A Robust Scheme for Cellular Power Control

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Abstract—**In this paper, we propose a robust filtering approach to distributed power allocation. A robust filter is used to predict the channel gain and interference under incomplete knowledge of the shadowing coefficient. Simulation results show the superior performance of the robust solution in comparison to a traditional Kalman-filter-based solution in terms of blocking probability.**

keywords: Robust filter, Kalman filter, power control, dynamic channel allocation, cellular network.

I. Introduction

The evolution of mobile wireless communications has triggered the interest in finding efficient power control algorithms. Many schemes have been investigated in the literature. Some initial distributed power control strategies that balance the signal-to-interference ratios were given in [1]–[5]. Later approaches [6], [7] incorporated QoS requirements. A Kalman filtering approach was given in [8] using the model introduced in [9]. This approach considers admission control as the central QoS issue. The basic assumption, however, is that the channel dynamics is accurately known, which in turns requires an accurate knowledge of the shadowing coefficient. Yet, even a good estimation algorithm for the shadowing coefficient will generally be prone to errors. We address this issue in this paper by considering a M/M/m/m queueing model for each cell and estimating the channel and the interference gains through a robust filtering algorithm. The robust filter will be used to combat the channel shadow fading uncertainty and to track the channel gain and users' interference in each BS.

The organization of the paper is as follows. In the next section we introduce the system model. In Sec. III, we derive certain interference statistics that can be used to initialize the robust filter given in Sec. V. In Sec. IV, we propose a power control scheme and in Sec. V we derive a robust power control algorithm. Sec. VI gives simulation results.

II. System model

Consider a cellular network with a limited number of users in each cell. The queuing model adopted for each cell is the Erlang M/M/m/m model. The users connect to the Base Station (BS) using the TDMA/FDMA multiple access scheme. The Signal-to-Interference-plus-Noise-Ratio (SIR) for user i on an uplink channel is

$$\Gamma_i = \frac{G_{ii} p_i}{\sum_{j \in \mathbf{A}} G_{ij} p_j + \eta_i} \qquad (1)$$

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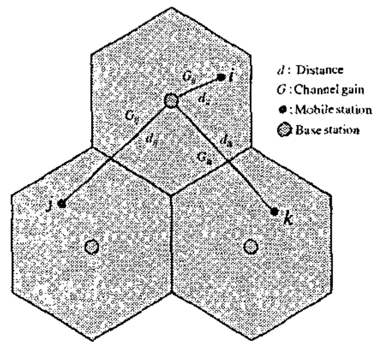

Fig. 1. Cellular system.

where G_{ij} is the channel gain from the jth user to the intended BS of the ith user, p_i is the transmitting power from the ith user, $\eta_i > 0$ is the thermal noise power at the receiver of the BS that user i is connected to, and $\mathbf{A}$ is the set of all users that are interfering with user i from other cells (say N_I of them). We use the model proposed in [9] for the channel gain from the ith user to its BS. In this model, G_{ii} has a lognormal distribution, i.e.,

$$G_{ii} = S_0 d_{ii}^{-\beta} 10^{\alpha/10} \qquad (2)$$

where S_0 is a function of the carrier frequency, β is the path loss exponent (PLE), and d_{ii} is the distance of the Mobile Station (MS) from the BS. The value of β depends on the physical environment and changes between 2 and 6 (usually 4), while, α is a zero mean Gaussian random variable with variance σ^2, which usually ranges between 6 and 12. Based on these statistics, the probability density function (pdf) of G_{ii}, conditioned on knowledge of d_{ii} takes the form

$$f_{G_{ii}}(g) = \frac{1}{\sigma' g \sqrt{2\pi}} e^{-\frac{(\ln(g)-\mu)^2}{2\sigma'^2}}$$

where $\mu = \ln(S_0) - \beta \ln(d_{ii})$ and $\sigma' = \frac{\sigma}{10} \ln 10$. This expresion shows that G_{ii} has a lognormal distribution with mean $e^{\frac{(\sigma \ln 10/10)^2}{2}} S_0 d_{ii}^{-\beta}$.

