ROBUST TIME-DELAY AND AMPLITUDE ESTIMATION FOR CDMA LOCATION FINDING

NABIL R. YOUSEF*, LOUAY M. A. JALLOUL†, AND ALI H. SAYED‡

Abstract - *Wireless location finding is gaining considerable attention in the field of wireless communications. This is due to a recent government mandate for wireless service providers to locate users in emergency situations within a high precision. In this paper, we introduce a new estimation scheme for the time and amplitude of arrival of a code division multiple access signal transmitted over a single path Rayleigh fading channel for location finding applications. Optimum choices of the estimation scheme parameters are derived. The paper also presents a new noise and fading biases equalization technique for amplitude of arrival estimation that enhances its estimation precision significantly. The paper gives simulation and field trial results that show significant robustness of the proposed scheme to high noise levels and fast channel fading.*

I. Introduction

Wireless location has recently been receiving great attention in the field of wireless communications. This is mainly in response to a 1996 mandate by the U.S. Federal Communications Commission (FCC), which requires all wireless service providers to provide public safety answering points with information to locate an emergency 911 caller within an accuracy of 125 meters for at least 67 percent of the cases, by the year 2001 [1]. Several infrastructure-based techniques for wireless location finding have been developed during the last few years (see, e.g., [2, 3]). These techniques are based on combining estimates of the time of arrival (TOA) and/or amplitude of arrival (AOA) of the signal transmitted from the mobile station (MS) when received at different base stations (BSs). For CDMA cellular systems, this construction faces several obstacles due to the low signal-to-noise ratio (SNR) nature of a CDMA signal, which significantly degrades the precision of the TOA and AOA estimates and thus the location accuracy. The accuracy is further degraded for MSs moving at high speeds [4]. Moreover, most of the TOA/AOA estimation algorithms that already exist in the literature have been designed mainly for code acquisition or tracking purposes (see e.g., [5, 6]). In these applications, coarse estimates for the channel time delays and amplitudes are sufficient for online signal decoding using rake receivers. Using the same algorithms for wireless location applications is not adequate for two main reasons.

First, channel fading is usually assumed constant during the relatively short estimation period of these algorithms, and is therefore totally ignored. This assumption cannot be made for wireless location applications where the estimation period could be much longer. Secondly, the low precision of the coarse estimates provided by these algorithms does not generally satisfy the FCC requirements [1]. For these reasons, conventional TOA/AOA estimation algorithms lack the needed robustness to fast channel fading and high noise levels.

The purpose of this paper is to introduce a robust adaptive CDMA-TOA/AOA estimation scheme that combats the effects of high noise levels and fast channel fading on the accuracy of TOA/AOA estimation over a single path Rayleigh fading channel. This will be achieved by exploiting the fading channel autocorrelation model and an estimate of the maximum Doppler frequency of the Rayleigh fading channel.

II. The Proposed Scheme

Consider a received sequence $\{r(n)\}_{n=1}^{K}$ that arises from the model

$$r(n) = A\,x(n)s(n-\tau^o) + v(n) \ , \tag{1}$$

where $\{x(n)\}$ is the channel Rayleigh fading complex gain sequence, which is assumed to be ergodic and of unity power, and $v(n)$ is a zero-mean additive white Gaussian noise of variance σ_v^2. Moreover, $\{s(n)\}$ is a known pulse-shaped real-valued CDMA spreading sequence. Our objective is to estimate the discrete TOA or delay (τ^o) and the static AOA (A).

Figure 1 shows a block diagram of the proposed CDMA TOA/AOA searcher.[1] In the proposed

*N. R. Yousef and A. H. Sayed are with the Adaptive and Nonlinear Systems Laboratory, Dept. of Electrical Engineering, University of California, Los Angeles, CA 90095. E-mail: nabil@ee.ucla.edu.

