DESIGN OF HALF- AND FULL-DUPLEX RELAY SYSTEMS BASED ON THE MMSE FORMULATION

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ABSTRACT

In this paper, half- and full-duplex relay processing matrices and source-destination beamforming vectors are jointly optimized based on a minimum mean-square-error (MMSE) formulation under inequality constraints on the transmit power of the source and the relays. Through BER simulations, we illustrate that the full-duplex method performs better than the half-duplex method in various signal-to-noise ratio (SNR) scenarios, and it can be a promising candidate for future relay networks.

***Index Terms*—** Minimum mean-square-error (MMSE), beamforming, multiple-input multiple-output (MIMO), full-duplex relay.

1. INTRODUCTION

The half-duplex (HD) relay structure, which receives and transmits data separately, is easier to implement since it does not suffer from cross interference (echo) between the retransmit and receive signals. However, two time slots are required for relaying the signal (see Fig. 1). Recently, by using multiple antennas at the relay, a spatial pre-nulling method has been proposed to suppress cross interference and employ the full-duplex (FD) relay without additional time or frequency resources [1]. Therefore, FD relay systems have become more practical than before. While HD relay systems are studied diligently [2–6], comparatively less work has been done for FD relay systems.

In this paper, we model FD amplify-and-forward (AF) relay systems assuming cross interference prenulled at the relay [1]. Optimal FD and HD relay processing matrices and source-destination beamforming vectors are designed by adopting a minimum-mean-square (MSE) criterion with power constraints at the source and the relay under the assumption of full channel state information (CSI) at each node. Although full CSI is a challenging task, demanding signaling and feedback procedures between nodes, it is within the range of possibility in slow varying or static channels. Closed form Karush-Kuhn-Tucker (KKT) conditions [7] are derived to solve the constrained MSE optimization problems, and iterative algorithms are then developed to achieve optimal MMSE performance. In simulation, we compare the bit-error-rate (BER) performance of the designed HD and FD relay systems.

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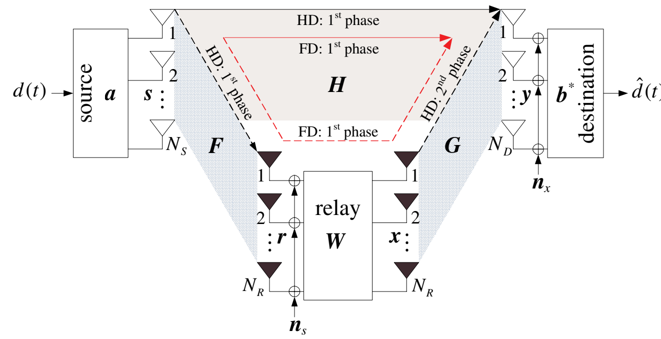

Fig. 1. Amplify-and-forward MIMO relay system model illustrating half- and full-duplex (HD and FD) relay communications on direct path (solid line) and relay path (dashed line).

2. RELAY SYSTEM AND SIGNAL MODEL

A two-hop MIMO relay system model is shown in Fig. 1. The source, relay, and destination layers have N_S, N_R, and N_D antennas, respectively, where N_S and N_D are greater than or equal to two for beamforming purposes while $N_R \geq 1$. For convenience of illustration, the relay is shown with separate transmit and receive antennas. The channel matrix of the direct channel between the source and the destination is represented by $\boldsymbol{H} \in \mathbb{C}^{N_D \times N_S}$ and the relay channel matrices of the first- and second-hops are represented by $\boldsymbol{F} \in \mathbb{C}^{N_R \times N_S}$ and $\boldsymbol{G} \in \mathbb{C}^{N_D \times N_R}$, respectively. The elements of $\boldsymbol{H}$, $\boldsymbol{F}$ and $\boldsymbol{G}$ are i.i.d and zero-mean complex Gaussian random variables with variances of σ_H^2, σ_F^2, and σ_G^2, respectively. It is assumed that every channel remains static during one transmission data block (or frame). Data symbols at time t are denoted by $d(t)$ so that the transmitted symbol vector from the source, $\boldsymbol{s}(t) \in \mathbb{C}^{N_S \times 1}$, is given by

$$\boldsymbol{s}(t) = \boldsymbol{a} d(t) \tag{1}$$

where $\mathrm{E}\,|d(t)|^2 = 1$ and $\boldsymbol{a} \in \mathbb{C}^{N_S \times 1}$ is a transmit beamforming vector.

