MULTIUSER TWO-WAY RELAYING METHOD FOR BEAMFORMING SYSTEMS

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ABSTRACT

Relay transceiver processing for multiuser two-way communications is optimized based on zero-forcing (ZF) and minimum mean-square-error (MMSE) formulations under relay power constraints. Co-channel interference (CCI) from multiusers and self-interference (SI) from two-way communications are efficiently mitigated by the proposed relay processing with multiple antennas. The system performance is examined in relation to the system BER by computer simulation. Numerical results indicate that the beamforming method is more attractive than the spatial multiplexing method for the two-way communications.

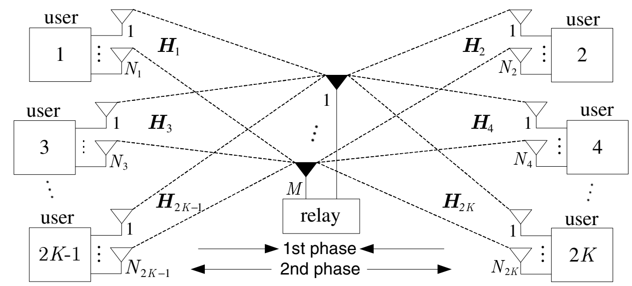

Fig. 1. Multiuser two-way relay system model illustrated with $2K$-user and one-relay nodes.

1. INTRODUCTION

Two-way relay methods (bidirectional communications) have been studied in [1–3] to improve spectral efficiency. In two-way, two users exchange data through two phases: multiple access phase and broadcast phase. In the multiple access phase (first-phase), the relay receives data from the two users, and in the broadcast phase (second-phase), the relay broadcasts the retransmit signals to the two users. In the second-phase, the signals transmitted through the previous phase interfere with information detection (thus causing self-interference (SI) [1]). Fortunately, the users can cancel the SI from their received signals since they know their own transmitted signals. To assist SI canceling and to improve system performance, bit-level coding techniques, such as XOR and superposition coding, were proposed for the decode-and-forward (DF) relay system in [1, 2]. On the other hand, linear preprocessing at the amplify-and-forward (AF) relay was proposed in [3].

The two-way DF relay communication has been extended to multiuser systems in [4, 5]. The co-channel interference (CCI) between multiple pairs is canceled by using either orthogonal sequences [4], i.e., code division multiple access (CDMA) techniques, or orthogonal space [5], i.e., zero-forcing (ZF) based space division multiple access (SDMA). Orthogonal frequencies can also be used to mitigate CCI, i.e., orthogonal frequency division multiple access (OFDMA). These multiple access systems can decompose multiuser pairs to independent multiple pairs without CCI and conventional SI cancelation techniques can then be applied for each user pair.

In this paper, we focus on AF relay systems and we propose a multiuser two-way relay process using multiple antennas as shown in Fig. 1. The proposed relay processing technique can mitigate both SI and CCI by steered beams through the multiple antennas at the relay. The transceiver processing matrix is optimized based on both ZF and minimum mean-square-error (MMSE) formulations. Our results can be considered as an extension of [3] to beamforming multiuser systems with SDMA. It is shown that beamforming at the users is more competitive than spatial multiplexing for two-way AF relay communications.

<u>Notation</u>. Throughout this paper, for any vector or matrix, the superscript 'T' and '$*$' denote transposition and complex conjugate transposition, respectively; $(\boldsymbol{W})^{-1}$, $(\boldsymbol{W})^{+}$, and $\mathrm{tr}(\boldsymbol{W})$ represent the inverse, pseudoinverse, and trace of matrix $\boldsymbol{W}$, respectively; 'E' stands for expectation of a random variable; for any vector $\boldsymbol{w}$ and matrix $\boldsymbol{W}$, the notations $\|\boldsymbol{w}\|$ and $\|\boldsymbol{W}\|_F$ denote 2-norm of $\boldsymbol{w}$ and Frobenius-norm of $\boldsymbol{W}$, respectively; $\mathbf{0}_w$ is w-dimensional zero column vector; and $\boldsymbol{I}_w$ is a w-dimensional identity matrix.

