ALAMOUTI SPACE-TIME CODED RELAY STRATEGY FOR WIRELESS NETWORKS

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ABSTRACT

A relay strategy for Alamouti space-time coded transmissions is proposed. The relay structure is optimized to maximize the received SNR and the receiver is able to exploit the orthogonal structure of the code to ensure temporal and spatial diversity.

1. INTRODUCTION

Relay strategies help improve the performance of wireless networks by increasing coverage and by reducing end-to-end path losses. MIMO techniques and cooperative relaying may be combined to provide spatial and temporal transmit diversity and path loss compensation [1],[2]. There have been several useful works on relay strategies in the literature. For example, the performance limit of a SISO link in a cooperative relay network is studied in [3], while virtual antenna arrays are studied in [4] in a cooperative MIMO broadband relay network. Moreover, the work [5] discusses a relay system that consists of two relays and a source-destination pair that uses space-time coding, while [6] shows that a multiple relay network with an appropriate cooperative code construction guarantees full spatial diversity gain. In [7] we proposed and analyzed a multi-relay minimum mean square error (MMSE) strategy. In this paper we incorporate Alamouti space-time coding [8] in a distributed fashion. In comparison to the relay schemes for space-time codes proposed in [5, 6], we optimize the relay structure in order to achieve maximum SNR at the receiver node. In addition, the receiver is able to exploit the structure of the Alamouti code and the temporal and spatial diversity of the code. An analysis of the power consumption per relay node is performed under Rayleigh conditions. It is shown that the power consumption per node is inversely proportional to the number of the relay nodes (N) for large enough N. Simulation results illustrate the conclusions.

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2. ALAMOUTI CODED TRANSMISSIONS

Consider a wireless link with one transmitter and one receiver – see Fig. 1. It is assumed that the transmitter has two antennas and the receiver has one antenna. A 2×1 Alamouti space-time code is used to transmit data. Let $\boldsymbol{h}$ denote the 2×1 channel vector between the transmitter and the receiver. A quasi-static fading condition is assumed for each channel tap so that the channel realizations remain fixed for the duration of a single frame. Let h_1 and h_2 denote the first and second elements of $\boldsymbol{h}$; they stand for the channel coefficients from the first and second transmit antennas to the receive antenna. The channel gains h_1 and h_2 are assumed to have a Rayleigh distributed amplitude with variance 1 and a uniformly distributed phase between 0 and 2π. The received data vector over two consecutive time instants is given by

$$\boldsymbol{y} = \begin{pmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{pmatrix} \boldsymbol{h} + \boldsymbol{v} \tag{1}$$

where s_1 and s_2 are the transmitted symbols and $(.)^*$ denotes complex conjugation. The 2×1 vector $\boldsymbol{y}$ consists of the received signals $\{y_1, y_2\}$ over two consecutive transmissions and $\boldsymbol{v}$ is a 2×1 complex Gaussian noise with covariance matrix $\sigma_v^2 \boldsymbol{I}$. In order to recover the transmitted symbols, relation (1) may be written as

$$\begin{pmatrix} y_1 \\ y_2^* \end{pmatrix} = \begin{pmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2^* \end{pmatrix} \tag{2}$$

so that the least-squares estimates of $\{s_1, s_2\}$ are given by

$$\begin{pmatrix} \hat{s}_1 \\ \hat{s}_2 \end{pmatrix} = \frac{1}{|h_1|^2 + |h_2|^2} \begin{pmatrix} h_1^* & -h_2 \\ h_2^* & h_1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2^* \end{pmatrix} \tag{3}$$

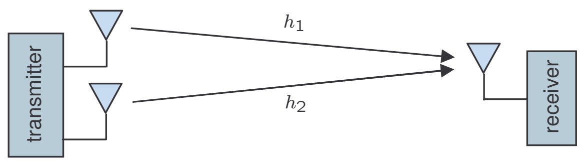

Fig. 1. A 2-transmit 1-receive Alamouti coded system.