2.1. Half-Duplex Relay

HD relay requires two-separate phases to receive and retransmit without interference. In the first phase t_1, the received signal vector $\boldsymbol{r}(t_1) \in \mathbb{C}^{N_R \times 1}$ at the relay is

$$\boldsymbol{r}(t_1) = \boldsymbol{F}\boldsymbol{s}(t_1) + \boldsymbol{n}_s(t_1) \tag{2}$$

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where $\boldsymbol{n}_s(t_1) \in \mathbb{C}^{N_R \times 1}$ is a zero-mean additive white Gaussian noise (AWGN) vector; $\mathrm{E}\, \boldsymbol{n}_s(t_1)\boldsymbol{n}_s^*(t_1) = \sigma_{n_s}^2 \boldsymbol{I}_{N_R}$; the superscript '$*$' means complex conjugate transposition; and $\boldsymbol{I}_q$ is a q-dimensional identity matrix. At the same phase, the received signal at the destination from the source is

$$\boldsymbol{y}(t_1) = \boldsymbol{H}\boldsymbol{s}(t_1) + \boldsymbol{n}_x(t_1). \tag{3}$$

where $\boldsymbol{n}_x(t_1) \in \mathbb{C}^{N_D \times 1}$ is an AWGN vector and $\mathrm{E}\, \boldsymbol{n}_x(t_1)\boldsymbol{n}_x^*(t_1) = \sigma_{n_x}^2 \boldsymbol{I}_{N_D}$. In the next phase t_2, the relay multiplies $\boldsymbol{r}(t_1)$ by the relay processing matrix $\boldsymbol{W} \in \mathbb{C}^{N_R \times N_R}$, and forwards $\boldsymbol{x}(t_2) \in \mathbb{C}^{N_R \times 1}$ to the destination, where

$$\boldsymbol{x}(t_2) = \boldsymbol{W}\boldsymbol{r}(t_1) \tag{4}$$

Meanwhile, it is assumed that the destination does not transmit a signal to avoid asynchronous problems at the destination between the source and relay signals. Therefore, at the destination, the received signal vector $\boldsymbol{y}(t_2) \in \mathbb{C}^{N_D \times 1}$ is

$$\boldsymbol{y}(t_2) = \boldsymbol{G}\boldsymbol{x}(t_2) + \boldsymbol{n}_x(t_2) \tag{5}$$

The destination combines two consecutive received signals $\boldsymbol{y}(t_1)$ and $\boldsymbol{y}(t_2)$ as follows:

$$\hat{d}_{HD} = \boldsymbol{b}^* \begin{bmatrix} \boldsymbol{y}(t_1) \\ \boldsymbol{y}(t_2) \end{bmatrix} = \boldsymbol{b}_1^*\boldsymbol{y}(t_1) + \boldsymbol{b}_2^*\boldsymbol{y}(t_2) \tag{6}$$

by using a receive beamforming vector $\boldsymbol{b}^* \triangleq \begin{bmatrix} \boldsymbol{b}_1^* & \boldsymbol{b}_2^* \end{bmatrix} \in \mathbb{C}^{1 \times 2N_D}$, where $\boldsymbol{b}_1^* \in \mathbb{C}^{1 \times N_D}$ and $\boldsymbol{b}_2^* \in \mathbb{C}^{1 \times N_D}$ combine the direct and relay path signals, respectively.