We shall neglect the effect of fast fading and assume that the power update algorithm has a large time period. The correlation between the logarithm of the channel power gains at two time instant separated by k samples is given by

$$R_{G_{ii}}(k) = \sigma_s^2 a^{|k|} \quad \text{where} \quad a = 10^{-\frac{vT}{D}} \tag{3}$$

with σ_s^2 ranging between 3 and 10 dB, and where v is the mobile station velocity, and T is the time period for channel probing. Moreover, D is the distance at which the normalized correlation reaches the value $\frac{1}{10}$. We now proceed to derive some interference statistics that will be used for the initialization of the proposed power control algorithm.

III. Channel gains and interference statistics

For simplicity, we assume a uniform distribution of MS's inside a circle of radius R, i.e.,

$$\begin{aligned} f_{d_{ii}}(r) &= 1/(0.9R) \qquad 0.1R \le r \le R \\ E(G_{ii}|d_{ii}) &= S_0' d_{ii}^{-\beta} \end{aligned}$$

so that

$$\begin{aligned} E(G_{ii}) &= \int_{0.1R}^{R} E(G_{ii}|d_{ii} = r) f_{d_{ii}}(r) dr \\ &= \frac{S_0' R^{-\beta}(1 - (0.1)^{1-\beta})}{0.9(1-\beta)} \end{aligned}$$

where $f_{d_{ii}}(r)$ is the probability density function of the distance of a mobile station from its base station, and

$$S_0' = e^{\frac{(\sigma \ln 10/10)^2}{2}} S_0$$

We also consider a uniform distribution for d_{ij}, the distance from the jth interfering MS to the base station of user i, in the range of R to $5R$, and assume that mobile stations farther than $5R$ do not interfere. Then

$$\begin{aligned} f_{d_{ij}}(r) &= 1/4R, \qquad R \le r \le 5R \\ E(G_{ij}|d_{ij}) &= S_0' d_{ij}^{-\beta} \end{aligned}$$

so that

$$\begin{aligned} E(G_{ij}) &= \int_{R}^{5R} E(G_{ij}|d_{ij} = r) f_{d_{ij}}(r) dr \\ &= \frac{S_0' R^{-\beta}(5^{1-\beta} - 1)}{4(1-\beta)} \end{aligned}$$

We can now derive the expected value of the interference at any given time instant. First, we make the following assumptions:

- The transmitted power of user j is independent of the channel gain of user i to its BS.
- The sum of the averages of all user powers is a constant depending only on the number of users.
- The location of a mobile station follows a time-invariant probability density function.

The interference for user i is given by,

$$I_i = \sum_{j \in \mathbf{A}} G_{ij} p_j + \eta_i \tag{4}$$

The conditional mean of I_i given that a total of N_I users cause interference is given by

$$\begin{aligned} E(I_i|N_I) &= E\left(\sum_{j \in \mathbf{A}} G_{ij} p_j + \eta_i\right) &(5) \\ &= \sum_{j \in \mathbf{A}} E(G_{ij}) E(p_j) + \eta_i &(6) \\ &= \sum_{j \in \mathbf{A}} \frac{S_0' R^{-\beta}(5^{-\beta+1} - 1)}{4(1-\beta)} \bar{p_j} + \eta_i &(7) \\ &= \frac{S_0' R^{-\beta}(5^{-\beta+1} - 1)}{4(1-\beta)} \sum_{j \in \mathbf{A}} \bar{p_j} + \eta_i &(8) \end{aligned}$$

where $\bar{p} = E(p_j)$. Now, from the assumptions, we have that

$$\sum_{j \in \mathbf{A}} \bar{p_j} = N_I \bar{P} \tag{9}$$

where $\bar{P}$ is a measure for the average amount of power per user in the system, It follows that

$$E(I_i|N_I) = \frac{S_0' R^{-\beta}(5^{-\beta+1} - 1)}{4(1-\beta)} N_I \bar{P} + \eta_i \tag{10}$$

For the Erlang model for each cell, we assume that there is a finite number of servers or free slots. The mean time between two call arrivals is $\frac{1}{\lambda}$ and the mean time between two departures is $\frac{1}{\mu}$. If we suppose that the FDMA/TDMA multiple access scheme has n_f frequency slots and n_t time slots in each cell, we then have $n_s = n_t \times n_f$ users that can be served in each cell. Therefore, in steady state, the probability of having N_I interfering users for a particular user i can be obtained as follows. The interference for a particular user i is caused by other users in the same frequency and time slot from other cells. Let $P(k)$ be the probability of having k active users in a cell different from that of user i,

$$P(k) = \frac{\frac{(\lambda/\mu)^k}{(k)!}}{\sum_{v=0}^{n_s} \frac{(\lambda/\mu)^v}{v!}}, \quad 0 \le k \le n_s$$