3. RELAYED ALAMOUTI SPACE-TIME CODE FORMULATION

Now consider the same wireless link with N relay terminals in between–see Fig 2. The purpose of the relays is to compensate for path loss and to improve spatial diversity. Each relay node is assumed to have one transmit and one receive antenna. Let $\boldsymbol{H}_s$ denote the $2 \times N$ channel matrix between the 2-antenna transmitter and the N 1-antenna relay nodes. We partition $\boldsymbol{H}_s$ as

$$\boldsymbol{H}_s = \begin{pmatrix} \boldsymbol{h}_{s,1} \\ \boldsymbol{h}_{s,2} \end{pmatrix} \quad (2 \times N) \tag{4}$$

where $\boldsymbol{h}_{s,1}$ and $\boldsymbol{h}_{s,2}$ are $1 \times N$ row channel vectors from the first and second antennas of the transmitter to the relay nodes, respectively. Moreover, let $\boldsymbol{h}_t$ denote the $N \times 1$ (column) channel vector from the N relay nodes to the receiver. The entries of $\boldsymbol{H}_s$ and $\boldsymbol{h}_t$ are assumed to have a Rayleigh distributed amplitude with variance 1 and a uniformly distributed phase between 0 and 2π. The received data at the relay terminals are

$$\begin{pmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \end{pmatrix} = \begin{pmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{pmatrix} \begin{pmatrix} \boldsymbol{h}_{s,1} \\ \boldsymbol{h}_{s,2} \end{pmatrix} + \begin{pmatrix} \boldsymbol{v}_{s,1} \\ \boldsymbol{v}_{s,2} \end{pmatrix} \tag{5}$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are $1 \times N$ vectors; they denote the signals received by the relay terminals during the first and second time frames, respectively. Let

$$\boldsymbol{R} = \begin{pmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \end{pmatrix} \quad (2 \times N)$$

The relays transform the received data matrix $\boldsymbol{R}$ using some $N \times N$ linear transformation $\boldsymbol{F}$, say as

$$\boldsymbol{X} = \boldsymbol{R}\boldsymbol{F} \tag{6}$$

or, equivalently,

$$\begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{pmatrix} = \begin{pmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{pmatrix} \begin{pmatrix} \boldsymbol{h}_{s,1} \\ \boldsymbol{h}_{s,2} \end{pmatrix} \boldsymbol{F} + \begin{pmatrix} \boldsymbol{v}_{s,1} \\ \boldsymbol{v}_{s,2} \end{pmatrix} \boldsymbol{F}$$

where $\boldsymbol{F}$ is to be determined in order to enforce some optimal performance as will be explained later. The signals at the destination over two consecutive time instants are then given by

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{h}_t + \boldsymbol{v}_t \tag{7}$$

i.e.,

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} s_1 & s_2 \\ s_2^* & -s_1^* \end{pmatrix} \begin{pmatrix} \boldsymbol{h}_{s,1} \\ \boldsymbol{h}_{s,2} \end{pmatrix} \boldsymbol{F}\boldsymbol{h}_t + \begin{pmatrix} \boldsymbol{v}_{s,1} \\ \boldsymbol{v}_{s,2} \end{pmatrix} \boldsymbol{F}\boldsymbol{h}_t + \begin{pmatrix} v_{t,1} \\ v_{t,2} \end{pmatrix} \tag{8}$$

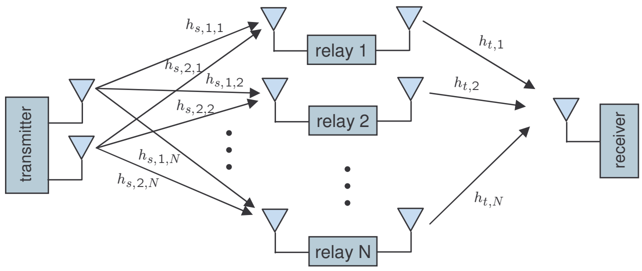

Fig. 2. Alamouti space-time coded system with N relay nodes.