2.2. Full-Duplex Relay

Through one phase, the FD relay receives (2) and retransmits (4) at the same time, i.e., $t_2 \triangleq t_1$; therefore, at the destination, including the direct signal in (2), the received signal vector at the destination can be written as

$$\boldsymbol{y}(t_1) = \boldsymbol{H}\boldsymbol{s}(t_1) + \boldsymbol{G}\boldsymbol{x}(t_1) + \boldsymbol{n}_x(t_1) \in \mathbb{C}^{N_S \times 1}. \tag{7}$$

The destination combines the received signal as follows:

$$\hat{d}(t_1)_{FD} = \boldsymbol{b}^*\boldsymbol{y}(t_1) \tag{8}$$

by using a N_D-dimensional receive beamforming vector $\boldsymbol{b}^* \in \mathbb{C}^{1 \times N_D}$.

Notation. Throughout this paper, for any vector or matrix, the superscript 'T' denotes transposition. Moreover, $\mathrm{tr}(\boldsymbol{Q})$ represents the trace of matrix $\boldsymbol{Q}$; 'E' stands for expectation of a random variable; for any scalar q, vector $\boldsymbol{q}$, and matrix $\boldsymbol{Q}$, the notations $|q|$, $\|\boldsymbol{q}\|$, and $\|\boldsymbol{Q}\|_F$ denote the absolute value of q, 2-norm of $\boldsymbol{q}$, and Frobenius-norm of $\boldsymbol{Q}$, respectively; and for simplicity, the time index t is henceforth omitted whenever convenient.

3. MMSE PROBLEM FORMULATION

In this paper, we aim to jointly design a set of beamforming vectors and a relay processing matrix in order to minimize the MSE under transmit power constraints at the source and the relay layers.

3.1. Half-Duplex Relay

Using (1)–(6), the overall signal model can be written as

$$\begin{aligned} \hat{d}_{HD} = &\left(\boldsymbol{b}_1^*\boldsymbol{H}\boldsymbol{a} + \boldsymbol{b}_2^*\boldsymbol{G}\boldsymbol{W}\boldsymbol{F}\boldsymbol{a}\right)d \\ &+ \left(\boldsymbol{b}_1^*\boldsymbol{n}_{x1} + \boldsymbol{b}_2^*\boldsymbol{n}_{x2} + \boldsymbol{b}_2^*\boldsymbol{G}\boldsymbol{W}\boldsymbol{n}_s\right) \end{aligned} \tag{9}$$

where $\boldsymbol{n}_{x1} = \boldsymbol{n}_x(t_1)$ and $\boldsymbol{n}_{x2} = \boldsymbol{n}_x(t_2)$. When the transmit power of the source signal and the relay signal are limited by P_S and P_R, respectively and independently, the desired MMSE problem is as follows:

$$\boxed{\begin{aligned} &\arg \min_{\{\boldsymbol{a},\boldsymbol{W},\boldsymbol{b}_1,\boldsymbol{b}_2\}} \mathrm{E}\,|d - \hat{d}_{HD}|^2 \\ &\text{s.t. } \mathrm{E}\,\|\boldsymbol{s}\|^2 \le P_S \text{ and } \mathrm{E}\,\|\boldsymbol{x}\|^2 \le P_R \end{aligned}} \tag{10}$$

The minimization problem (10) with two inequality constraints can be transformed into (11) at the bottom of this page with two non-negative Lagrange multipliers λ_S and λ_R. By using the signal models (1), (4), and (9), the Lagrange cost J in (11) can be written as follows:

$$\begin{aligned} J = 1 &- 2\,\mathrm{Re}(\boldsymbol{b}_1^*\boldsymbol{H}\boldsymbol{a}) - 2\,\mathrm{Re}(\boldsymbol{b}_2^*\boldsymbol{G}\boldsymbol{W}\boldsymbol{F}\boldsymbol{a}) \\ &+ 2\,\mathrm{Re}(\boldsymbol{a}^*\boldsymbol{H}^*\boldsymbol{b}_1\boldsymbol{b}_2^*\boldsymbol{G}\boldsymbol{W}\boldsymbol{F}\boldsymbol{a}) + |\boldsymbol{a}^*\boldsymbol{H}^*\boldsymbol{b}_1|^2 \\ &+ |\boldsymbol{a}^*\boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}_2|^2 + \sigma_{n_s}^2\|\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}_2\|^2 \\ &+ \sigma_{n_x}^2(\|\boldsymbol{b}_1\|^2 + \|\boldsymbol{b}_2\|^2) + \lambda_S(\|\boldsymbol{a}\|^2 - P_S) \\ &+ \lambda_R(\|\boldsymbol{W}\boldsymbol{F}\boldsymbol{a}\|^2 + \sigma_{n_s}^2\|\boldsymbol{W}\|_F^2 - P_R) \end{aligned} \tag{12}$$