†L. M. A. Jalloul is with MOTOROLA, Advanced Radio Technology Group, 1441 W. Shure Dr., Arlington Hts. IL 60004. E-mail: jalloul@cig.mot.com.

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[1]This structure was studied and tested at MOTOROLA in Arlington Heights, IL, during a 1998 Summer internship by the first author. A justification and derivation of the structure from a maximum likelihood estimation point of view is provided in [7].

scheme, the received sequence $\{r(n)\}$ is multiplied by a replica of the pulse-shaped spreading sequence $\{s(n-\tau)\}$, for different values of τ. The resulting sequence is first averaged coherently over an interval of N samples, and then averaged non-coherently for M samples to build a power delay profile, $J(\tau)$. The averaging intervals N and M are positive integers that satisfy $K = NM$, and the value of N will be picked adaptively in an optimal manner by using an estimate of the maximum Doppler frequency of the fading channel ($\hat{f}_D$), which can be estimated using different well-known techniques (see, e.g., [8]).

The searcher then picks the maximum of $J(\tau)$ and assigns its index to the TOA estimate, according to

$$\widehat{\tau^o} = \arg\max_{\tau} J(\tau) \ . \quad (2)$$

The searcher also equalizes the peak value of $J(\tau)$ by subtracting two fading and noise biases, which are estimated by means of the upper and lower branches of the scheme of Figure 1. The output of this correction procedure is taken as an estimate for the AOA.

We now explain how the optimal value for N, as well as the fading and noise biases correction factors, are calculated.

III. Parameter Optimization

The output of the noncoherent averaging process, $J(\tau)$, is seen from Figure 1 to be equal to

$$J(\tau) = \frac{1}{M}\sum_{m=1}^{M}\left|\frac{1}{N}\sum_{n=(m-1)N+1}^{mN} r(n)s(n-\tau)\right|^2 \ . \quad (3)$$

We will consider the case of an infinite received sequence length (so that $M \to \infty$). This situation is actually typical in wireless location applications where the duration of the received sequence is usually of the order of a fraction of a second and can, therefore, be assumed to be sufficiently long for analysis purposes. In this case, expression (3) can be approximated by

$$J(\tau) = \lim_{M\to\infty}\frac{1}{M}\sum_{m=1}^{M}\left|\frac{1}{N}\sum_{n=(m-1)N+1}^{mN} r(n)s(n-\tau)\right|^2$$

which, by the law of large numbers, can be written as

$$J(\tau) = \mathrm{E}\left|\frac{1}{N}\sum_{n=1}^{N} r(n)s(n-\tau)\right|^2$$

in terms of the expectation operator. Using (1), we obtain

$$J(\tau) = \mathrm{E}\left|\frac{1}{N}\sum_{n=1}^{N}\left[Ax(n)s(n-\tau^o)s(n-\tau) + v'(n,\tau)\right]\right|^2$$

where $v'(n,\tau) = v(n)s(n-\tau)$. For mathematical tractability of the analysis, we impose the following assumption.

A.1 *The sequence $\{s(n-\tau)\}$ is identically statistically independent (i.i.d), and is independent of the channel fading gain sequence $\{x(n)\}$.*

This assumption is feasible in CDMA systems, where the transmitted spreading sequences are usually chosen to be i.i.d. Then, it is straightforward to show that at $\tau = \tau^o$, the value of $J(\tau)$ becomes

$$J(\tau^o) = A^2\left[\frac{R_x(0)}{N} + \sum_{i=1}^{N-1}\frac{2(N-i)R_x(i)}{N^2}\right] + \frac{\sigma_v^2}{N} \quad (4)$$

where $R_x(i)$ is the autocorrelation function of the sequence $\{x(n)\}$, which is given by (see, *e.g.*, [9]):

$$R_x(|i|) = J_o\left(2\pi f_D T_s\, i\right) \ ,$$

where $J_o(\cdot)$ is the first order Bessel function, T_s is the sampling period of the received sequence $\{r(n)\}$, and f_D is the maximum Doppler frequency of the Rayleigh fading channel.