The probability of having one of these users in the same slot as user i is

$$Q(k) = \frac{\binom{n_s - 1}{k-1}}{\binom{n_s}{k}} = \frac{k}{n_s}$$

The unconditional probability of having a user interfering with user i is therefore

$$\begin{aligned} Z &= \sum_{k=1}^{n_s} P(k) Q(k) \\ &= \sum_{k=1}^{n_s} \frac{\frac{(\lambda/\mu)^k}{k!}}{\sum_{v=0}^{n_s} \frac{(\lambda/\mu)^v}{v!}} \cdot \frac{k}{n_s} \\ &= \frac{\lambda}{\mu n_s} \left(1 - \frac{\frac{(\lambda/\mu)^{n_s}}{(n_s)!}}{\sum_{v=0}^{n_s} \frac{(\lambda/\mu)^v}{v!}}\right) \end{aligned}$$

Now the probability of having m interfering users with user i over all cells is

$$P(N_I = m) = \binom{N_s}{m} Z^m (1-Z)^{N_s - m}$$

where $N_s + 1$ is the total number of BSs. The average number of users that generate interference is then given by the mean of a binomial distribution

$$E(N_I) = N_s \times Z = \left(\frac{\lambda N_s}{\mu n_s}\right)\left(1 - \frac{\frac{(\lambda/\mu)^{n_s}}{(n_s)!}}{\sum_{v=0}^{n_s} \frac{(\lambda/\mu)^v}{v!}}\right)$$

Hence,

$$\begin{aligned} E(I_i) &= E(E(I_i|N_I)) \\ &= E\left(\frac{S_0' R^{-\beta}(5^{-\beta+1}-1)}{4(1-\beta)} N_I \bar{P}\right) + \eta_i \\ &= \left(\frac{S_0' R^{-\beta}(5^{-\beta+1}-1)}{4(1-\beta)} \bar{P}\right) E(N_I) + \eta_i \\ &= \left(\frac{S_0' R^{-\beta}(5^{-\beta+1}-1)}{4(1-\beta)} \bar{P}\right)\left(\frac{\lambda N_s}{\mu n_s}\right) \\ &\times \left(1 - \frac{\frac{(\lambda/\mu)^{n_s}}{(n_s)!}}{\sum_{v=0}^{n_s} \frac{(\lambda/\mu)^v}{v!}}\right) + \eta_i \end{aligned}$$

The knowledge of $E(G_{ii})$ and $E(I_i)$ will help us speed the convergence of the power control algorithm.

IV. Power Control Strategy

Every new user has to satisfy an SIR threshold condition in order to be continuously served by the system. For every user i, we should have

$$\Gamma_i > \gamma_i$$

where γ_i is the necessary SIR for user i to be able to use an idle channel. After the initial channel assignment, whenever the SIR drops below $\gamma_{\min}$, the algorithm tries to find a new idle channel with SIR more than $\gamma_{\min}$ or the call drops otherwise. As in [10], we can describe the feasibility condition in terms of a set of equations as

$$\mathbf{p} \geq \mathbf{T}(\mathbf{Z} - \mathbf{I})\mathbf{p} + \mathbf{u}$$

where

$$\begin{aligned} \mathbf{p} &= [p_i] \quad (N \times 1 \text{ vector}) \\ \mathbf{Z} &= [z_{ij}] \text{ and } z_{ij} = \frac{G_{ij}}{G_{ii}} \\ \mathbf{T} &= \operatorname{diag}(\gamma_1, \cdots, \gamma_N) \\ \mathbf{u} &= [u_i] = \left[\frac{\gamma_i \eta_i}{G_{ii}}\right] \\ \mathbf{I} &= N \times N \text{ identity matrix} \end{aligned}$$

where N is the number of users in the system. Now if we assume we know the channel power gains $\{G_{ij}\}$ and the noise powers $\{\eta_j\}$, then the minimum feasible power is

$$\hat{\mathbf{p}} = (\mathbf{I} - \mathbf{T}(\mathbf{Z} - \mathbf{I}))^{-1}\mathbf{u}$$