where we assume that the data symbols, channel elements, and noises are independent of one another and $\mathrm{Re}(\cdot)$ takes the real value of its argument. Although J in (12) is not guaranteed to be jointly convex over all variables $\{\boldsymbol{a}, \boldsymbol{W}, \boldsymbol{b}_1, \boldsymbol{b}_2\}$, it can be easily shown that it is convex over each of the variables from the fact that the Hessian matrices with respect to each variable are positive definite matrices. Therefore, alternating minimization procedures, where variables are optimized one at a time while keeping all others fixed [10], are applicable to get a feasible local optimal solution. Differentiating (12) with respect to $\boldsymbol{a}$, $\boldsymbol{W}$, $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, λ_S and λ_R, and equating the derivatives to zero (KKT conditions), we can solve for the optimum beamforming vectors and relay processing matrix. Using the techniques of complex matrix derivatives [8] and linear algebra [9], the derivative of J with respect to $\boldsymbol{W}$ is obtained and equated to zero. As a result, we get

$$\begin{aligned} &\boldsymbol{G}^*\boldsymbol{b}_2(1 - \boldsymbol{b}_1^*\boldsymbol{H}\boldsymbol{a})\boldsymbol{a}^*\boldsymbol{F}^* \\ &= (\boldsymbol{G}^*\boldsymbol{b}_2\boldsymbol{b}_2^*\boldsymbol{G} + \lambda_R\boldsymbol{I}_{N_R})\,\boldsymbol{W}\left(\boldsymbol{F}\boldsymbol{a}\boldsymbol{a}^*\boldsymbol{F}^* + \sigma_{n_s}^2\boldsymbol{I}_{N_R}\right). \end{aligned} \tag{13}$$

Using the matrix inversion lemma [8], when $\lambda_R > 0$, the solution is obtained as

$$\boxed{\boldsymbol{W} = \frac{(1 - \boldsymbol{b}_1^*\boldsymbol{H}\boldsymbol{a})\boldsymbol{G}^*\boldsymbol{b}_2\boldsymbol{a}^*\boldsymbol{F}^*}{\left(\|\boldsymbol{G}^*\boldsymbol{b}_2\|^2 + \lambda_R\right)\left(\|\boldsymbol{F}\boldsymbol{a}\|^2 + \sigma_{n_s}^2\right)}} \tag{14}$$

Here, we can also show that (14) is a minimum Frobenius-norm solution of (13) when $\lambda_R = 0$. For the optimal transmit beamforming vector, the derivative of J with respect to $\boldsymbol{a}$ is obtained [8] and

$$\arg \min_{\{\boldsymbol{a},\boldsymbol{W},\boldsymbol{b}_1,\boldsymbol{b}_2,\lambda_S,\lambda_R\}} \underbrace{\mathrm{E}\,|d - \hat{d}_{HD}|^2 + \lambda_S\left(\mathrm{E}\,\|\boldsymbol{s}\|^2 - P_S\right) + \lambda_R\left(\mathrm{E}\,\|\boldsymbol{x}\|^2 - P_R\right)}_{J} \tag{11}$$

equated to zero. As a result, we get

$$\begin{aligned}\big(\|\boldsymbol{H}^*\boldsymbol{b}_1 + \boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}_2\|^2\boldsymbol{I}_{N_S} + \lambda_S\boldsymbol{I}_{N_S} + \lambda_R\boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{W}\boldsymbol{F}\big)\boldsymbol{a} \\ = \boldsymbol{H}^*\boldsymbol{b}_1 + \boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}_2.\end{aligned} \tag{15}$$