2. RELAY SYSTEM AND SIGNAL MODEL

There are $2K$ user nodes and one relay node in multiuser two-way communications as shown in Fig. 1. The $2K$ users result in K pairs of two users each performing two-way communication. Without loss of generality, it is assumed that the $(2k-1)$th and the $(2k)$th users communicate with each other ($k \in \{1, \cdots, K\}$ for user pairs) through two phases. In the first-phase, the $2K$ users transmit their data simultaneously to the relay node. In the second-phase, the relay retransmits (broadcasts) the received data to the destination users. Each kth user has N_k antennas and the relay has M antennas. The $M \times N_k$ MIMO channel matrix between the kth user and the relay node is represented by $\boldsymbol{H}_k \in \mathbb{C}^{M \times N_k}$ where the (m, n)th element is the channel gain between the nth antenna of the user and the mth antenna of the relay. The elements of $\boldsymbol{H}_k$ are i.i.d and zero-mean complex Gaussian random variables with unit variance. Here, we assume (i) the kth node can estimate its own MIMO channel

This work was supported in part by NSF grants ECS-0601266 and ECS-0725441 and by the Korea Research Foundation Grant funded by the Korean Government [KRF-2008-357-D00179].

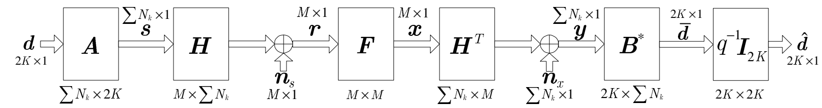

Fig. 2. Beamforming, relaying and equalization model in multiuser MIMO two-way channels.

matrix $\boldsymbol{H}_k$ by training through the second-phase whenever required, and (ii) the relay node can estimate the multiuser channel state information (CSI), i.e., $\{\boldsymbol{H}_k \text{ for all } k\}$, by training through the first-phase. It is also assumed that every channel remains static during the two phases. Let $d_k(t)$ denote the data symbol at time t for the kth user. The kth user performs transmit beamforming with unit-norm vector $\boldsymbol{a}_k \in \mathbb{C}^{N_k \times 1}$ and transmits the vector

$$\boldsymbol{s}_k(t) = \boldsymbol{a}_k d_k(t) \tag{1}$$

to the relay node, through the first-phase. The received signal at the relay is then

$$\boldsymbol{r}(t) = \sum_{k=1}^{2K} \boldsymbol{H}_k \boldsymbol{s}_k(t) + \boldsymbol{n}_s(t) \tag{2}$$

where $\boldsymbol{n}_s(t) \in \mathbb{C}^{M \times 1}$ is a zero-mean additive white Gaussian noise (AWGN) vector and $\mathrm{E}\, \boldsymbol{n}_s(t)\boldsymbol{n}_s^*(t) = \sigma_{n_s}^2 \boldsymbol{I}_M$. The relay multiplies $\boldsymbol{r}(t)$ by the relay processing matrix $\boldsymbol{F} \in \mathbb{C}^{M \times M}$, and forwards $\boldsymbol{x}(t) \in \mathbb{C}^{M \times 1}$ during the second-phase, where

$$\boldsymbol{x}(t) = \boldsymbol{F}\boldsymbol{r}(t). \tag{3}$$

Since the first and second-phase channels are symmetric and static, the received signal vector $\boldsymbol{y}_k(t) \in \mathbb{C}^{N_k \times 1}$, at the kth user, can be written as

$$\boldsymbol{y}_k(t) = \boldsymbol{H}_k^T \boldsymbol{x}(t) + \boldsymbol{n}_{x,k}(t) \tag{4}$$

