COOPERATIVE SPECTRUM SENSING VIA COHERENCE DETECTION

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ABSTRACT

Efficient and reliable spectrum sensing plays a critical role in cognitive radio networks. This paper proposes a cooperative sensing scheme that detects the existence of a common signal component in the signals received by multiple geographically distributed radios. The scheme assumes that signals received by different radios display strong coherence if they have a common source. Detection of this coherence in a wireless environment is studied, especially when the transmitted signal is distorted by multipath channels.

***Index Terms*—** Cognitive radio, coherence detection, spectrum sensing.

1. INTRODUCTION

Cognitive radio has recently emerged as a useful technology to improve the efficiency of spectrum utilization [1]. Traditionally, the spectrum is assigned by the Federal Communications Commission (FCC) to specific users or applications, and each user can only utilize its pre-assigned bandwidth for communication. This discipline causes some bandwidth to be overcrowded while some other bandwidth may be seriously under-utilized. The concept of cognitive radio aims at providing a flexible way of spectrum management, permitting secondary users to temporally access spectrum that is not currently used by legacy users. In this regard, the FCC has taken a number of steps towards allowing low-power devices to operate in the broadcast TV bands that are not being used by TV channels. To promote this development, IEEE has established the IEEE 802.22 Working Group to develop a standard for a cognitive radio-based device in TV bands.

A key challenge in the development of the IEEE 802.22 standard is that a cognitive radio should be able to reliably detect the presence of TV signals in a fading environment. Otherwise, the radio may use the frequency band that is occupied by a TV channel, and cause serious interference to the TV receivers nearby. Many sensing or detection schemes have been recently reported in the IEEE 802.22 community. These schemes can be classified into two categories: single-user sensing and cooperative sensing. Due to the large variations in the received signal strength that are caused by path loss and fading, single-user sensing has proven to be unreliable, which consequently triggered the FCC to require geolocation-based methods for identifying unused frequency bands. The geolocation approach is suitable for registered TV bands; however, its cost and operational overhead prevent its wide use in the opportunistic access to occasional "white spaces" in the spectrum. Cooperative sensing relies on multiple radios to detect the presence of primary users, and provides a reliable solution for cognitive radio networks [2–5]. In this paper, we consider the problem of detecting the presence of a common signal component from the signals received by multiple geographically distributed sensors. If the signals received by these radios exhibit strong cross-correlation (coherence), it has a high probability that the spectrum is being occupied. This sensing technique minimizes the amount of prior information required to perform spectrum sensing.

Throughout this paper, we adopt the following definitions and notation. The network consists of M cognitive radios that are monitoring the frequency band of interest (see Fig. 1). The two hypotheses corresponding to the signal-absent and signal-present events are defined as:

$\mathcal{H}_0$: target signal is absent (i.e., spectrum is vacant);
$\mathcal{H}_1$: target signal is present (i.e., spectrum is occupied).

The performance of detecting $\mathcal{H}_0$ against $\mathcal{H}_1$ is measured by the probability of false alarm and the probability of miss detection. False alarm refers to the error of accepting $\mathcal{H}_1$ when $\mathcal{H}_0$ is true, while miss detection refers to the error of accepting $\mathcal{H}_0$ when $\mathcal{H}_1$ is true.

2. COHERENCE DETECTION – LINE-OF-SIGHT CASE

For the case of line-of-sight propagation, the signals received by different radios are attenuated and delayed replicas of the target signal. We consider the following hypothesis testing

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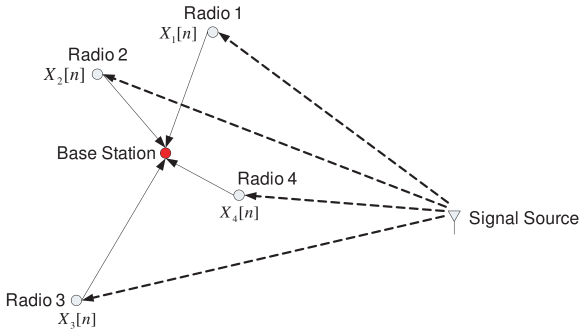

