A STOCHASTIC POWER CONTROL ALGORITHM FOR WIRELESS NETWORKS

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ABSTRACT

In this paper, we propose a power control strategy for distributed wireless networks by using a quadratic state-space control design. Simulation results show the effectiveness of the proposed power control algorithm.

keywords: Power control, Rate adaptation, Wireless networks, Quadratic stochastic control.

I. INTRODUCTION

The evolution of mobile wireless communication and sensor networks has triggered an interest in efficient power control strategies. Several power control algorithms have already been investigated in the literature. For example, the strategies proposed in [1], [2] balance the signal to interference ratios in a distributed way. Related approaches from [3] and [4] include QoS requirements, while the Kalman filtering approach from [5] uses admission control as the central QoS issue. However, these solutions may not perform well when the desired rate of transmission varies due to a rate adaptation algorithm or a congestion control algorithm that might be in place in the wireless network. The motivation for allowing such variable rates of transmission through rate adaptation is due to the availability and affordability of newer wireless devices that support multiple data rates. An effective power control algorithm should meet a required Signal-to-Interference-Ratio (SIR) and at the same time guarantee a desired rate commanded by the rate adaptation algorithm. In addition, the power control algorithm has to be responsive to congestion conditions in the network and, therefore, should be able to adjust the power levels accordingly. We address these issues in this paper and in [6], [7] by assuming an M/M/m/m queueing model for each wireless node in terms of its task arrival in the form of queries, data packets that need to be routed, and any other messages that need to be handled. We also allow the channel and interference gains to vary. Extensive simulations illustrate the performance of the proposed power control algorithm.

II. SYSTEM MODEL

Consider a wireless network operating under dynamic network conditions. The nodes are organized into local clusters or cells with one node acting as the cell master node. The master node receives data from all cell members, arranges and processes them, and then routes them to individual master nodes in adjacent cells. The master nodes can perform additional important tasks like routing and congestion control for high data rate communications, as well as handle some data processing and network information for the other nodes connected to them. Master nodes communicate with each other using selective frequency bands. Being a master node is power consuming and the nodes may take turns as master nodes in some randomized fashion similar to that proposed in [10]. Moreover, the multiple access control protocol that we adopt in this paper is as follows. The space is divided into virtual geographical cells. A time slot is allocated to a node that wishes to communicate in a cell, and the nodes communicating in the same time slot in other cells cause interference with this cell. Given the above, the Signal-to-Interference-plus-Noise-Ratio (SIR) for node i at time k on an uplink channel is defined by

$$\gamma_i(k) = \frac{G_{ii}(k)p_i(k)}{\sum_{j\in \mathbf{A}} G_{ij}(k)p_j(k) + \sigma_i^2} \tag{1}$$

where G_{ij} denotes the channel gain from the $j-th$ node to the intended master node of the $i-th$ node, p_i is the transmission power from the $i-th$ node, and σ_i^2 is the power of white Gaussian noise at the receiver of the master node that node i is connected to. Moreover, $\mathbf{A}$ is the set of nodes that are interfering with node i. A schematic representation with three cells, three master nodes, and active and interfering nodes is shown in Figure 1.

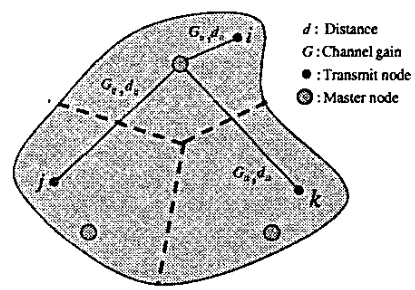

Fig. 1. A schematic representation with three cells, three master nodes, and active and interfering nodes.
figure

