 - r_2^2 + r_1^2) \\ \frac{1}{2}(K_3^2 - r_3^2 + r_1^2) \\ \vdots \\ \frac{1}{2}(K_n^2 - r_n^2 + r_1^2) \\ x_1 + r_1\cos\alpha_1 \\ y_1 + r_1\sin\alpha_1 \\ \vdots \\ x_n + r_n\cos\alpha_n \\ y_n + r_n\sin\alpha_n \end{pmatrix}$$

Measurements are subject to errors. These errors consist generally of two components: a line-of-sight (LOS) term that arises from measurement noise, and a non-line-of-sight term that arises from the temporal and spatial variations of the channel, and the distribution of scatterers. The work [8] presents an algorithm for equalizing the NLOS problem by using a constrained optimization formulation that exploits the topology of the cellular network. This formulation can be used to improve the accuracy of the location estimate.

V. Simulation Results

An uplink DS-CDMA system is used to evaluate the performance of the algorithms. We use the cdma2000 standard [9] to model the users within the cell and simulate the link (any DS-CDMA standard can be used). For simplicity, we assume a processing gain of 64 for all users. The known pilot sequence transmitted by each user is used for training purposes at the base station. The CDMA chip-rate in the simulation is 4MHz, and a 4-element antenna array is employed at the base station. The antenna spacing is assumed to be half a wavelength. The mobile users are uniformly distributed in a 120 degrees sector. This is due to the fact that in current cell designs, the cell is divided into 3 sectors of each 120 degrees. We choose the coherent correlation length (K) to be equal to the processing gain.

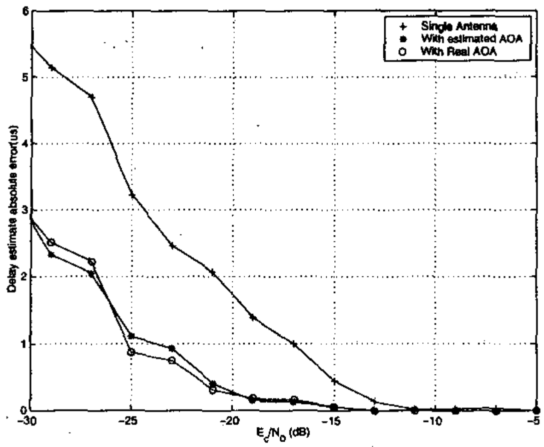

Fig. 4. Standard deviation of TOA estimation error versus E_c/N_0

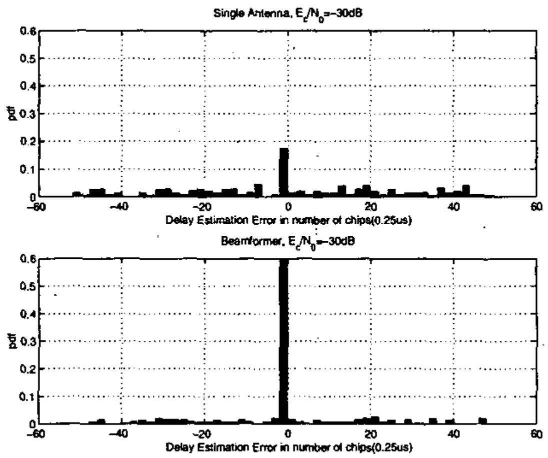

Fig. 5. Distribution of the error in TOA estimation at E_c/N_0 of -30dB

A. TOA Estimation

In this subsection we evaluate the performance of the TOA estimation technique presented in Sec. II. Therefore, in this part, we assume a single path channel with Rayleigh distribution and Doppler frequency of 100Hz. This doppler frequency corresponds to a velocity of about 40mph at the carrier frequency of 1.8GHz. The error in TOA estimation versus the chip energy-to-noise ratio (E_c/N_0) is shown in Figure 4 for single antenna and multiple antenna scenarios. As shown in the figure, using antenna arrays at the base station improves the TOA estimation significantly. In the multiple antenna case, two different beamformers are simulated. One is the ideal beamformer using the real value for AOA, and the other is using the LS estimation of the AOA to form the beam. As shown in Figure 3, the degradation in performance due to AOA estimation error is small. This also validates the accuracy of the AOA estimation algorithm for chip energy-to-noise ratios as low as -30dB.