Equation (4) shows that, when $M \to \infty$, the value of the power delay profile at $\tau = \tau^o$ is composed of two terms. The first term is proportional to A^2, while the second term is proportional to σ_v^2. Thus, a performance index that we might maximize is the signal-to-noise ratio (SNR), defined as the ratio of the signal and noise terms at $\tau = \tau^o$. This SNR (S) is given, from (4), by

$$S = \frac{A^2}{\sigma_v^2}\left(R_x(0) + \sum_{i=1}^{N-1}\frac{2(N-i)R_x(i)}{N}\right) \ .$$

The SNR at the input of our scheme is $\frac{A^2}{\sigma_v^2}$. Thus, the SNR gain introduced by our estimation algorithm (S_G) is given by

$$S_G = \frac{S}{A^2/\sigma_v^2} = R_x(0) + \sum_{i=1}^{N-1}\frac{2(N-i)R_x(i)}{N} \ . \quad (5)$$

Figure 2 is a plot of this SNR gain as a function of N for a Rayleigh fading channel for various values of f_D. It can be seen that there is a value of the coherent averaging period, N_{opt}, that maximizes the SNR gain. Increasing N beyond this optimum value, the SNR gain oscillates and then asymptotically approaches a fixed value that depends on f_D. We can also see that the optimal value of N decreases monotonically as f_D increases.

The optimal value of the coherent averaging period (N_{opt}) is obtained by maximizing the SNR gain given in (5) with respect to N. It is easy to see that N_{opt} can be obtained by solving the equation

$$\sum_{i=1}^{N_{opt}-1} iR_x(i) = 0 \ . \quad (6)$$

This equation shows that the coherent averaging interval N *should be adapted* according to the channel autocorrelation function.