Fig. 1. Cognitive radio network for spectrum sensing.

problem:

$$\mathcal{H}_0 : X_m[n] = W_m[n],$$
$$\mathcal{H}_1 : X_m[n] = A_m S[n-\tau_m] + W_m[n],\ m = 1, 2, \ldots, M,$$

where $X_m[n]$ is the signal acquired by the m^{th} radio at time n, $S[n]$ is the *unknown* target signal, A_m is the *unknown* amplitude of the target signal, τ_m is the *unknown* propagation delay, and $W_m[n]$ is the observation noise. Assume that $W_m[n]$ are i.i.d. Gaussian distributed with mean zero and variance σ_W^2. To detect $\mathcal{H}_1$ against $\mathcal{H}_0$, we apply the Generalized Likelihood Ratio test [6]. Under $\mathcal{H}_1$, the Maximum Likelihood estimates of A_m, τ_m and $S[n]$ are given by the solutions to the following problem:

$$\begin{aligned}&\langle \widehat{A}_m, \widehat{\tau}_m, \widehat{S}[n]\rangle \\ &= \underset{A_m,\,\tau_m,\,S[n]}{\arg\min} \sum_{m=1}^{M}\sum_{n=1}^{N}\left(X_m[n] - A_m S[n-\tau_m]\right)^2,\end{aligned}$$

where N is the number of signal samples acquired by each receiver. To solve this problem, we notice that for fixed τ_m, the estimate of A_m is given by

$$\widehat{A}_m = \frac{\sum_{n=1}^{N} X_m[n] S[n-\tau_m]}{\sum_{n=1}^{N}\left(S[n-\tau_m]\right)^2}.$$

The estimates of τ_m and $S[n]$ are thus given by solving

$$\langle \widehat{\tau}_m,\, \widehat{S}[n]\rangle = \underset{\tau_m,\,S[n]}{\arg\max} \sum_{m=1}^{M} \frac{\left(\sum_{n=1}^{N} X_m[n] S[n-\tau_m]\right)^2}{\sum_{n=1}^{N}\left(S[n-\tau_m]\right)^2}.$$

With proper alignment for given τ_m, we could rewrite the above equation as

$$\begin{aligned}\langle \widehat{\tau}_m,\, \widehat{S}[n]\rangle &= \underset{\tau_m,\,S[n]}{\arg\max} \sum_{m=1}^{M} \frac{\left(\sum_{n=1}^{N} X_m[n+\tau_m] S[n]\right)^2}{\sum_{n=1}^{N}\left(S[n]\right)^2} \\ &= \underset{\tau_m,\,\mathbf{s}}{\arg\max} \frac{\mathbf{s}^T\mathbf{A}(\tau_1,\ldots,\tau_M)\mathbf{s}}{\mathbf{s}^T\mathbf{s}}\end{aligned}$$

where

$$\mathbf{s} = \left[\ S[1] \quad S[2] \quad \ldots \quad S[N]\ \right]^T,$$
$$\mathbf{A}(\tau_1,\ldots,\tau_M) = \sum_{m=1}^{M} \widetilde{\mathbf{x}}_m(\tau_m)\widetilde{\mathbf{x}}_m^T(\tau_m),$$
$$\widetilde{\mathbf{x}}_m(\tau_m) = \left[\ X_m[1+\tau_m] \quad \ldots \quad X_m[N+\tau_m]\ \right]^T.$$

For fixed τ_m, the optimal $\mathbf{s}$ is given by the solution to the following problem:

$$\widehat{\mathbf{s}} = \arg\max_{\mathbf{s}} \mathbf{s}^T\mathbf{A}(\tau_1,\ldots,\tau_M)\mathbf{s} \ \text{ subject to } \ \|\mathbf{s}\| = 1.$$