where $\boldsymbol{n}_{x,k}(t) \in \mathbb{C}^{N_k \times 1}$ is an AWGN vector and $\mathrm{E}\, \boldsymbol{n}_{x,k}(t)\boldsymbol{n}_{x,k}^*(t) = \sigma_{n_x}^2 \boldsymbol{I}_{N_k}$. The kth user combines its own received signal (4) by using a receive beamforming vector $\boldsymbol{b}_k^* \in \mathbb{C}^{1 \times N_k}$ to get

$$\bar{d}_k(t) = \boldsymbol{b}_k^* \boldsymbol{y}_k(t) \tag{5}$$

where $\|\boldsymbol{b}_k\|^2 = 1$. Finally, the signal $\bar{d}_k(t)$ is equalized by a scalar gain q_k to yield the estimate $\hat{d}_k(t)$:

$$\hat{d}_k(t) = q_k^{-1} \bar{d}_k(t). \tag{6}$$

Here, note that $\hat{d}_k(t)$ is the estimate of $d_{k-1}(t)$ or $d_{k+1}(t)$ from a corresponding data exchange. For notational convenience, the time index t is henceforth omitted.

3. MULTIUSER TWO-WAY RELAY SYSTEM DESIGN

In this section, we consider the design of the relay processing matrix $\boldsymbol{F}$ under the assumption that the transmit- and receive-beamforming vectors $\{\boldsymbol{a}_k, \boldsymbol{b}_k\}$ are predetermined. Joint optimization of $\boldsymbol{F}$ and $\{\boldsymbol{a}_k, \boldsymbol{b}_k\}$ remains as future work.

3.1. Multiuser Signal Model

From (1) and (2), the received signal at the relay can be rewritten in matrix form as follows:

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{A}\boldsymbol{d} + \boldsymbol{n}_s \tag{7}$$

with the multiuser transmit signal vector $\boldsymbol{d} = [d_1 \cdots d_{2K}]^T \in \mathbb{C}^{2K \times 1}$ satisfying $\mathrm{E}\, \boldsymbol{d}\boldsymbol{d}^* = \sigma_d^2 \boldsymbol{I}_{2K}$; the multiuser channel matrix $\boldsymbol{H} = \left[\, \boldsymbol{H}_1 \cdots \boldsymbol{H}_{2K} \,\right] \in \mathbb{C}^{M \times \sum N_k}$; and the multiuser transmit-beamforming matrix:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{0}_{N_1} & \cdots & \boldsymbol{0}_{N_1} \\ \boldsymbol{0}_{N_2} & \boldsymbol{a}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{0}_{N_{2K-1}} \\ \boldsymbol{0}_{N_{2K}} & \cdots & \boldsymbol{0}_{N_{2K}} & \boldsymbol{a}_{2K} \end{bmatrix} \in \mathbb{C}^{\sum N_k \times 2K}. \tag{8}$$

Constructing the vector $\bar{\boldsymbol{d}} = [\bar{d}_1\ \bar{d}_2 \cdots \bar{d}_{2K}]^T \in \mathbb{C}^{2K \times 1}$ with $\bar{d}_k$ from (5), the multiuser received signal is represented as follows:

$$\bar{\boldsymbol{d}} = \boldsymbol{B}^* \boldsymbol{H}^T \boldsymbol{x} + \boldsymbol{B}^* \boldsymbol{n}_x \tag{9}$$

with the multiuser noise vector $\boldsymbol{n}_x = [\boldsymbol{n}_{x,1}^T \cdots \boldsymbol{n}_{x,2K}^T]^T \in \mathbb{C}^{\sum N_k \times 1}$ and the multiuser receive-beamforming matrix

$$\boldsymbol{B}^* = \begin{bmatrix} \boldsymbol{b}_1^* & \boldsymbol{0}_{N_2}^T & \cdots & \boldsymbol{0}_{N_{2K}}^T \\ \boldsymbol{0}_{N_1}^T & \boldsymbol{b}_2^* & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{0}_{N_{2K}}^T \\ \boldsymbol{0}_{N_1}^T & \cdots & \boldsymbol{0}_{N_{2K-1}}^T & \boldsymbol{b}_{2K}^* \end{bmatrix} \in \mathbb{C}^{2K \times \sum N_k}. \tag{10}$$