Let $f_i(k)$ denote the flow rate at node i at time k. We shall assume that each node in the network employs a flow-rate control algorithm of the following form (see, e.g., [11]):

$$\boxed{f_i(k+1) = f_i(k) + \mu[d(k) - c(k)f_i(k)]} \tag{2}$$

where μ is a positive step-size, $c(k)$ is a measure of the amount of congestion in the network, and $d(k)$ controls the amount of rate increase per iteration. In the absence of congestion (i.e., when $c(k) = 0$), the rate is increased by $\mu d(k)$. When congestion is present, the change in the rate is decreased by $\mu c(k)f_i(k)$. The

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parameter $d(k)$ is modelled as a random variable with mean m_d and variance σ_d^2. In Appendix A we explain how $c(k)$ can be estimated. Now in view of Shannon's capacity formula, the flow rate $f_i(k)$ demands an SIR level $\gamma_i'(k)$ that is given by

$$f_i(k) = \frac{1}{2}\log_2[1 + \gamma_i'(k)] \tag{3}$$

This means that we need to design the power sequence $p_i(k)$ such that the resulting SIR level $\gamma_i(k)$ that is obtained from (1) approaches the desired SIR level $\gamma_i'(k)$ from (3).

Let $\bar{x}_i(k)$ denote the dB value of a variable $x_i(k)$, namely $\bar{x}_i(k) = 10\log x_i(k)$. Now, usually, during normal network operation, $\gamma_i'(k) \gg 1$ and, hence, $f_i(k)$ in (3) is proportional to $\bar{\gamma}_i'(k)$. Substituting this fact into (2) we find that the desired SIR level (in dB scale) should vary according to the rule

$$\boxed{\bar{\gamma}_i'(k+1) = [1 - \mu c(k)]\bar{\gamma}_i'(k) \; + \; \mu' d(k)} \tag{4}$$

where $\mu' = 20\mu/\log_2(10)$. We assume initially that each node in the network adjusts its power according to the power control algorithm:

$$\boxed{\bar{p}_i(k+1) = \bar{p}_i(k) + \alpha_i[\bar{\gamma}_i'(k) - \bar{\gamma}_i(k)]} \tag{5}$$

where α_i is some given step-size parameter that is allowed to vary from one node to another. Let

$$\beta_i(k) = \frac{G_{ii}(k)}{\sum_{j\in\mathcal{A}} G_{ij}(k)p_j(k) \; + \; \sigma_i^2}$$

denote the scaling factor that determines how $p_i(k)$ affects the achieved $\gamma_i(k)$ in (1), i.e.,

$$\gamma_i(k) = \beta_i(k)p_i(k)$$

or, equivalently, in dB scale,

$$\boxed{\bar{\gamma}_i(k) = \bar{\beta}_i(k) \; + \; \bar{p}_i(k)} \tag{6}$$

We shall refer to $\bar{\beta}_i(k)$ as the effective channel gain. We assume that $\bar{\beta}_i(k)$ varies according to the model:

$$\bar{\beta}_i(k+1) = \bar{\beta}_i(k) + n_i(k) \tag{7}$$

where $n_i(k)$ is a zero-mean disturbance of variance σ_n^2 and is independent of $\bar{p}_i(k)$. Substituting this model for $\bar{\beta}_i(k)$ into (6), we find that the achieved $\bar{\gamma}_i(k)$ varies according to the rule:

$$\boxed{\bar{\gamma}_i(k+1) = (1-\alpha_i)\bar{\gamma}_i(k) + \alpha_i\bar{\gamma}_i'(k) + n_i(k)} \tag{8}$$

III. STOCHASTIC POWER CONTROL ALGORITHM

Our objective then is to design the power control sequence $\{p_i(k)\}$ such that the actual SIR levels $\{\gamma_i(k)\}$ will tend to the desired SIR levels $\{\gamma_i'(k)\}$. We shall address this design problem by formulating a quadratic control problem as follows. First, we drop the node index i for simplicity of notation (it is to be understood that the resulting control mechanism is implemented at each node). Second, we introduce the two-dimensional state vector:

$$x_k \triangleq \begin{bmatrix} \bar{\gamma}(k) \\ \bar{\gamma}'(k) \end{bmatrix}$$