In addition to the variance of the TOA estimation error, there is another criterion to evaluate the performance of location estimation algorithms, called *Failure Rate*. Failure rate is defined as the probability at which the algorithm fails to estimate the location with an acceptable accuracy. The simulation results for single antenna and multiple antenna receivers are shown in Figure 5. In this plot, an accuracy of 1 CDMA chip duration space ($0.25\mu s$) in TOA estimation is assumed as an acceptable estimation. This amount of TOA estimation error translates into 75 meters error in location estimation. As shown in the figure, using an antenna array decreases the failure rate in TOA estimation significantly. At $E_c/N_0 = -30$dB, using an antenna array improves the failure rate from 82 percent down to 41 percent.

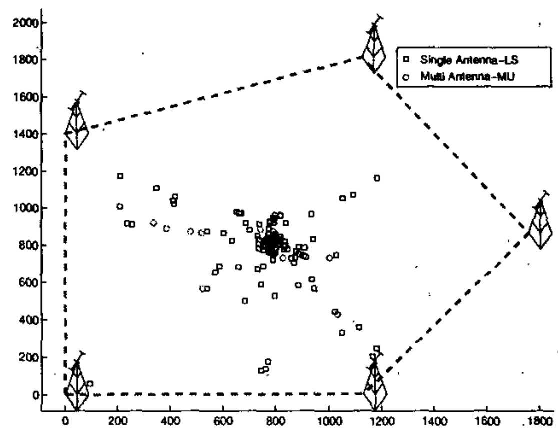

Fig. 6. Location of estimated points using two different methods. The SNR at base stations are 2.44,-4.64,7.52,2.96 and 1.92 dB. The simulation is over 400 different channels and the processing gain is 16. The actual mobile station station location is at (793,810).

B. Location Using Multiple Base Stations

In this subsection, we consider the complete location estimating architecture. First, we simulate CDMA transmissions of a MS in the presence of interfering users for different scenarios, then we simulate the proposed TOA and AOA estimators on each BS, and finally we use results of the Sec. IV to combine the individual estimates provided by each base station to find the MS location. Fig. 6 shows the estimated points by using two different methods. In this figure we simulated the case of five BSs and line-of-sight channels for all BSs and four antennas for each BS. Fig. 7 shows the corresponding estimation error for 65 and 95 percent outage using two different methods for line-of-sight and non-line-of-sight [8] situation. In this figure, distance between BSs is 1200 meter and 4-element antenna arrays are used by each base station. Fig. 8 and Fig. 9 show the outage values for different scenarios. Fig. 9 depicts the effect of processing gain and Fig. 8 illustrates the effect of NLOS using multiple antennas. In all the plots, MU and MA denote multi-user-cancellation and multiple-antennas. Moreover, LS and SA stand for least-squares and single-antenna.

VI. Conclusions

Combining multiple-antennas reception with non-line of sight equalization, and exploiting the topology of the cellular

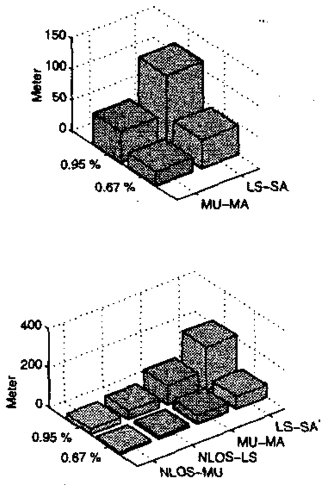

Fig. 7. Outage bar vs. different methods in location estimation for 3 active users. For the top figure the SNR at the base stations are -1.71,-1.71,4.8,-0.40, and -1.34 dB. The simulation is over 600 different channels and processing gain is 64. For the lower figure there are four BSs and three of them are NLOS. The simulation is over 300 different channels and processing gain equals 64. The NLOS equalization proposed in [8] is used.