which leads to

$$\begin{pmatrix} y_1 \\ y_2^* \end{pmatrix} = \begin{pmatrix} \boldsymbol{h}_{s,1}\boldsymbol{F}\boldsymbol{h}_t & \boldsymbol{h}_{s,2}\boldsymbol{F}\boldsymbol{h}_t \\ -\boldsymbol{h}_t^*\boldsymbol{F}^*\boldsymbol{h}_{s,2}^* & \boldsymbol{h}_t^*\boldsymbol{F}^*\boldsymbol{h}_{s,1}^* \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} \boldsymbol{v}_{s,1}\boldsymbol{F}\boldsymbol{h}_t \\ \boldsymbol{h}_t^*\boldsymbol{F}^*\boldsymbol{v}_{s,2}^* \end{pmatrix} + \begin{pmatrix} v_{t,1} \\ v_{t,2}^* \end{pmatrix} \tag{9}$$

or, more compactly,

$$\bar{\boldsymbol{y}} = \bar{\boldsymbol{H}}\boldsymbol{s} + \bar{\boldsymbol{v}}_s + \bar{\boldsymbol{v}}_t \tag{10}$$

It can be verified that the modified channel matrix $\bar{\boldsymbol{H}}$ has the same orthogonal property as the original channel matrix appearing in (2), i.e.,

$$\bar{\boldsymbol{H}}^*\bar{\boldsymbol{H}} = \tag{11}$$

$$\begin{pmatrix} |\boldsymbol{h}_{s,1}\boldsymbol{F}\boldsymbol{h}_t|^2 + |\boldsymbol{h}_{s,2}\boldsymbol{F}\boldsymbol{h}_t|^2 & 0 \\ 0 & |\boldsymbol{h}_{s,1}\boldsymbol{F}\boldsymbol{h}_t|^2 + |\boldsymbol{h}_{s,2}\boldsymbol{F}\boldsymbol{h}_t|^2 \end{pmatrix}$$

Then the least-squares estimates of $\{s_1, s_2\}$ can be obtained from (9) as

$$\begin{aligned} \begin{pmatrix} \hat{s}_1 \\ \hat{s}_2 \end{pmatrix} &= \frac{1}{|\boldsymbol{h}_{s,1}\boldsymbol{F}\boldsymbol{h}_t|^2 + |\boldsymbol{h}_{s,2}\boldsymbol{F}\boldsymbol{h}_t|^2} \bar{\boldsymbol{H}}^*\bar{\boldsymbol{y}} \\ &= \boldsymbol{s} + \frac{\bar{\boldsymbol{H}}^*\bar{\boldsymbol{v}}_s + \bar{\boldsymbol{H}}^*\bar{\boldsymbol{v}}_t}{|\boldsymbol{h}_{s,1}\boldsymbol{F}\boldsymbol{h}_t|^2 + |\boldsymbol{h}_{s,2}\boldsymbol{F}\boldsymbol{h}_t|^2} \end{aligned} \tag{12}$$

We now define the SNR performance as

$$\text{SNR} = \frac{E\|\boldsymbol{s}\|^2}{E\left\| \frac{\bar{\boldsymbol{H}}^*\bar{\boldsymbol{v}}_s + \bar{\boldsymbol{H}}^*\bar{\boldsymbol{v}}_t}{|\boldsymbol{h}_{s,1}\boldsymbol{F}\boldsymbol{h}_t|^2 + |\boldsymbol{h}_{s,2}\boldsymbol{F}\boldsymbol{h}_t|^2} \right\|^2} \tag{13}$$

and we are interested in choosing $\boldsymbol{F}$ in order to maximize the SNR. We would like each node to rely only on its own received signal. Thus we limit $\boldsymbol{F}$ to be a diagonal matrix in order to limit communication among the relay nodes. Let

$z = Fh_t$. Then the noise power is given by

$$\sigma_v^2 \triangleq E\left\|\frac{\bar{H}^*\bar{v}_s + \bar{H}^*\bar{v}_t}{|h_{s,1}Fh_t|^2 + |h_{s,2}Fh_t|^2}\right\|^2 = \tag{14}$$

$$E\left\|\frac{\begin{pmatrix} z^*h_{s,1}^* & -h_{s,2}z \\ z^*h_{s,2}^* & h_{s,1}z \end{pmatrix}\begin{pmatrix} v_{s,1}z \\ z^*v_{s,2}^* \end{pmatrix}}{|h_{s,1}z|^2 + |h_{s,2}z|^2}\right\|^2$$