Using (3) and (7) in (9) yields the multiuser received signal model as

$$\bar{\boldsymbol{d}} = \boldsymbol{B}^* \boldsymbol{H}^T \boldsymbol{F} \boldsymbol{H} \boldsymbol{A} \boldsymbol{d} + \boldsymbol{B}^* \boldsymbol{H}^T \boldsymbol{F} \boldsymbol{n}_s + \boldsymbol{B}^* \boldsymbol{n}_x. \tag{11}$$

To reduce the channel effect from (11), equalization is performed with q as

$$\hat{\boldsymbol{d}} = q^{-1} \bar{\boldsymbol{d}} \tag{12}$$

where the vector $\hat{\boldsymbol{d}} = [\hat{d}_1\ \hat{d}_2 \cdots \hat{d}_{2K}]^T \in \mathbb{C}^{2K \times 1}$ with $\hat{d}_k$ in (6). Besides, due to the specific structure of the transmit- and receive-beamforming matrices in (8) and (10), it is difficult to jointly design the $\{\boldsymbol{A}, \boldsymbol{B}\}$ with the $\boldsymbol{F}$. To avoid this problem, we assume that the transmit- and receive-beamforming matrices $\{\boldsymbol{A}, \boldsymbol{B}\}$ are predetermined and satisfy certain power constrains. The transmit beamforming power of the source users is constrained by P_U as follows:

$$\mathrm{E}\, \|\boldsymbol{s}\|^2 = \sigma_d^2\, \mathrm{tr}\,(\boldsymbol{A}^* \boldsymbol{A}) = 2\sigma_d^2 K \triangleq P_U. \tag{13}$$

Similarly, for the receive beamforming,

$$\mathrm{tr}\,(\boldsymbol{B}^* \boldsymbol{B}) = 2K. \tag{14}$$

With the predetermined beamforming matrices $\{\boldsymbol{A}, \boldsymbol{B}\}$, the relay transceiver processing matrix $\boldsymbol{F}$ will be designed by using both ZF and MMSE criteria. Meanwhile, the equalization factor q is determined. Refer to Fig. 2, which shows the overall procedure for beamforming, relaying, and equalizing for multiuser two-way systems.

3.2. ZF-Based Design

Considering the ZF criterion to eliminate CCIs and SIs, and ignoring noise, the effective channel matrix of $\hat{\boldsymbol{d}}$ in (12) should be

$$\boxed{q^{-1}\boldsymbol{B}^*\boldsymbol{H}^T\boldsymbol{F}\boldsymbol{H}\boldsymbol{A}=\boldsymbol{P}} \tag{15}$$

where $\boldsymbol{P}$ is a $2K$-dimensional block diagonal matrix with kth block diagonal matrix $\left[\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right]$ exchanging data of the $(2k-1)$th and the $(2k)$th user. To avoid ambiguity, the number of relay antennas should be greater than or equal to the number of users, i.e., $M \geq 2K$, since the rank on the left hand side of (15) should be the same as that on the right hand side, i.e., $\min(M, 2K) = 2K$. Consequently, the minimum norm solution of (15) is obtained as

$$\boldsymbol{F}=q\bar{\boldsymbol{F}} \tag{16}$$

where

$$\bar{\boldsymbol{F}}=\left(\boldsymbol{B}^*\boldsymbol{H}^T\right)^+\boldsymbol{P}\left(\boldsymbol{H}\boldsymbol{A}\right)^+. \tag{17}$$

Next, the transmit power constraint is considered as:

$$\boxed{\mathrm{E}\left\|\boldsymbol{x}\right\|^2 \leq P_R} \tag{18}$$

From (3) and (7), the constraint in (18) is rewritten as

$$\mathrm{E}\left\|\boldsymbol{F}(\boldsymbol{H}\boldsymbol{A}\boldsymbol{d}+\boldsymbol{n}_s)\right\|^2 \leq P_R \tag{19}$$