Then combining (4) and (8) we arrive at the state-space model:

$$x_{k+1} = \begin{bmatrix} 1-\alpha & \alpha \\ 0 & 1-\mu c(k) \end{bmatrix} x_k + \begin{bmatrix} n(k) \\ \mu' d(k) \end{bmatrix}$$

or, more compactly,

$$x_{k+1} = A_k x_k + w_k \tag{9}$$

where the 2×2 coefficient matrix A_k is given by

$$\boxed{A_k = \begin{bmatrix} 1-\alpha & \alpha \\ 0 & 1-\mu c(k) \end{bmatrix}} \tag{10}$$

and where w_k is a 2×1 random vector with covariance matrix

$$\boxed{Q = Ew_k w_k^T = \begin{bmatrix} \sigma_n^2 & \\ & \mu'^2\sigma_d^2 \end{bmatrix}} \tag{11}$$

In order to drive $\gamma(k)$ towards the desired level $\gamma'(k)$ we shall introduce a control sequence $\{u_k\}$ into (9) as follows:

$$\boxed{x_{k+1} = A_k x_k + Bu_k + w_k} \tag{12}$$

for some 2×2 matrix B and 2×1 control sequence u_k to be designed. For example, let

$$Bu_k = \begin{bmatrix} u_p(k) \\ u_f(k) \end{bmatrix}$$

denote the individual entries of Bu_k to be designed. Then the inclusion of the term Bu_k in (12) simply amounts to adding a control signal $u_p(k)$ to the power update (5). Likewise, the control signal $u_f(k)$ is added to the desired SIR update (4) (and, consequently, $(\log_2(10)/20)u_f(k)$ into the rate-flow update (2)).

In addition to employing a control sequence $\{u_k\}$, we shall assume for generality that we also have access to output measurements that are related to the state vector as follows:

$$\boxed{y_k = Cx_k + v_k} \tag{13}$$

for some known matrix C and where v_k denotes measurement noise with covariance matrix R,

$$\boxed{R = Ev_k v_k^T}$$

Usually, $C = I$ so that the entries of y_k correspond to noisy measurements of the actual and desired SIR levels, $\{\bar{\gamma}(k), \bar{\gamma}'(k)\}$.

We then seek a control sequence $\{u_k\}$ that minimizes the following stochastic quadratic cost function:

$$\mathcal{J} = \lim_{N\to\infty} \frac{1}{N} E\left[\sum_{k=0}^{N} \left(\lambda\|Lx_k\|^2 \; + \; \|u_k\|^2\right)\right]$$

where λ is a positive regularization parameter (chosen by the designer) and $L = [\ 1 \quad -1\ ]$ (or some other more general choice). This particular choice of L results in

$$Lx_k = \bar{\gamma}(k) - \bar{\gamma}'(k)$$

so that $\|Lx_k\|^2$ is a measure of the energy of the difference between $\{\bar{\gamma}(k), \bar{\gamma}'(k)\}$. In this way, the cost function $\mathcal{J}$ defined above is such that it seeks to minimize, on average, the squared Euclidean distance between the successive actual and desired SIR levels, as well as the energy of the control sequence itself. By varying the parameter λ we can give more or less weight to the term $\|Lx_k\|^2$ relative to $\|u_k\|^2$; larger values of λ give more relevance to $\|Lx_k\|^2$. The problem we are faced with is therefore to select the control sequence $\{u_k\}$ in order to minimize the cost function $\mathcal{J}$ subject to the state-space constraint (12). Moreover, the solution $\{u_k\}$ should be a function of the available measurements $\{y_k\}$ only. The solution to this stochastic control problem is well-known [12], [13] and is known as the LQG (Linear Quadratic Gaussian) solution. The solution is given by the following measurement feedback form. Start with $\hat{x}_k = 0$,

$$P_0 = Ex_0x_0^T \triangleq \Pi_0, \quad P_\infty^c = 0$$

and iterate for all $k \geq 0$:

$$\begin{aligned}
K_{c.k} &= (I + B^T P_k^c B)^{-1} B P_k^c A_k \\
K_{p.k} &= A_k P_k C^T (R + C P_k C)^{-1} \\
u_k &= -K_{c.k} \hat{x}_k \\
\hat{x}_{k+1} &= (A_k - K_{p.k} C)\hat{x}_k \ + \ K_{p.k} y_k \ + \ B u_k \\
P_{k+1} &= A_k P_k A_k^T + BQB^T - K_{p,k}(R + CP_kC)^T K_{p.k} \\
P_k^c &= A_k^T P_{k+1}^c A_k + \lambda L^T L - K_{c.k}^T (I + B^T P_{k+1}^c B) K_{c.k}
\end{aligned}$$

More specifically, the solution employs two Riccati recursions: one is for P_k and runs forward in time while the other is for P_k^c and runs backwards in time. The variable P_k is used to compute the gain matrix $K_{p,k}$, which in turn is used to estimate the state vector from the observations y_k. On the other hand, the variable P_k^c is used to compute the gain matrix $K_{c,k}$, which is used to determine the optimal control sequence u_k. It should be noted that all matrix variables involved in the above recursions are 2×2 and, hence, the computational complexity involved in evaluating the solution is not significant.

The structure of the solution can be simplified if we assume that the network is operating under stationary conditions. In this case, the congestion control function $c(k)$ can be assumed to have reached some steady-state value, say c, and, consequently, the coefficient matrix A_k becomes a constant matrix A,

$$\boxed{A = \begin{bmatrix} 1-\alpha & \alpha \\ 0 & 1-\mu c \end{bmatrix}} \tag{14}$$

Under these circumstances, we can simplify the construction of the control sequence $\{u_k\}$ by replacing the Riccati recursions for $\{P_k, P_k^c\}$ by Riccati *equations*, i.e., by replacing $\{P_k, P_k^c\}$ by their positive-definite steady-state values $\{P, P^c\}$, which are obtained by solving the equations [1]:

$$\begin{aligned}
P &= APA^T + BQB^T - K_p(R + CPC)^T K_p \quad (15) \\
P^c &= A^T P^c A + \lambda L^T L - K_c^T (I + B^T P^c B) K_c \quad (16)
\end{aligned}$$

where

$$\begin{aligned}
K_c &= (I + B^T P^c B)^{-1} B P^c A \quad (17) \\
K_p &= APC^T (R + CPC)^{-1} \quad (18)
\end{aligned}$$

[1] Unique positive semi-definite solutions $\{P, P^c\}$ are guaranteed to exist under mild stabilizability assumptions on the pairs $\{A, B\}$ and $\{A, L\}$, and detectability assumptions on the pairs $\{A, C\}$ and $\{A, B\}$. Moreover, the resulting closed-loop solution (19)–(20) will be stable - see [13].

Moreover,

$$\begin{aligned}
u_k &= -K_c \hat{x}_k \quad (19) \\
\hat{x}_{k+1} &= (A - K_p C)\hat{x}_k \ + \ K_p y_k \ + \ B u_k \quad (20)
\end{aligned}$$

In summary, we arrive at the following statement.

Quadratic power and rate control algorithm. Given positive scalars $\{\lambda, \alpha, \mu\}$ and 2×2 matrices $\{A, B, C, L, R, Q\}$, the following is an optimal joint power and rate-flow control strategy:

1. Compute u_k as in (19) by solving the 2×2 Riccati equations (15)-(16) for the positive definite solutions $\{P, P^c\}$. Partition Bu_k as

$$Bu_k = \begin{bmatrix} u_p(k) \\ u_f(k) \end{bmatrix}$$

2. Update the rate flow and the power at the relevant node as follows. Let $\kappa = (\log_2(10))/20$. Then

$$\begin{aligned}
\bar{\gamma}'(k) &= f_i(k)/\kappa \quad (21) \\
\bar{p}_i(k+1) &= \bar{p}_i(k) + \alpha_i[\bar{\gamma}_i'(k) - \bar{\gamma}_i(k)] \ + \ u_p(k) \quad (22) \\
f_i(k+1) &= f_i(k) + \mu[d(k) - cf_i(k)] \ + \ \kappa u_f(k) \quad (23)
\end{aligned}$$

where c is the steady-state value of the congestion control sequence $c(k)$, which is estimated as explained in Appendix A.