$$+E\left\|\frac{\begin{pmatrix} z^*h_{s,1}^* & -h_{s,2}z \\ z^*h_{s,2}^* & h_{s,1}z \end{pmatrix}\begin{pmatrix} v_{t,1} \\ v_{t,2}^* \end{pmatrix}}{|h_{s,1}z|^2 + |h_{s,2}z|^2}\right\|^2$$

where the expectation is over the noise terms, since we are assuming quasi static channels over the duration of two data frames. Since the noises are assumed to be independent, the expectation of all cross terms over the noises are zero so that

$$\sigma_v^2 = \frac{2(\sigma_{v_s}^2\|z\|^2 + \sigma_{v_t}^2)}{|h_{s,1}z|^2 + |h_{s,2}z|^2} \tag{15}$$

Let $\beta = \|z\|^2$. Then (15) can be rewritten as

$$\sigma_v^2 = \frac{2(\sigma_{v_s}^2\beta + \sigma_{v_t}^2)}{z^*(h_{s,1}^*h_{s,1} + h_{s,2}^*h_{s,2})z} \tag{16}$$

Finally, using (13), the SNR is

$$\text{SNR} = \frac{\sigma_s^2 z^*(h_{s,1}^*h_{s,1} + h_{s,2}^*h_{s,2})z}{2(\sigma_{v_s}^2\beta + \sigma_{v_t}^2)} \tag{17}$$

where the unknown F is embedded in z and β.

4. MAXIMUM SNR RELAY STRATEGY

We shall select F in order to solve

$$\hat{F} = \arg\max_{F} \text{ SNR} \quad \text{s.t. } \|z\|^2 = \beta \tag{18}$$

with a constraint on $\|z\|^2$ (i.e., we shall fix β). Introduce the singular value decomposition (SVD) of the rank 2 matrix $h_{s,1}^*h_{s,1} + h_{s,2}^*h_{s,2}$, i.e.,

$$h_{s,1}^*h_{s,1} + h_{s,2}^*h_{s,2} = U\Sigma V^T \tag{19}$$

where U and V are $N \times N$ unitary matrices, and Σ is a diagonal matrix with generally two nonzero singular values $\sigma_1 \geq \sigma_2$. We are interested in choosing a vector z such that $\|z\|^2 = \beta$ and (17) is maximized. The optimum z is given by

$$z = \sqrt{\beta}\, u_1 \tag{20}$$

where u_1 is the first column of U, which is the singular vector corresponding to the largest singular value σ_1. Let $F = \text{diag}(f)$, where f is a column vector. Then we must have

$$z \triangleq Fh_t = \text{diag}(h_t)f = \sqrt{\beta}u_1 \tag{21}$$

which allows us to solve for the relay vector f as

$$\boxed{f = \sqrt{\beta}\text{diag}(h_t)^{-1}u_1} \tag{22}$$

In other words, each element of f is given by

$$\boxed{f_i = \sqrt{\beta}\frac{u_{1,i}}{h_{t,i}}} \quad i = 1, ..., N \tag{23}$$

and the resulting maximum SNR is

$$\text{SNR} = \frac{\sigma_s^2\sigma_1\beta}{2(\sigma_{v_s}^2\beta + \sigma_{v_t}^2)} \tag{24}$$

5. PERFORMANCE ANALYSIS

In the above relay strategy, the transmitted data over two time instants by the ith relay node is

$$\begin{pmatrix} x_{1,i} \\ x_{2,i} \end{pmatrix} = \sqrt{\beta}\frac{u_{1,i}}{h_{t,i}}\begin{pmatrix} r_{1,i} \\ r_{2,i} \end{pmatrix} \tag{25}$$

where $r_{1,i}$ and $r_{2,i}$ are the received signals at the ith relay node at the first and second time frames, respectively. Assuming a fixed channel, the power that is spent by each relay node for the duration of two frames is therefore

$$P_i = \beta|u_{1,i}|^2\frac{|r_{1,i}|^2 + |r_{2,i}|^2}{|h_{t,i}|^2} \tag{26}$$

In order to examine how power consumption changes as the number of relay terminal increases, we approximate the relay matrix F for large N. Thus note first that, for a reasonably large number of relay terminals and for independent channels we can assume that