The $\{\hat{\mathbf{p}}_i\}$ can be estimated recursively in a decentralized algorithm format as follows [10]:

$$p_i(n) = \frac{\gamma_i}{G_{ii}} I_i(n-1) = p_i(n-1)\frac{\gamma_i}{\Gamma_i(n-1)} \tag{11}$$

In dB scale, (11) can be written as (the subscript d means dB scale values):

$$p_{id}(n) = p_{id}(n-1) + \gamma_{id} - \Gamma_{id}(n-1)$$

It can be shown that under constant channel gains, this algorithm converges.

V. State Space Model and Robust Filtering

Usually, the gains $\{G_{ij}\}$ are not known accurately. We now describe a procedure to determine the $\{p_i\}$ under incomplete knowledge of the channel gains. A first-order Markov random model for the channel gains that results from (3) is as follows (see, e.g., [8]):

$$G_{iid}(n) = \bar{G}_{iid}(n) + \delta G_{iid}(n) \tag{12}$$

$$\delta G_{iid}(n) = a\delta G_{iid}(n-1) + v_{G_d}(n) \tag{13}$$

where $v_{G_d}(n)$ is white zero-mean Gaussian noise, $a = 10^{\frac{-vT}{D}}$, $G_{iid}(n)$ is the channel gain from the ith MS to its BS in dB scale, and $\bar{G}_{iid}$ is the bias value. We can use a similar first-order Markov model for the interference:

$$I_{id}(n) = \bar{I}_{id}(n) + \delta I_{id}(n) \tag{14}$$

$$\delta I_{id}(n) = a\delta I_{id}(n-1) + v_{I_d}(n) \tag{15}$$

where $v_{I_d}(n)$ is white zero-mean Gaussian noise. From (12)–(15) we get

$$\begin{aligned} G_{iid}(n) &= aG_{iid}(n-1) + (1-a)\bar{G}_{iid}(n) + v_{G_d}(n) \\ I_{id}(n) &= aI_{id}(n-1) + (1-a)\bar{I}_{id} + v_{I_d}(n) \end{aligned}$$

or, equivalently,

$$\begin{pmatrix} G_d(n) \\ \bar{G}_d(n) \end{pmatrix} = \begin{pmatrix} a & 1-a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} G_d(n-1) \\ \bar{G}_d(n-1) \end{pmatrix} + \begin{pmatrix} v_{G_d}(n) \\ \bar{v}_{G_d}(n) \end{pmatrix}$$

with measurement equation

$$y_G(n) = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} G_d(n) \\ \bar{G}_d(n) \end{pmatrix} + v_{GG_d}(n)$$

where $v_{GG_d}(n)$ is zero-mean white noise. Similarly, for the interference, we have

$$\begin{aligned} \begin{pmatrix} I_d(n) \\ \bar{I}_d(n) \end{pmatrix} &= \begin{pmatrix} a & 1-a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} I_d(n-1) \\ \bar{I}_d(n-1) \end{pmatrix} + \begin{pmatrix} v_{I_d}(n) \\ \bar{v}_{I_d}(n) \end{pmatrix} \\ y_I(n) &= \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} I_d(n) \\ \bar{I}_d(n) \end{pmatrix} + v_{II_d}(n) \end{aligned}$$

where $v_{II_d}(n)$ is zero-mean white Gaussian noise. Moreover $\bar{v}_{G_d}(n)$ and $\bar{v}_{I_d}(n)$ are independent of each other and of $v_{II_d}(n)$ and $v_{GG_d}(n)$. The value a is usually not accurately known.