When $\lambda_R \neq 0$ and $\lambda_S \neq 0$, we get $\boldsymbol{a}$ as

$$\begin{aligned}\boldsymbol{a} =& \big(\|\boldsymbol{H}^*\boldsymbol{b}_1+\boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}_2\|^2\boldsymbol{I}_{N_S}+\lambda_S\boldsymbol{I}_{N_S}+\lambda_R\boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{W}\boldsymbol{F}\big)^{-1} \\ &\times (\boldsymbol{H}^*\boldsymbol{b}_1 + \boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}_2)\end{aligned} \tag{16}$$

which can be reformulated as:

$$\boxed{\boldsymbol{a} = \frac{\boldsymbol{H}^*\boldsymbol{b}_1 + \boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}_2}{\|\boldsymbol{H}^*\boldsymbol{b}_1 + \boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}_2\|^2 + \lambda_R\,\|\boldsymbol{W}\boldsymbol{F}\|_F^2 + \lambda_S}} \tag{17}$$

The equality between (16) and (17) can be shown by using (14), the matrix inversion lemma, and $\|\boldsymbol{Q}\|_F^2 = \mathrm{tr}(\boldsymbol{Q}\boldsymbol{Q}^*)$. Here, we can also show that (17) is a minimum 2-norm solution of (15) when $\lambda_R = 0$ and/or $\lambda_S = 0$. Similarly, equating the derivative of J with respect to $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ to zero, we obtain

$$\boxed{\boldsymbol{b}_1 = \frac{(1 - \boldsymbol{a}^*\boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}_2)\boldsymbol{H}\boldsymbol{a}}{\|\boldsymbol{H}\boldsymbol{a}\|^2 + \sigma_{n_x}^2}} \tag{18}$$

and

$$\boxed{\boldsymbol{b}_2 = \frac{(1 - \boldsymbol{a}^*\boldsymbol{H}^*\boldsymbol{b}_1)\boldsymbol{G}\boldsymbol{W}\boldsymbol{F}\boldsymbol{a}}{\|\boldsymbol{G}\boldsymbol{W}\boldsymbol{F}\boldsymbol{a}\|^2 + \sigma_{n_s}^2\|\boldsymbol{G}\boldsymbol{W}\|_F^2 + \sigma_{n_x}^2}} \tag{19}$$

Finally, by equating the derivatives of J with respect to λ_R and λ_S to zero, the equalities

$$P_R = \|\boldsymbol{W}\boldsymbol{F}\boldsymbol{a}\|^2 + \sigma_{n_s}^2\|\boldsymbol{W}\|_F^2, \tag{20}$$

and

$$P_S = \|\boldsymbol{a}\|^2, \tag{21}$$

are obtained. Substituting (14) into (20) leads to

$$\boxed{\begin{aligned}\lambda_R = \Bigg(&\frac{\sqrt{\|\boldsymbol{F}\boldsymbol{a}\|^4\,\|\boldsymbol{G}^*\boldsymbol{b}_2\|^2 + \sigma_{n_s}^2\,\|\boldsymbol{G}^*\boldsymbol{b}_2\boldsymbol{a}^*\boldsymbol{F}^*\|_F^2}}{\sqrt{P_R}(1-\boldsymbol{b}_1^*\boldsymbol{H}\boldsymbol{a})^{-1}\left(\|\boldsymbol{F}\boldsymbol{a}\|^2 + \sigma_{n_s}^2\right)} \\ &- \|\boldsymbol{G}^*\boldsymbol{b}_2\|^2\Bigg)^+\end{aligned}} \tag{22}$$

where $(v)^+ = \max(0, v)$. Substituting (17) into (21) yields

$$\boxed{\begin{aligned}\lambda_S = \Bigg(&\frac{\|\boldsymbol{H}^*\boldsymbol{b}_1 + \boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}_2\|}{\sqrt{P_S}} \\ &- \|\boldsymbol{H}^*\boldsymbol{b}_1 + \boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}_2\|^2 - \lambda_R\,\|\boldsymbol{W}\boldsymbol{F}\|_F^2\Bigg)^+\end{aligned}} \tag{23}$$