$$h_{s,1}h_{s,2}^* \approx 0 \tag{27}$$

Moreover, and in view of (27), we may approximate u_1, the first column of U, as follows. Writing

$$h_{s,1}^*h_{s,1} + h_{s,2}^*h_{s,2} = \tag{28}$$

$$\begin{pmatrix} \frac{h_{s,1}^*}{\|h_{s,1}\|} & \frac{h_{s,2}^*}{\|h_{s,2}\|} \end{pmatrix}\begin{pmatrix} \|h_{s,1}\|^2 & \\ & \|h_{s,2}\|^2 \end{pmatrix}\begin{pmatrix} \frac{h_{s,1}}{\|h_{s,1}\|} \\ \frac{h_{s,2}}{\|h_{s,2}\|} \end{pmatrix} \tag{29}$$

and calling upon (27) we can set

$$\sigma_1 \approx \max(\|h_{s,1}\|^2, \|h_{s,2}\|^2)$$
$$\sigma_2 \approx \min(\|h_{s,1}\|^2, \|h_{s,2}\|^2)$$

and

$$\boldsymbol{u}_1 \approx \begin{cases} \frac{\boldsymbol{h}_{s,1}^*}{\|\boldsymbol{h}_{s,1}\|}, & \|\boldsymbol{h}_{s,1}\|^2 \geq \|\boldsymbol{h}_{s,2}\|^2 \\ \frac{\boldsymbol{h}_{s,2}^*}{\|\boldsymbol{h}_{s,2}\|}, & \|\boldsymbol{h}_{s,1}\|^2 < \|\boldsymbol{h}_{s,2}\|^2 \end{cases} \quad (30)$$

$$\boldsymbol{u}_2 \approx \begin{cases} \frac{\boldsymbol{h}_{s,2}^*}{\|\boldsymbol{h}_{s,2}\|}, & \|\boldsymbol{h}_{s,1}\|^2 \geq \|\boldsymbol{h}_{s,2}\|^2 \\ \frac{\boldsymbol{h}_{s,1}^*}{\|\boldsymbol{h}_{s,1}\|}, & \|\boldsymbol{h}_{s,1}\|^2 < \|\boldsymbol{h}_{s,2}\|^2 \end{cases}$$

Now let us assume without loss of generality and for convenience of presentation, that $\|\boldsymbol{h}_{s,1}\|^2 \geq \|\boldsymbol{h}_{s,2}\|^2$. Then the ith element of $\boldsymbol{u}_1$ is given by

$$u_{1,i} \approx \frac{h_{s,1,i}^*}{\sqrt{\|\boldsymbol{h}_{s,1}\|^2}} \quad (31)$$

In order to investigate how the power consumption changes as the number of relay terminals increases, we may use $E|u_{1,i}|^2$ for large N instead of $|u_{1,i}|^2$ in (26) as an approximation, so that

$$\begin{aligned} P_i &= \beta |u_{1,i}|^2 \frac{|r_{1,i}|^2 + |r_{2,i}|^2}{|h_{t,i}|^2} \\ &\approx \beta E|u_{1,i}|^2 \frac{|r_{1,i}|^2 + |r_{2,i}|^2}{|h_{t,i}|^2} \\ &= \beta \frac{1}{N} \frac{|r_{1,i}|^2 + |r_{2,i}|^2}{|h_{t,i}|^2} \end{aligned} \quad (32)$$

since

$$E|u_{1,i}|^2 = E\left(\frac{|h_{s,1,i}|^2}{\|\boldsymbol{h}_{s,1}\|^2}\right) \approx \frac{\sigma_{h_s}^2}{N.\sigma_{h_s}^2} = \frac{1}{N} \quad (33)$$

As a result, it follows that for a fixed β, increasing the number of relay terminals decreases the power consumption for each relay terminal while keeping the SNR constant. Moreover, it can be seen from (24) that the SNR is related to the choice of β; increasing β increases the SNR and decreasing β decreases the SNR.