Using the linearity and cyclic properties of the trace function, i.e., $\omega_1\,\mathrm{tr}(\boldsymbol{W}_1)+\omega_2\,\mathrm{tr}(\boldsymbol{W}_2)=\mathrm{tr}(\omega_1\boldsymbol{W}_1+\omega_2\boldsymbol{W}_2)$ and $\mathrm{tr}(\boldsymbol{W}_1\boldsymbol{W}_2)=\mathrm{tr}(\boldsymbol{W}_2\boldsymbol{W}_1)$, respectively, we can get q from (19) as

$$\boxed{q=\sqrt{\frac{P_R}{\mathrm{tr}\left(\bar{\boldsymbol{F}}\boldsymbol{\Omega}\bar{\boldsymbol{F}}^*\right)}}} \tag{20}$$

where $\boldsymbol{\Omega}=\sigma_d^2\boldsymbol{H}\boldsymbol{A}\boldsymbol{A}^*\boldsymbol{H}^*+\sigma_{n_s}^2\boldsymbol{I}_M$. Consequently, the ZF-based relaying matrix can be obtained from (16), (17), and (20) as

$$\boxed{\boldsymbol{F}_{ZF}=\sqrt{\frac{P_R}{\mathrm{tr}\left(\bar{\boldsymbol{F}}\boldsymbol{\Omega}\bar{\boldsymbol{F}}^*\right)}}\bar{\boldsymbol{F}}}$$

3.3. MMSE-Based Design

Noting the data exchange with $\boldsymbol{P}$, the optimization problem for minimizing the MSE under the relay power constraint is written as

$$\boxed{\arg\min_{\{\boldsymbol{F},q\}}\mathrm{E}\left\|\boldsymbol{P}\boldsymbol{d}-\hat{\boldsymbol{d}}\right\|^2 \text{ s.t. } \mathrm{E}\left\|\boldsymbol{F}\boldsymbol{r}\right\|^2 \leq P_R} \tag{21}$$

The optimal solution of $\boldsymbol{F}$ in (21) can be obtained from the Karush-Kuhn-Tucker (KKT) conditions [6]. By using the signal models (7) and (12), the minimization problem (21) with constraint can be transformed into (22) with a non-negative Lagrange multiplier λ, at the bottom of this page. The Lagrange cost J in (22) can be expanded as

$$\begin{aligned}J=&\,2\sigma_d^2K-q^{-1}\sigma_d^2\,\mathrm{tr}\left(\boldsymbol{P}\boldsymbol{B}^*\boldsymbol{H}^T\boldsymbol{F}\boldsymbol{H}\boldsymbol{A}\right)\\&-q^{-1}\sigma_d^2\,\mathrm{tr}\left(\boldsymbol{A}^*\boldsymbol{H}^*\boldsymbol{F}^*(\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{P}\right)\\&+q^{-2}\sigma_d^2\,\mathrm{tr}\left(\boldsymbol{A}^*\boldsymbol{H}^*\boldsymbol{F}^*(\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{B}^*\boldsymbol{H}^T\boldsymbol{F}\boldsymbol{H}\boldsymbol{A}\right)\\&+q^{-2}\sigma_{n_s}^2\,\mathrm{tr}\left(\boldsymbol{F}^*(\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{B}^*\boldsymbol{H}^T\boldsymbol{F}\right)+q^{-2}\sigma_{n_x}^2\,\mathrm{tr}\left(\boldsymbol{B}\boldsymbol{B}^*\right)\\&+\lambda\left(\sigma_d^2\,\mathrm{tr}\left(\boldsymbol{A}\boldsymbol{H}^*\boldsymbol{F}^*\boldsymbol{F}\boldsymbol{H}\boldsymbol{A}\right)+\sigma_{n_s}^2\,\mathrm{tr}\left(\boldsymbol{F}^*\boldsymbol{F}\right)-P_R\right)\end{aligned}$$