It is easy to verify that $\widehat{\mathbf{s}}$ is the eigenvector of $\mathbf{A}(\tau_1,\ldots,\tau_M)$ that is associated with the maximum eigenvalue. That is,

$$\max \frac{\mathbf{s}^T\mathbf{A}(\tau_1,\ldots,\tau_M)\mathbf{s}}{\mathbf{s}^T\mathbf{s}} = \lambda_{\max}\left(\mathbf{A}(\tau_1,\ldots,\tau_M)\right).$$

It immediately follows that the Maximum Likelihood estimate of τ_m is given by

$$\widehat{\tau}_m = \arg\max_{\tau_m} \lambda_{\max}\left(\mathbf{A}(\tau_1,\ldots,\tau_M)\right),$$

and the associated Generalized Log-Likelihood ratio (GLLR) is given by

$$\text{GLLR} = \frac{1}{2\sigma_W^2}\max_{\tau_m} \lambda_{\max}\left(\mathbf{A}(\tau_1,\ldots,\tau_M)\right).$$

The test is thus given by

$$\text{Accept } \mathcal{H}_1 \text{ if } \max_{\tau_m} \lambda_{\max}\left(\mathbf{A}(\tau_1,\ldots,\tau_M)\right) \geq \eta,$$
$$\text{Accept } \mathcal{H}_0 \text{ if } \max_{\tau_m} \lambda_{\max}\left(\mathbf{A}(\tau_1,\ldots,\tau_M)\right) < \eta,$$

where η is the predetermined threshold in order to achieve a desirable P_{FA} and P_{MISS}. To determine a proper value for η, we need to study the distributions of

$$\max_{\tau_m} \lambda_{\max}\left(\mathbf{A}(\tau_1,\ldots,\tau_M)\right)$$

under $\mathcal{H}_0$ and $\mathcal{H}_1$. In the test, $\max_{\tau_m}\lambda_{\max}(\mathbf{A}(\tau_1,\ldots,\tau_M))$ can be computed by the following two methods:
(1) (*Optimal*) Perform an exhaustive search over all possible τ_m, $m = 1, 2, \ldots, M$, to find $\max_{\tau_m}\lambda_{\max}(\mathbf{A}(\tau_1,\ldots,\tau_M))$.
(2) (*Heuristic*) Let

$$\mathbf{A}_m(\tau_1,\ldots,\tau_M) = \sum_{i=1}^{m} \widetilde{\mathbf{x}}_i(\tau_m)\widetilde{\mathbf{x}}_i^T(\tau_m)$$

for $m = 1, 2, \ldots, M$. Start with $\tau_1 = 0$ and then find τ_m, $m = 2, 3, \ldots, M$, in a sequential manner. Given the previously found $\tau_1, \tau_2, \ldots, \tau_{m-1}$, the next τ_m is found by solving

$$\max_{\tau_m} \lambda_{\max}\left(\mathbf{A}_m(\tau_1,\ldots,\tau_M)\right).$$

Method 1 performs an exhaustive search and is computationally expensive, while Method 2 finds τ_m for each radio one by one and might not be optimal. In Method 2, we first find the optimal τ_2 to align the received signals from the 1^{st} and 2^{nd} radios. Then, we find the optimal τ_3 to align the signal from the 3^{rd} radio with the signals from the 1^{st} and 2^{nd} radios. We keep repeating this procedure until the M^{th} radio.