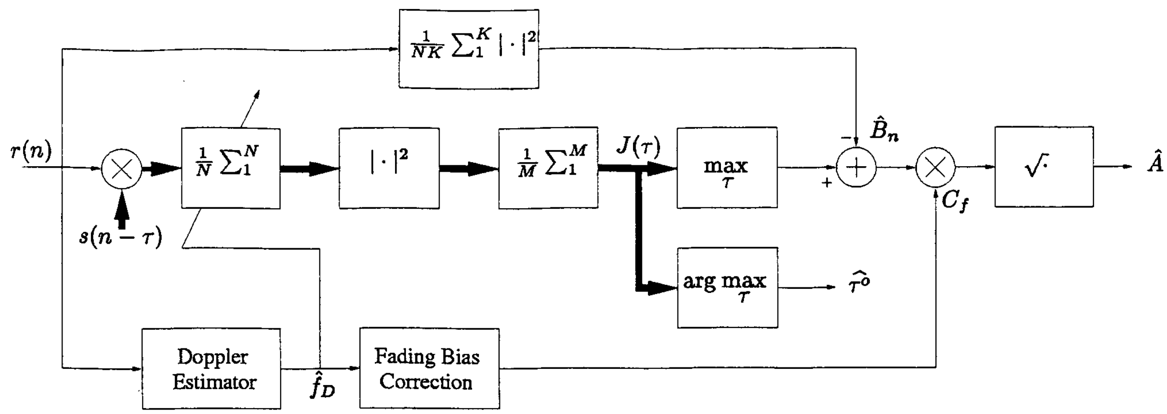

Figure 1: *A new CDMA location searcher scheme.*

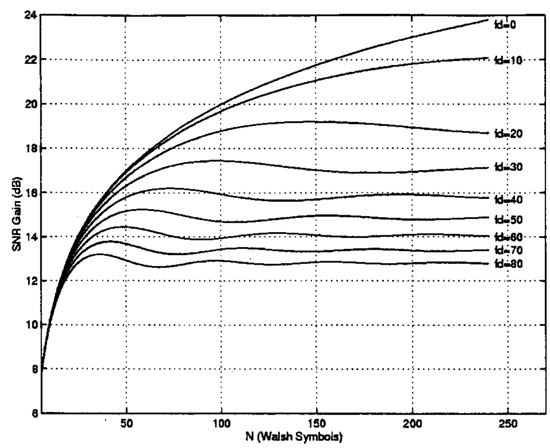

Figure 2: *SNR gain vs. the coherent averaging period N.*

IV. Amplitude Estimation

We now show how to estimate the AOA. For this purpose, observe from (4) that the value of the power delay profile, $J(\tau)$, at $\tau = \tau^o$, is dependent on A^2. However, the expression for $J(\tau^o)$ suffers from two biases. The first bias is an additive noise bias that increases with the noise variance and is given by

$$B_n = \frac{\sigma_v^2}{N} \ . \tag{7}$$

The second bias is a multiplicative fading bias that depends on the autocorrelation function and is given by

$$B_f = \frac{R_x(0)}{N} + \sum_{i=1}^{N-1} \frac{2(N-i)R_x(i)}{N^2} \ . \tag{8}$$

It is clear that B_f is less than or equal to unity (it is unity for static channels, which explains why previous conventional designs ignored this bias as fading was not considered in these designs — see [5]. The value of B_f is also unity for $N = 1$.) For our purposes, it is more convenient to work with the inverse of B_f, say $C_f = 1/B_f$, i.e.,

$$C_f = \left[\frac{R_x(0)}{N} + \sum_{i=1}^{N-1} \frac{2(N-i)R_x(i)}{N^2}\right]^{-1} \ . \tag{9}$$

We shall refer to C_f as the fading correction factor. Figure 3 plots C_f versus N for a Rayleigh fading channel. It is clear that the need for this correction factor increases for higher Doppler frequencies.

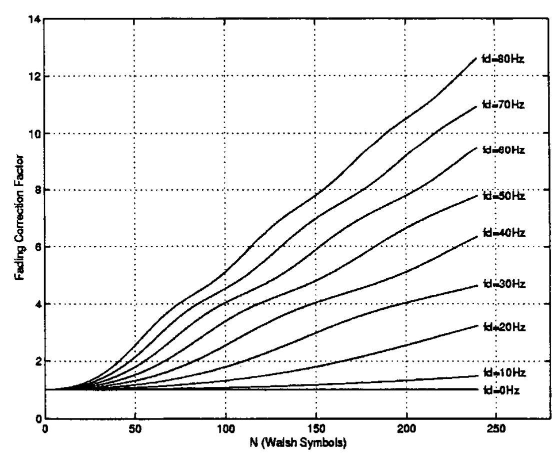

Figure 3: *Fading correction factor C_f versus f_D.*

If $\{B_n, C_f\}$ were known, then A can be obtained from (4) via

$$A = \sqrt{C_f \left[J(\tau^o) - B_n\right]} \ .$$

The value of C_f can be found from (9)[2]. As for B_n, its value can be estimated as follows. Note first that the noise variance σ_v^2 can be estimated *directly* from the received sequence $\{r(n)\}$ since, for CDMA signals,

[2] Actually, when f_D is estimated, we end up with an estimate for C_f.

the SNR is typically very low. In other words, we can get an estimate for σ_v^2 as follows

$$\widehat{\sigma_v^2} = \frac{1}{K}\sum_{i=1}^{K}|r(i)|^2 .$$

Then, an estimate for B_n is given, from (7), by

$$\widehat{B}_n = \frac{\widehat{\sigma_v^2}}{N} = \frac{1}{NK}\sum_{i=1}^{K}|r(i)|^2. \qquad (10)$$

With $\{\widehat{B}_n, C_f\}$ so computed, we obtain an estimate for A via the expression

$$\widehat{A} = \sqrt{C_f\left[J\left(\widehat{\tau^o}\right) - \widehat{B}_n\right]} . \qquad (11)$$

Expressions (2), (3), (6), (9), (10), and (11) completely define our estimation algorithm.