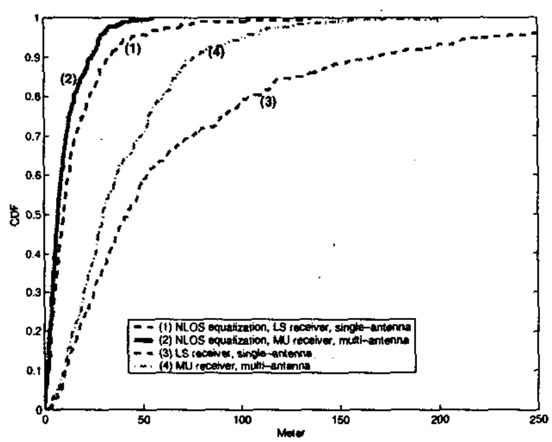

Fig. 8. Outage curve vs. four different methods in location estimation for 4 active users. Three BSs out of four are NLOS. The simulation is over 300 different channels and processing gain is 32.

network, can lead to significant improvements in wireless location accuracy.

References

[1] FCC, "Revision of the commission's rules to ensure compatibility with enhanced 911 emergency calling systems," *Report and Order and Further Notice of Proposed Rulemaking*, Washington D.C. Federal Communications Commission, 1996.

[2] J. Caffery and G. L. Stuber, "Subscriber location in CDMA cellular networks," *IEEE Trans. on Vehicular Technology*, vol. 47, pp. 406–415, May 1998.

[3] N. J. Thomas, D. G. M. Cruickshank and D. I. Laurenson, "Performance of a TDOA-AOA hybrid mobile location system," *3G Mobile Communication Technologies*, No. 477, pp. 216–220, March 2001.

[4] N. R. Yousef and A. H. Sayed, "A new adaptive estimation algorithm for wireless location finding systems," *Proc. Asilomar Conference*, vol. 1, pp. 491–495, October 1999.

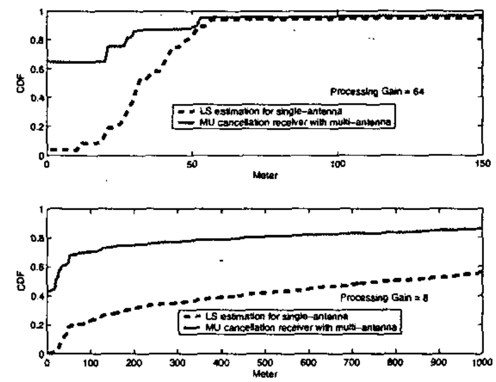

Fig. 9. Outage curve vs. two different methods in location estimation for 3 active users. The SNR at base stations are 3.96,-0.6,0.4,0.92 and -5.68 dB. The simulation is over 600 different channels and processing gain for top figure is 64 and low figure is 8.

[5] A. Tarighat, N. Khajehnouri and A. H. Sayed, "Improved wireless location accuracy using antenna arrays and interference cancellation," *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Hong Kong, April 2003.

[6] J. C. Liberti and T. S. Rappaport, *Smart Antennas for Wireless Communication*, Englewood Cliffs, NJ: Prentice-Hall, 1999.

[7] J. Proakis, *Digital Communications*, McGraw-Hill, New York, 2000.

[8] N. Khajehnouri and A. H. Sayed, "A non-line-of-sight equalization scheme for wireless cellular location," *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Hong Kong, April 2003.

[9] 3GPP2, "Third Generation Partnership Project2," http://www.3gpp2.org, 2000.