3. COHERENCE DETECTION – MULTIPATH CASE

For multipath propagation, the wireless channels are modeled as filters with finite impulse response (FIR). The signals received by each radio under $\mathcal{H}_0$ and $\mathcal{H}_1$ are represented by

$$\begin{aligned}&\mathcal{H}_0 : X_m[n] = W_m[n],\\ &\mathcal{H}_1 : X_m[n] = (H_m[n] \star S[n]) + W_m[n],\ m = 1, 2, \ldots, M,\end{aligned}$$

where $\star$ stands for the convolution operation and $H_m[n], m = 1, 2, \ldots, M$, are the impulse response functions of the propagation channels from the radio transmitter to the M receivers. Note that the line-of-sight case in Section 2 is a special case of the multipath case, where $H_m[n]$ degenerates to the time-delay operators. By using matrix notation, we have

$$\begin{aligned}&\mathcal{H}_0 : \mathbf{x}_m = \mathbf{w}_m,\\ &\mathcal{H}_1 : \mathbf{x}_m = \mathbf{B}_m\mathbf{s} + \mathbf{w}_m,\ m = 1, 2, \ldots, M,\end{aligned}$$

where

$$\begin{aligned}\mathbf{x}_m &= \begin{bmatrix} X_m[1] & X_m[2] & \ldots & X_m[N] \end{bmatrix}^T,\\ \mathbf{w}_m &= \begin{bmatrix} W_m[1] & W_m[2] & \ldots & W_m[N] \end{bmatrix}^T,\\ \mathbf{s} &= \begin{bmatrix} S[1] & S[2] & \ldots & S[N] \end{bmatrix}^T,\end{aligned}$$

and $\mathbf{B}_m$ is defined at the top of next page and L is the length of the channel responses. The Log-Likelihood ratio (LLR) is

$$\mathrm{LLR} = -\frac{1}{2\sigma_W^2}\sum_{m=1}^{M}\|\mathbf{x}_m - \mathbf{B}_m\mathbf{s}\|^2 + \frac{1}{2\sigma_W^2}\sum_{m=1}^{M}\|\mathbf{x}_m\|^2.$$

Let

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_M \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \vdots \\ \mathbf{B}_M \end{bmatrix}.$$

The expression for LLR can be rewritten as

$$\mathrm{LLR} = -\frac{1}{2\sigma_W^2}\|\mathbf{x} - \mathbf{B}\mathbf{s}\|^2 + \frac{1}{2\sigma_W^2}\|\mathbf{x}\|^2.$$

Since $\mathbf{B}$ and $\mathbf{s}$ are unknown, their Maximum Likelihood estimate is given by

$$\langle\widehat{\mathbf{B}}, \widehat{\mathbf{s}}\rangle = \arg\min_{\mathbf{B},\mathbf{s}} \|\mathbf{x} - \mathbf{B}\mathbf{s}\|^2.$$

We note that

(1) For fixed $\mathbf{B}$, the corresponding optimal estimate of $\mathbf{s}$ is given by

$$\widehat{\mathbf{s}} = \left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{x}. \tag{2}$$

(2) For fixed $\mathbf{s}$, the corresponding optimal estimate of $\mathbf{B}_m$ can be found by

$$\widehat{\mathbf{h}}_m = \left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T\mathbf{x}_m,\ \ m = 1, 2, \ldots, M, \tag{3}$$

where

$$\mathbf{h}_m = \begin{bmatrix} H_m[0] & H_m[1] & \ldots & H_m[L-1] \end{bmatrix}^T, \tag{4}$$

$$\mathbf{S} = \tag{5}$$

$$\begin{bmatrix} S[1] & 0 & 0 & \ldots & 0 \\ S[2] & S[1] & 0 & \ldots & 0 \\ S[3] & S[2] & S[1] & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ S[L] & S[L-1] & S[L-2] & \ldots & S[1] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ S[N] & S[N-1] & S[N-2] & \ldots & S[N-L+1] \end{bmatrix}_{N\times L}.$$