IV. SIMULATION RESULTS

We use the model proposed in [9] for the channel gain from the $i-th$ node to its master node. The G_{ii} has a lognormal distribution. Specifically, G_{ii} is modelled as

$$G_{ii} = S_0 d_{ii}^{-\beta} 10^{\alpha/10} \tag{24}$$

where S_0 is a function of the carrier frequency, β is the path loss exponent (PLE), and d_{ii} is the distance of the node from its intended master node. The value of β depends on the physical environment and changes between 2 and 6 (mostly 4). Moreover, α is a zero mean Gaussian random variable with variance σ^2, which usually ranges between 6 and 12. Based on these statistics, G_{ii} has the following Lognormal distribution:

$$f_{G_{ii}}(g) = \frac{1}{\sigma' g\sqrt{2\pi}} e^{-\frac{(\ln(g)-\mu)^2}{2\sigma'^2}}$$

where $\mu = \ln(S_0) - \beta \ln(d_{ii})$ and $\sigma' = \sigma \ln 10/10$. In other words, $\ln G_{ii}$ has a Gaussian distribution with mean $\log(S_0) - \beta \log(d_{ii})$ in dB scale. We shall neglect the effect of fast fading since the power update algorithm has a large time period. For the shadowing effect, we shall use the model proposed in [9], which assumes that the correlation between the $\ln G_{ii}$ at two different time instants separated by k samples is given by

$$R_{G_{ii}}(k) = \sigma_s^2 a^{|k|}, \quad a = 10^{-\frac{vT}{D}}$$

where σ_s^2 ranges between 3 and 10 dB, v is the velocity, and T is the time period for channel probing. Moreover, D is the distance at which the normalized correlation reaches $\frac{1}{10}$. We will assume the velocity of the nodes to be small so that we can approximate $a \approx 1$.

We consider 9 cells each with 8 time slots so that each cell has the capacity for 8 nodes. Queries through nodes arrive at the system with a poison distribution with arrival rate of λ and the service time or holding time for each user is an exponential distribution with average holding time of $\frac{1}{\mu}$. We consider a traffic load between 5.5 and 11 Erlang per cell, where the ratio of arrival rate to average service time $\frac{\lambda}{\mu \times 9}$ denotes the traffic in Erlang per cell. New nodes need to have 12 dB SIR to get admission into the system. The system tries to keep the SIR close to 10 dB. Whenever the SIR drops below 8 dB the system drops the node. To have uniform power distribution between nodes we change the head node randomly between the nodes. Maximum acceptable power that a node can transmit is the amount of power that causes SIR to be 20dB without any other user interference at a distance of 25 meters. Figure 2 shows the error variance between the actual and the desired SIR with and without the control law.

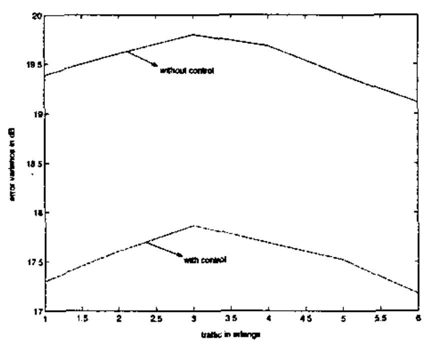

Fig. 2. Error variance between the actual and desired SIR with and without the proposed control law.
figure

V. CONCLUSIONS

In this paper, we proposed a distributed power control algorithm for wireless networks that caters to a desired rate of transmission between nodes. The algorithm was obtained by formulating a quadratic control problem and by solving the ensuing optimization problem by means of two 2×2 Riccati equations. A system level simulation environment shows the superior performance of the solution. More sophisticated rate and power control algorithms that cater to uncertainties in network dynamics are given in [6] and [7].