V. Simulation and Field Trial Results

A direct spread sequence is generated and filtered using a $T_c/8$ upsampled version of the IS-95 pulse-shaping filter. The resulting signal is passed through a single path Rayleigh fading channel and then white Gaussian noise, which accounts for both multiple access interference and thermal noise [9], is added. The received signal is sampled with a sampling period of $T_c/8$, then used to obtain $\hat{\tau}$ and $\hat{A}$ according to (2) and (11), respectively. The search window used in the simulations is 25 chips wide and the carrier frequency f_c is 900 MHz. Each simulation point is the statistical average of 4000 runs. Figures (4) and (5) show the mean absolute TOA and the mean square AOA estimation errors versus the chip energy-to-noise ratio (E_c/N_o) for various values of f_D, respectively. Note that the estimation error decreases with E_c/N_o and increases with f_D. It is also clear that for values of E_c/N_o higher than -40 dB, a mean absolute TOA of 0.1 μsec and an AOA MSE of -15 dB could be achieved. This estimation accuracy is considerably higher than the closest known estimation techniques (see, e.g., [6]).

In order to appreciate the effect of the new amplitude bias equalization technique on the accuracy of the AOA estimate, the AOA MSE is plotted in Figure 6 versus E_c/N_o, with and without each of the two correction factors. It is seen that noise correction only is not enough for high chip energy-to-noise ratios. This is due to the fact that at very low noise levels, the noise bias becomes negligible with respect to the fading bias. It is also seen that for very low chip energy-to-noise ratios, the uncorrected amplitude estimate is more precise than the corrected estimate. This is explained as follows. At very low chip energy-to-noise ratios, the error in noise estimation is boosted

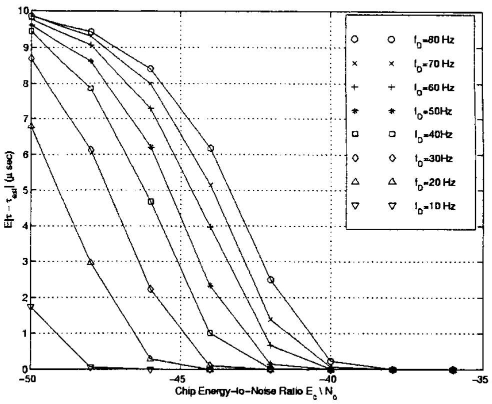

Figure 4: *Mean absolute TOA error vs.* E_c/N_o *for* $M =$ 128.

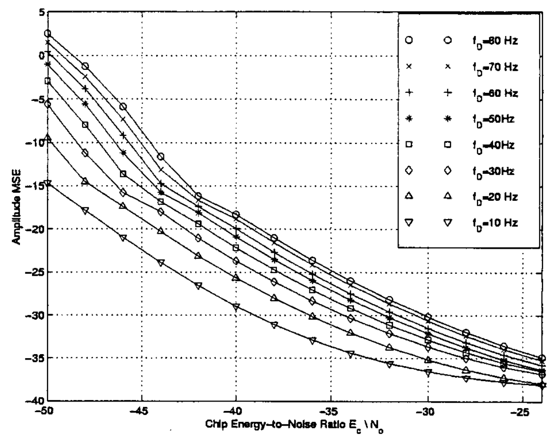

Figure 5: *AOA MSE error vs.* E_c/N_o *for* M*=128.*

up by the fading correction factor causing more errors in the estimate. However, the estimation error in this region is very large and is not of interest. It is clear from the figure that the proposed correction technique improves the precision of the AOA estimate significantly.

In a field trial performed in Chamburg, IL [10], using our proposed estimation scheme, a root mean square location error of 57 meters off the actual location obtained using a global positioning system (GPS) unit was reported. The field trial was performed in various non line-of-site situations. This promising result emphasizes the potential of the proposed scheme in the context of CDMA wireless location finding.