3.2. Full-Duplex Relay

Using (1), (2), (4), (7), and (8), the overall signal model can be written as

$$\hat{d}_{FD} = \boldsymbol{b}^*\big(\boldsymbol{H}\boldsymbol{a} + \boldsymbol{G}\boldsymbol{W}\boldsymbol{F}\boldsymbol{a}\big)d + \boldsymbol{b}^*\big(\boldsymbol{n}_x + \boldsymbol{G}\boldsymbol{W}\boldsymbol{n}_s\big) \tag{24}$$

and the desired MMSE problem is as follows:

$$\boxed{\begin{aligned}&\arg\min_{\{\boldsymbol{a},\boldsymbol{W},\boldsymbol{b}\}} \mathrm{E}\,|d - \hat{d}_{FD}|^2 \\ &\text{s.t. } \mathrm{E}\,\|\boldsymbol{s}\|^2 \le P_S \text{ and } \mathrm{E}\,\|\boldsymbol{x}\|^2 \le P_R\end{aligned}} \tag{25}$$

Substituting $\hat{d}_{FD}$ in (24) into (11) instead of $\hat{d}_{HD}$, the Lagrange cost J in (12) is rewritten as

$$\begin{aligned}J = 1 &- 2\,\mathrm{Re}(\boldsymbol{b}^*(\boldsymbol{G}\boldsymbol{W}\boldsymbol{F} + \boldsymbol{H})\boldsymbol{a}) + |\boldsymbol{b}^*(\boldsymbol{G}\boldsymbol{W}\boldsymbol{F} + \boldsymbol{H})\boldsymbol{a}|^2 \\ &+ \sigma_{n_s}^2\|\boldsymbol{W}^*\boldsymbol{G}^*\boldsymbol{b}\|^2 + \sigma_{n_x}^2\|\boldsymbol{b}\|^2 + \lambda_S\left(\|\boldsymbol{a}\|^2 - P_S\right) \\ &+ \lambda_R\left(\|\boldsymbol{W}\boldsymbol{F}\boldsymbol{a}\|^2 + \sigma_{n_s}^2\|\boldsymbol{W}\|_F^2 - P_R\right)\end{aligned} \tag{26}$$

Following the same optimization procedure for HD system, we can obtain the optimal solution of (25) as

$$\boldsymbol{W} = \frac{(1 - \boldsymbol{b}^*\boldsymbol{H}\boldsymbol{a})\boldsymbol{G}^*\boldsymbol{b}\boldsymbol{a}^*\boldsymbol{F}^*}{\left(\|\boldsymbol{G}^*\boldsymbol{b}\|^2 + \lambda_R\right)\left(\|\boldsymbol{F}\boldsymbol{a}\|^2 + \sigma_{n_s}^2\right)} \tag{27}$$

$$\boldsymbol{a} = \frac{(\boldsymbol{H}^* + \boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*)\boldsymbol{b}}{\|(\boldsymbol{H}^* + \boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*)\boldsymbol{b}\|^2 + \lambda_R\,\|\boldsymbol{W}\boldsymbol{F}\|_F^2 + \lambda_S} \tag{28}$$

$$\boldsymbol{b} = \frac{(\boldsymbol{H} + \boldsymbol{G}\boldsymbol{W}\boldsymbol{F})\boldsymbol{a}}{\|(\boldsymbol{H} + \boldsymbol{G}\boldsymbol{W}\boldsymbol{F})\boldsymbol{a}\|^2 + \sigma_{n_s}^2\|\boldsymbol{G}\boldsymbol{W}\|_F^2 + \sigma_{n_x}^2} \tag{29}$$

$$\lambda_R = \left(\frac{\sqrt{\|\boldsymbol{F}\boldsymbol{a}\|^4\|\boldsymbol{G}^*\boldsymbol{b}\|^2 + \sigma_{n_s}^2\|\boldsymbol{G}^*\boldsymbol{b}\boldsymbol{a}^*\boldsymbol{F}^*\|_F^2}}{\sqrt{P_R}(1 - \boldsymbol{b}^*\boldsymbol{H}\boldsymbol{a})^{-1}\left(\|\boldsymbol{F}\boldsymbol{a}\|^2 + \sigma_{n_s}^2\right)} - \|\boldsymbol{G}^*\boldsymbol{b}\|^2\right)^+ \tag{30}$$