6. SIMULATION

The performance of the proposed scheme is investigated with one source node and one destination node. We assume that all relay terminals are essentially at the same distance from the source and destination sensors. Using this assumption, the channels from the source to the relay terminals have the same second moment statistics as the channels from the relay nodes to the destination. Moreover, we use zero-mean unit variance complex Gaussian channel models for $\boldsymbol{h}_{s,1}$, $\boldsymbol{h}_{s,2}$ and $\boldsymbol{h}_t$ and the transmitted signal from the source sensor is assumed to be QPSK with unit power. We consider two different scenarios to study the performance. In the first scenario, we scale the factor β by the number

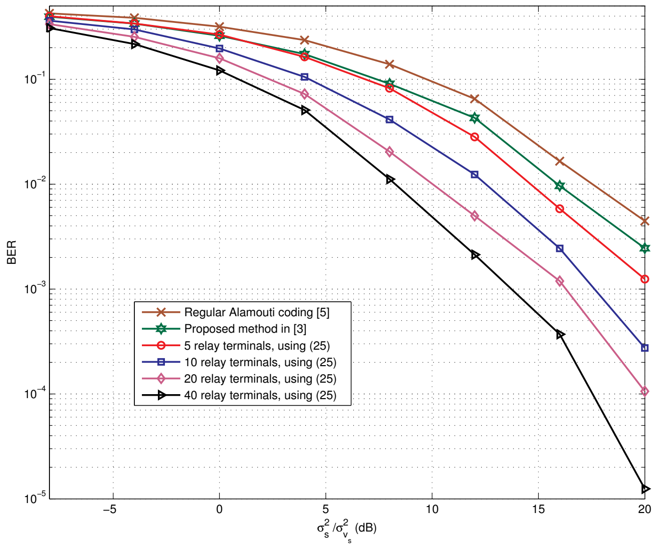

Fig. 3. The BER performance of the relay system when $\beta = N/10$.

of relay nodes, for example in the first simulation we have assumed $\beta = N/10$. As a result, using (32), the average power consumption per relay node is almost fixed for any number of the relay nodes. Fig. 3 compares the BER performance of the system for different number of relay nodes. Also we have investigated the BER performance of the system when no relay nodes are used. In such a case, since there is no relay in between, we can assume a path loss coefficient of 2.5 for twice the distance of the relay case, so the channel variance becomes $\frac{1}{2^{2.5}} = 0.18$ instead of 1. It can be seen that increasing the number of relay terminals improves the performance. The second scenario assumes fixed β for any number of relay terminals. As a result the BER performance will be the same for any number of relay nodes, but the per node power consumption decreases as we increase the number of relay terminals.

7. CONCLUSION

In this article, we have proposed a method to combine Alamouti space-time coding with cooperative relaying to achieve both spatial diversity and path loss compensation. The proposed scheme maximizes SNR and results in a relay strategy that decreases power consumption per relay node as the number of such nodes increases.

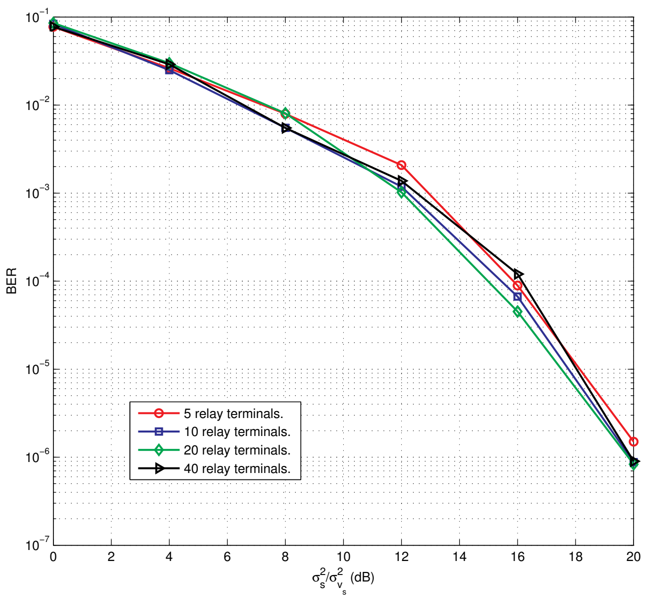

Fig. 4. The BER performance of the system when β is fixed for different number of relay terminals.

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