VI. Sensitivity to Doppler Estimation Errors

The simulation results given in the previous section assume perfect knowledge of the maximum Doppler frequency f_D. In this section, the effects of maximum

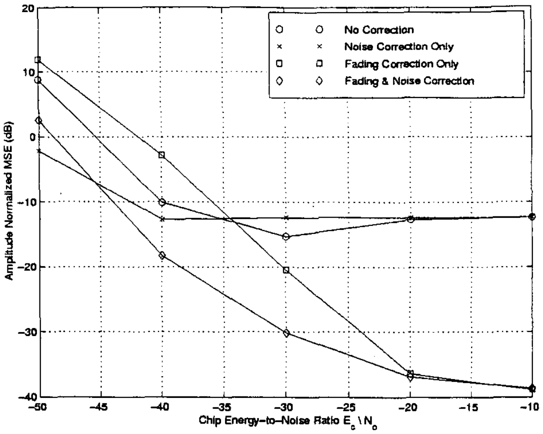

Figure 6: *AOA MSE with and without biases equalization.*

Doppler frequency estimation errors on the precision of the TOA and AOA estimators are shown. It was mentioned in Section III that the SNR gain becomes approximately insensitive to N when it increases beyond its optimal value. Thus, the TOA estimation accuracy, which depends only on the SNR gain, becomes insensitive to errors in the maximum Doppler frequency estimate for that range of N. On the other hand, a relatively accurate maximum Doppler frequency estimate is needed for precise amplitude estimation. This is due to the need for the amplitude fading correction factor.

The amplitude estimation error, $|\Delta A_{f_D}|$, due to an error $|\Delta f_D|$ in the maximum Doppler frequency estimate is given from (11) by

$$|\Delta A_{f_D}| = \left| A - \sqrt{C'_f\left[J(\tau^o) - B_n\right]} \right| \quad , \qquad (12)$$

where C'_f is the fading correction factor calculated for an estimated maximum Doppler frequency $(f_D + \Delta f_D)$, and f_D is the actual maximum Doppler frequency. Here, we assumed no errors in the TOA estimate. The absolute normalized amplitude estimation error is then

$$\left| \frac{\Delta A_{f_D}}{A} \right| = \left| 1 - \sqrt{\frac{C'_f}{C_f}} \right| \quad , \qquad (13)$$

where C_f is the fading correction factor calculated at the actual maximum Doppler frequency f_D. Figure 7 shows the absolute relative amplitude error percentage versus the relative maximum Doppler frequency error percentage for different values of f_D. It is shown in the figure that the absolute relative amplitude error increases by approximately 0.5% for every 1% increase in the relative maximum Doppler frequency error. It is also seen that the amplitude estimate sensitivity to maximum Doppler frequency errors does not change significantly with the value of f_D. Thus,

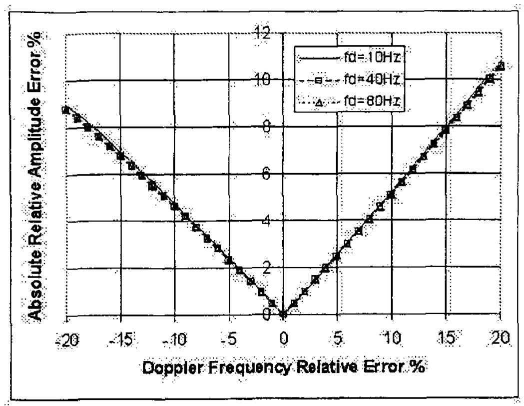

Figure 7: *Effect of Doppler estimation errors.*

we conclude that our estimation scheme is robust to errors in the maximum Doppler frequency estimation. This implies that a rough estimate of the MS speed would be adequate to obtain a reasonable accuracy in the TOA/AOA estimation process.

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