With the estimate $\widehat{\mathbf{B}}$ of $\mathbf{B}$, the GLLR is given by

$$\begin{aligned}\mathrm{GLLR} &= -\frac{1}{2\sigma_W^2}\|\mathbf{x} - \widehat{\mathbf{B}}\widehat{\mathbf{s}}\|^2 + \frac{1}{2\sigma_W^2}\|\mathbf{x}\|^2\\ &= \frac{1}{2\sigma_W^2}\mathbf{x}^T\widehat{\mathbf{B}}\big(\widehat{\mathbf{B}}^T\widehat{\mathbf{B}}\big)^{-1}\widehat{\mathbf{B}}^T\mathbf{x}.\end{aligned}$$

The test is thus given by

$$\begin{aligned}&\text{Accept } \mathcal{H}_1 \text{ if } \mathbf{x}^T\widehat{\mathbf{B}}\big(\widehat{\mathbf{B}}^T\widehat{\mathbf{B}}\big)^{-1}\widehat{\mathbf{B}}^T\mathbf{x} \geq \eta,\\ &\text{Accept } \mathcal{H}_0 \text{ if } \mathbf{x}^T\widehat{\mathbf{B}}\big(\widehat{\mathbf{B}}^T\widehat{\mathbf{B}}\big)^{-1}\widehat{\mathbf{B}}^T\mathbf{x} < \eta,\end{aligned}$$

for some predetermined η. Based on (2) and (3), we have Algorithm 1 to find a suboptimal $\widehat{\mathbf{B}}$ for the test.

4. SIMULATION RESULTS

The simulated network has $M = 4$ cognitive radios for spectrum sensing. The signal and noise are Gaussian distributed. The received signal variances σ_S^2 and noise variances σ_W^2 under $\mathcal{H}_1$ and $\mathcal{H}_0$ are listed in Table 1. The signal block length is $N = 100$. Fig. 2 shows the performance of the proposed detector with different threshold values for the line-of-sight propagation environment. Fig. 3 shows the performance of the proposed detector with different threshold values for the multipath propagation environment. The channel response is modeled as an FIR filter with length 4, and its taps are independently Rayleigh distributed with the total power normalized to be 1.

$$\mathbf{B}_m = \begin{bmatrix} H_m[0] & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ H_m[1] & H_m[0] & 0 & \dots & 0 & 0 & \dots & 0 \\ H_m[2] & H_m[1] & H_m[0] & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ H_m[L-1] & H_m[L-2] & H_m[L-3] & \dots & H_m[0] & 0 & \dots & 0 \\ 0 & H_m[L-1] & H_m[L-2] & \dots & H_m[1] & H_m[0] & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & & & \\ 0 & 0 & 0 & \dots & H_m[L-1] & H_m[L-2] & \dots & H_m[0] \end{bmatrix}_{N \times N} \tag{1}$$

	m=1	m=2	m=3	m=4
σ_S^2	-4 dB	-5 dB	-6 dB	-7 dB
σ_W^2	0 dB	0 dB	0 dB	0 dB

Table 1. Simulated signal and noise variances.

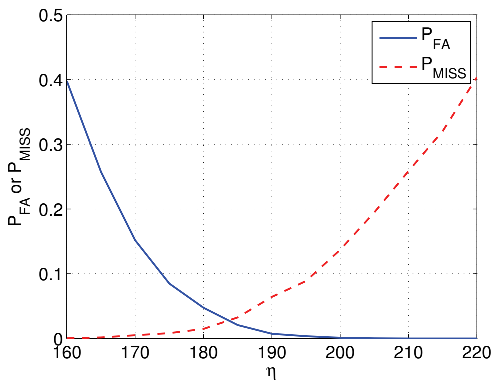

Fig. 2. Achievable P_{FA} and P_{MISS} with the proposed algorithm for a line-of-sight propagation environment. The noise and signal variances are defined in Table 1.