Hence, we shall use a robust filter that estimates the channel gains and the interference without complete knowledge of a. Let x_n denote either the vector $\begin{pmatrix} G_d(n-1) \\ \bar{G}_d(n-1) \end{pmatrix}$ or $\begin{pmatrix} I_d(n-1) \\ \bar{I}_d(n-1) \end{pmatrix}$ and let $y(n)$ denote $y_G(n)$ or $y_I(n)$, respectively. Then consider an $n-$dimensional state-space model of the form:

$$x_{n+1} = Ax_n + Bu_n \quad (16)$$
$$y(n) = Cx_n + v(n), \quad k \geq 0 \quad (17)$$

where $\{u_n, v(n)\}$ are uncorrelated white zero-mean random processes with unknown but bounded variances, say

$$Eu_n u_n^* \leq \rho_u I, \qquad Ev(n)v(n)^* \leq \rho_v$$

for some $\{\rho_u, \rho_v\}$. The initial condition x_0 is also a zero-mean random variable that is uncorrelated with $\{u_n, v(n)\}$ for all n. The state matrix A and the output matrix C are unknown but are assumed to lie inside a convex bounded polyhedral domain $\mathcal{K}$ described by p vertices as follows:

$$\mathcal{K} = \left\{ (A, C) = \sum_{i=1}^{i=p} \alpha_i (A_i, C_i), \;\; \alpha_i \geq 0, \;\; \sum_{i=1}^{i=p} \alpha_i = 1 \right\} \quad (18)$$

The vertices $A_i, i = 1, .., p$, are assumed to be bounded, say $\|A_i\| \leq \beta$.

A robust linear filter for estimating x_n has the form

$$\hat{x}_{n+1} = A_f \hat{x}_n + B_f y_n, \quad n \geq 0 \quad (19)$$

for some matrices A_f and B_f. One way to determine $\{A_f, B_f\}$ is as follows [13]. Start with a value $\gamma > 1$ that is close to 1.

1. Solve the following convex optimization problem over the variables $\{P_1, P_2, Q_1, Q_2, W\}$:

$$\min \; (\text{Tr}(B^T(P_1 + P_2)B + W)\frac{\gamma}{\gamma - 1} \quad (20)$$

subject to the conditions

$$\left(\begin{array}{cc|cc} \gamma_1^{-1}P_1 & 0 & A^T P_1 & \hat{J} \\ 0 & \gamma_2^{-1}P_2 & 0 & Q_1 \\ \hline P_1 A & 0 & P_1 & 0 \\ \hat{J}^T & Q_1^T & 0 & P_2 \end{array} \right) > 0$$

$$\begin{pmatrix} W & Q_2 \\ Q_2^T & P_2 \end{pmatrix} > 0$$

with $\gamma_1 > \gamma$, $\gamma_2 > \gamma$ and where

$$\hat{J} = -C^T Q_2 - Q_1 + A^T P_2$$

with $P_1 > I$ and $P_2 > I$ for all $\{A, C\}$ taking values in $[A_1, ..., A_p]$ and $[C_1, ..., C_p]$.

2. Compute the resulting cost of (20) and compare it with the previous cost.
3. If the new cost is less than the previous cost increment γ by $\delta\gamma$ (say .01) and go to step 1, otherwise go to step 4.
4. The filter parameters are given by

$$A_f = (Q_1 P_2^{-1})^T, \qquad B_f = (Q_2 P_2^{-1})^T \quad (21)$$

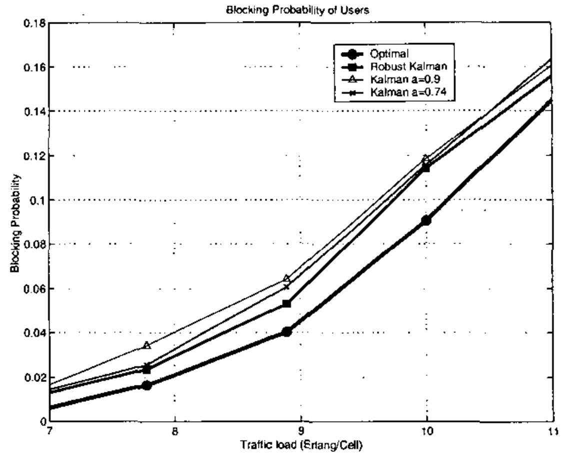

Fig. 2. Blocking Probability for a TDMA/FDMA system with a grid of 3 × 3 Bs's.