$$\begin{aligned}\lambda_S = \Bigg(&\frac{\|(\boldsymbol{H}^* + \boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*)\boldsymbol{b}\|}{\sqrt{P_S}} - \|(\boldsymbol{H}^* + \boldsymbol{F}^*\boldsymbol{W}^*\boldsymbol{G}^*)\boldsymbol{b}\|^2 \\ &- \lambda_R\,\|\boldsymbol{W}\boldsymbol{F}\|_F^2\Bigg)^+\end{aligned} \tag{31}$$

3.3. Iterative Algorithm

Since the optimum values $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{W}, \lambda_S, \lambda_R\}$ are functions of one another, these can be obtained via an iterative procedure where variables are computed one at a time while keeping all others fixed [10]. At the kth iteration, denoting the MSE, the beamforming vectors, and the relay matrix by J_k, $\{\boldsymbol{a}_k, \boldsymbol{b}_k\}$, and $\boldsymbol{W}_k$, respectively, the proposed iterative algorithm is described in Table 1. In each iterative step, the MSE, J_k, is monotonically diminishing. It is also obvious that the J_k is lower bounded by zero and the convergence of the J_k is then guaranteed. Therefore, the difference between J_{k-1} and J_k can be used as a stopping criterion with a positive design factor ϵ in Step 3.

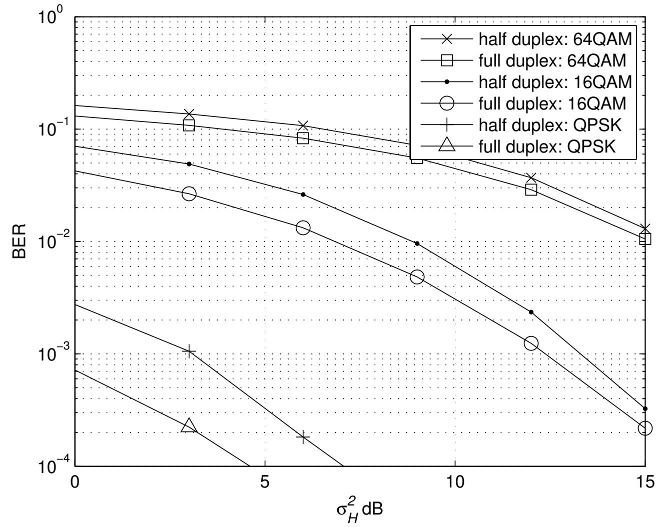
4. SIMULATION AND DISCUSSION

Computer simulations are conducted to examine the performance of the designed relay systems. The BER performance of the proposed systems are evaluated and discussed under the perfect CSI assumption when $N_S = N_R = N_D = 2$. The transmitted signals from the sources are modulated by 64-QAM and the modulated symbols are grouped into frames consisting of 100 symbols. For each frame, flat fading MIMO channel matrices $\boldsymbol{H}$, $\boldsymbol{F}$ and $\boldsymbol{G}$ are generated from independent Gaussian random variables. Channels are fixed during a frame, but they vary independently over frames. The results shown below are the averages over 10^5 independent trials. Assuming $\sigma_F^2 = 10\,\mathrm{dB}$, $P_S = P_R = P_D = 1$, and $\sigma_{n_s}^2 = \sigma_{n_x}^2 = 1$, the BER performance curves are evaluated over various σ_G^2 and σ_H^2.