APPENDIX A : CONGESTION ESTIMATION

We describe here a methodology by which $c(k)$ can be estimated at every iteration. Over a time period T, let

$$\Delta = \text{Source Bandwidth} - \text{Estimated Network Bandwidth} \qquad (25)$$

where "Source Bandwidth" is the flow rate at the source and the "Estimated Network Bandwidth" is the end-to-end network bandwidth as estimated in [14]. We can then evaluate an end to end congestion measure $q(k)$ as follows:

$$\begin{aligned} &\text{If} \quad \Delta > \Delta_{th}, \quad \text{then } q(k) = \frac{\Delta - \Delta_{th}}{\Delta} \\ &\text{else} \quad q(k) = 0 \end{aligned}$$

for some positive threshold Δ_{th}, and then compute $c(k)$ as:

$$t(k) = 1 - \psi^{-(\gamma_{ssd} - \hat{\gamma}_i(k))^+} \qquad (26)$$

$$c(k) = \epsilon q(k) + (1 - \epsilon) t(k) \qquad (27)$$

for some $0 < \epsilon < 1$ and $\psi > 1$, and where γ_{ssd} is a desired steady-state SIR level and $\hat{\gamma}_i(k)$ is a one step ahead predicted SIR at time k.

VI. REFERENCES

[1] J. Zander, Distributed cochannel interference control in cellular radio systems, *IEEE Trans. Vehicular Technology*, vol. 41, pp. 305–311, Mar. 1992.

[2] S. A. Grandhi, R. Vijayan and D. J. Goodman, Distributed power control in cellular radio systems, *IEEE Trans. Communications*, vol. 42, no. 2, pp. 226–228, Feb. 1994.

[3] S. C. Chen, N. Bambos and G. J. Pottie, Admission control schemes for wireless communication networks with adjustable transmitted powers, *Proc. IEEE INFOCOM*, Toronto, Canada, 1994.

[4] M. Andersin, Z. Rosberg and J. Zander, Gradual removals in cellular PCS with constrained power control and noise, *Wireless Networks* , vol. 2, no. 1 pp. 27–43, 1996.

[5] K. Shoarinejad, J. L. Speyer, and G. J. Pottie, A distributed scheme for integrated predictive dynamic channel and power allocation in cellular radio networks. *Proc. GLOBECOM*, pp 3623–3627, San Antonio, Texas, 2001.

[6] A. Subramanian and A. H. Sayed, Joint rate and power control for wireless networks, submitted for publication.

[7] A. Subramanian and A. H. Sayed, A robust power and rate control method for state-delayed wireless networks, submitted for publication.

[8] N. Bambos, S. C. Chen, and G. J. Pottie, Channel access algorithms with active link protection for wireless communication networks with power control, *IEEE/ACM Transactions on Networking*, vol. 8, no. 5, pp 583–597, Oct 2000.

[9] M. Gudmundson, Correlation model for shadow fading in mobile radio systems, *Electronic Letters* vol. 27, no. 23, pp. 2145–2146, Nov 1991.

[10] W. B. Heinzelman, A. P. Chandrakasan, and H. Balakrishnan, An application specific protocol architechture for wireless microsensor networks, IEEE transactions on wireless communications, vol. 1, no. 4, pp 660–670, Oct. 2002.

[11] F. P. Kelly, Models for a self managed internet, *Philosophical transactions of the Royal society*, A358, pp. 2335–2348, 2000.

[12] K. J. Aström, *Introduction to Stochastic Control Theory*, Academic Press, NY, 1970.

[13] T. Kailath, A. H. Sayed, and B. Hassibi, *Linear Estimation*, Prentice Hall, NJ, 2000.

[14] R. Wang, et.al., Adaptive badwidth share estimation in TCP Westwood, *Proc. GLOBECOM*, Taipei, Nov. 2002.