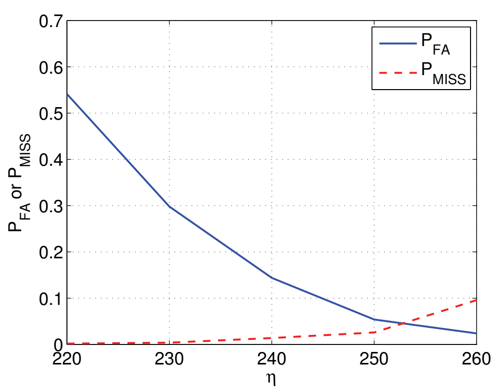

Fig. 3. Achievable P_{FA} and P_{MISS} with the proposed algorithm for a multipath propagation environment. The noise and signal variances are defined in Table 1.

Algorithm 1 Coherence Sensing in a Multipath Environment

0: Set $i = 0$.
0: Find an initial estimate $\widehat{\mathbf{B}}_{m,0}$, $m = 1, 2, \dots, M$, of $\mathbf{B}_m$ by using the algorithm proposed in Section 2. The initial estimate assumes a line-of-sight propagation environment.
1: **repeat**
2: $\quad i = i + 1$.
3: $\quad$ Find the optimal $\widehat{\mathbf{s}}_i$ for the previously obtained $\widehat{\mathbf{B}}_{i-1}$ by

$$\widehat{\mathbf{s}}_i = \left(\widehat{\mathbf{B}}_{i-1}^T \widehat{\mathbf{B}}_{i-1}\right)^{-1} \widehat{\mathbf{B}}_{i-1}^T \mathbf{x}.$$

4: $\quad$ Find the optimal $\widehat{\mathbf{B}}_{m,i}$ for the previously obtained $\widehat{\mathbf{s}}_i$ by

$$\widehat{\mathbf{h}}_{m,i} = (\widehat{\mathbf{S}}_i^T \widehat{\mathbf{S}}_i)^{-1} \widehat{\mathbf{S}}_i^T \mathbf{x}_m,$$

$\quad$ where $\widehat{\mathbf{B}}_{m,i}$ is determined by $\widehat{\mathbf{h}}_{m,i}$ according to (4), (1) and $\widehat{\mathbf{S}}_i$ is determined by $\widehat{\mathbf{s}}_i$ according to (5).
5: **until** there is no significant improvement in the objective function $\|\mathbf{x} - \widehat{\mathbf{B}}_i \widehat{\mathbf{s}}_i\|^2$.
6: If $\mathbf{x}^T \widehat{\mathbf{B}} \big(\widehat{\mathbf{B}}^T \widehat{\mathbf{B}}\big)^{-1} \widehat{\mathbf{B}}^T \mathbf{x} \geq \eta$, "$\mathcal{H}_1$: Target Signal is Present" is claimed; if $\mathbf{x}^T \widehat{\mathbf{B}} \big(\widehat{\mathbf{B}}^T \widehat{\mathbf{B}}\big)^{-1} \widehat{\mathbf{B}}^T \mathbf{x} < \eta$, "$\mathcal{H}_0$: Target Signal is Absent" is claimed.

5. REFERENCES

[1] J. Mitola III and G. Q. Maguire, "Cognitive radio: Making software radios more personal," *IEEE Pers. Commun.*, vol. 6, pp. 13–18, 1999.

[2] J. Unnikrishnan and V. V. Veeravalli, "Cooperative sensing for primary detection in cognitive radio," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 18–27, Feb. 2008.

[3] Z. Quan, S. Cui, and A. H. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 28–40, Feb. 2008.

[4] Y. Zeng, Y.-C. Liang, and R. Zhang, "Blindly combined energy detection for spectrum sensing in cognitive radio," *IEEE Signal Process. Lett.*, vol. 15, pp. 649–652, Feb. 2008.

[5] Q. Zou, S. Zheng, and A. H. Sayed, "Cooperative spectrum sensing via sequential detection for cognitive radio networks," in *Proc. International Workshop on Signal Processing Advances for Wireless Communications (SPAWC)*, Jun. 2009.

[6] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*, vol. 2, Prentice Hall, New Jersey, 1998.