VI. Simulation Results

In the simulation environment, we use a TDMA/FDMA multiuser scheme similar to that in [11]. We consider 9 cells in a 3×3 grid with 2 carrier frequency and 8 time slots in each cell so that each cell has the capacity for 16 users. Base stations are located at the center of each cell and the distance between two base stations is 800 meters. The AR(1) coefficient a is a function of velocity update time and shadowing correlation distance. In our simulations, a varies between 0.74 when the mobile has a maximum speed and 1 when it does not move. The maximum signal-to-noise ratio for each mobile station is 35 dB when the MS is at the corner of the BS and there is no other user in the system. In this case the distance between the MS and BS is about $400\sqrt{2}$. Users arrive at the system with a poison distribution with arrival rate of λ and the service time or holding time for each user is an exponential distribution with average holding time of $\frac{1}{\mu}$. We consider a traffic load between 5.5 and 11 Erlang per cell, where the ratio of arrival rate to average service time $\left(\frac{\lambda}{\mu \times 9}\right)$ denotes the traffic in Erlang per cell. The velocity of each user admitted into the system is uniformly distributed ranging between 0 and 85 Km/h and the direction is also a uniformly distributed random variable between 0 and 2π. But once the velocity and direction are determined initially, they are kept constant during the user's service time. New users need to have 12 dB SIR to get admission into the system. The system tries to keep the SIR close to 10 dB. Whenever the SIR drops below 10 dB the system tries to find a better channel with SIR superior to 10 dB. If the SIR falls below 8 dB the system waits for 5 seconds to see if a new FDMA/TDMA channel is available before the call is dropped. Fig. 1 compares the blocking probability of the proposed method against Kalman filtering and Fig. 2 and 3 show power statistics for the proposed method is closer to an optimal solution than Kalman filtering. The initial estimates of the channel gains and interference are set to the theoretical values derived in section III.

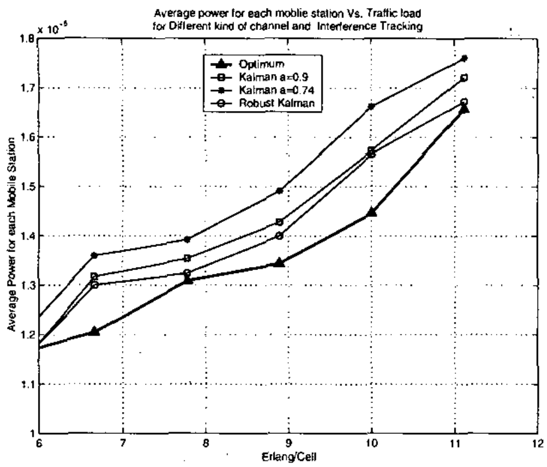

Fig. 3. Average power of mobile stations.

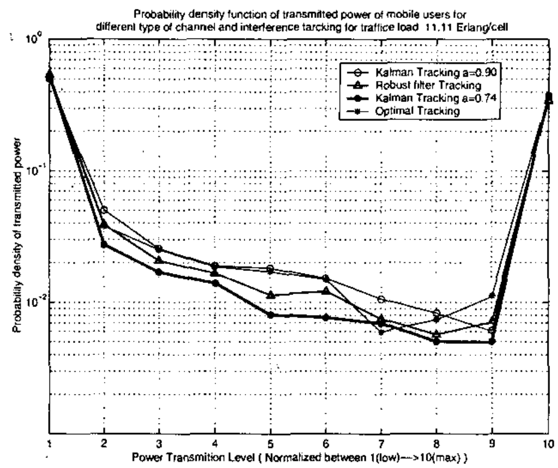

Fig. 4. PDF of transmitted power.

VII. Conclusion

In this paper, we proposed a robust predictive dynamic channel and power allocation scheme. A system level simulation environment shows the superior performance of the robust solution.

Parameter	Value	description
$SNR_{\max}$	35 dB	At the corner of cell
$a_{\max}$	1	when $v = 0$ km/h
$a_{\min}$	0.74	when $v = 85$ km/h
σ_s^2	8 dB	variance of the Gaussian part of lognormal distribution
σ_{Gd}^2	4 dB	variance of the Markov model white noise for Channel gain
σ_{Id}^2	4 dB	variance of the Markov model white noise for interference
σ_{GGd}^2	7 dB	variance of the measurement error for channel gain
σ_{iid}^2	7 dB	variance of the measurement error for interference
D	20 m	shadowing correlation distance
γ_{new}	12dB	SIR threshold for new user to enter to the system
γ_d	10 dB	SIR threshold for user to not reassign the channel
γ_{drop}	8 dB	SIR threshold for user to drop
D_b	800 m	distance between base stations

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