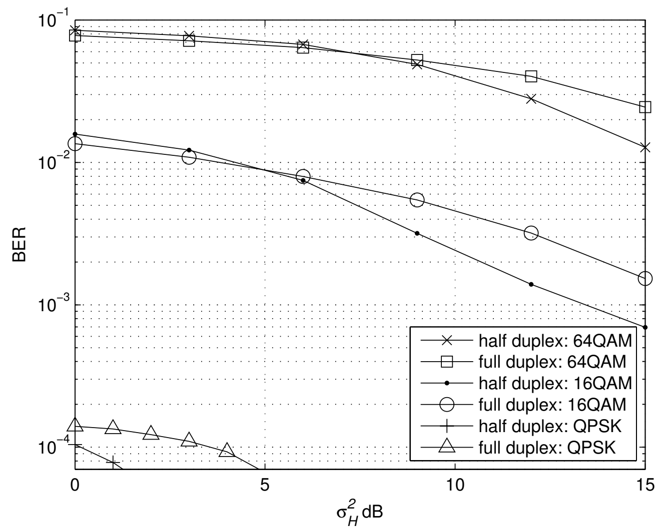
Table 1. Iterative Algorithm, (Half/ Full) Duplex

Step 1:	Initialization, $k = 0$
	$\boldsymbol{W}_0 = \boldsymbol{I}_{N_R}, \boldsymbol{b}_0 = [1 \cdots 1]$,
	$\lambda_S = \lambda_R = 0$, $J_0 = 0$.
Step 2:	Iteration: $k \leftarrow k + 1$
	$\boldsymbol{a}_k = f_{\boldsymbol{a}}\left(\boldsymbol{W}_{k-1}, \boldsymbol{b}_{k-1}, \lambda_S, \lambda_R\right)$ in (17/ 28)
	$\boldsymbol{W}_k = f_{\boldsymbol{W}}\left(\boldsymbol{a}_k, \boldsymbol{b}_{k-1}, \lambda_R\right)$ in (14/ 27)
	$\boldsymbol{b}_k = f_{\boldsymbol{b}}\left(\boldsymbol{W}_k, \boldsymbol{a}_k\right)$ in (18,19/ 29)
	$\lambda_R = f_{\lambda_R}\left(\boldsymbol{a}_k, \boldsymbol{b}_k\right)$ in (22/ 30)
	$\lambda_S = f_{\lambda_S}\left(\boldsymbol{W}_k, \boldsymbol{b}_k, \lambda_R\right)$ in (23/ 31)
	$J_k = f_J\left(\boldsymbol{W}_k, \boldsymbol{a}_k, \boldsymbol{b}_k, \lambda_S, \lambda_R\right)$ in (12/ 26)
Step 3:	If $0 \leq J_{k-1} - J_k \leq \epsilon$ go to Step 4
	otherwise go back Step 2.
	(In our simulation, we use $\epsilon = 0.0001$).
Step 4:	$\boldsymbol{a} = \boldsymbol{a}_k; \boldsymbol{b} = \boldsymbol{b}_k; \boldsymbol{W} = \boldsymbol{W}_k$.

In Fig. 2(a), when $\sigma_G^2 = 5$ dB, the FD relay system shows better BER performance than the HD relay system while, in Fig. 2(b), when $\sigma_G^2 = 15$ dB, the HD relay system shows better BER performance than the FD relay system in some σ_H^2 regimes. Generally, the poor performance of the HD method comes from the fact that the noise effect at the destination is twice as much as the FD method as shown in the signal model (9) and (24), i.e., $\{\boldsymbol{n}_{x1}, \boldsymbol{n}_{x2}\}$ in HD system and $\boldsymbol{n}_x$ in FD system. Furthermore, it is noticeable that the HD relay system consumes twice as much time resources as the FD relay system. Consequently, the FD relay system has good potential as a candidate for future relay networks.



(a)



(b)

Fig. 2. Comparison of BER performance when the $\sigma_F^2 = 10$ dB and $N_S = N_R = N_D = 2$. (a) $\sigma_G^2 = 5$ dB. (b) $\sigma_G^2 = 15$ dB.

5. CONCLUSION

In this paper, we designed jointly HD and FD relay processing matrices and source-destination beamforming vectors based on an MMSE formulation under inequality constraints on the transmit power of the source and the relay. From the simulation results, the designed FD relay system performs better than the designed HD relay system in various SNR regimes.

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