under the assumption that the data symbols, channel elements, and noises are independent of each other. Using the techniques of complex matrix derivatives [7], we get

$$\begin{aligned}\frac{\partial J}{\partial\boldsymbol{F}}=&-q^{-1}\sigma_d^2\boldsymbol{H}(\boldsymbol{B}^*)^T\boldsymbol{P}\boldsymbol{A}^T\boldsymbol{H}^T\\&+q^{-2}\sigma_d^2\boldsymbol{H}(\boldsymbol{B}\boldsymbol{B}^*)^T\boldsymbol{H}^*(\boldsymbol{H}\boldsymbol{A}(\boldsymbol{F}\boldsymbol{H}\boldsymbol{A})^*)^T\\&+q^{-2}\sigma_{n_s}^2\boldsymbol{H}(\boldsymbol{B}\boldsymbol{B}^*)^T\boldsymbol{H}^*(\boldsymbol{F}^*)^T\\&+\lambda\left(\sigma_d^2(\boldsymbol{H}\boldsymbol{A}(\boldsymbol{F}\boldsymbol{H}\boldsymbol{A})^*)^T+\sigma_{n_s}^2(\boldsymbol{F}^*)^T\right)\end{aligned} \tag{23}$$

Equating (23) to zero, we arrive at

$$\begin{aligned}\left((\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{B}^*\boldsymbol{H}^T+\lambda q^2\boldsymbol{I}_M\right)\boldsymbol{F}\left(\sigma_d^2\boldsymbol{H}\boldsymbol{A}\boldsymbol{A}^*\boldsymbol{H}^*+\sigma_{n_s}^2\boldsymbol{I}_M\right)\\=q\sigma_d^2(\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{P}\boldsymbol{A}^*\boldsymbol{H}^*.\end{aligned} \tag{24}$$

When $\lambda \neq 0$ and $\sigma_{n_s}^2 \neq 0$, $\boldsymbol{F}$ in (24) can be obtained as

$$\begin{aligned}\boldsymbol{F}=&\,q\sigma_d^2\left((\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{B}^*\boldsymbol{H}^T+\lambda q^2\boldsymbol{I}_M\right)^{-1}(\boldsymbol{H}^*)^T\\&\times\boldsymbol{B}\boldsymbol{P}\boldsymbol{A}^*\boldsymbol{H}^*\left(\sigma_d^2\boldsymbol{H}\boldsymbol{A}\boldsymbol{A}^*\boldsymbol{H}^*+\sigma_{n_s}^2\boldsymbol{I}_M\right)^{-1}.\end{aligned} \tag{25}$$

Similarly, by equating the derivative of J with respect to q and λ to zero, we obtain

$$\begin{aligned}q=&\,2\,\mathrm{tr}\left(\boldsymbol{A}^*\boldsymbol{H}^*\boldsymbol{F}^*(\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{B}^*\boldsymbol{H}^T\boldsymbol{F}\boldsymbol{H}\boldsymbol{A}\right.\\&\left.+\sigma_{n_s}^2\sigma_d^{-2}\boldsymbol{F}^*(\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{B}^*\boldsymbol{H}^T\boldsymbol{F}+\sigma_{n_x}^2\sigma_d^{-2}\boldsymbol{B}\boldsymbol{B}^*\right)\\&\times\overset{-1}{\mathrm{tr}}\left(\boldsymbol{P}\boldsymbol{B}^*\boldsymbol{H}^T\boldsymbol{F}\boldsymbol{H}\boldsymbol{A}+\boldsymbol{A}^*\boldsymbol{H}^*\boldsymbol{F}^*(\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{P}\right)\end{aligned} \tag{26}$$

and

$$\sigma_d^2\,\mathrm{tr}\left(\boldsymbol{A}^*\boldsymbol{H}^*\boldsymbol{F}^*\boldsymbol{F}\boldsymbol{H}\boldsymbol{A}\right)+\sigma_{n_s}^2\,\mathrm{tr}\left(\boldsymbol{F}^*\boldsymbol{F}\right)=P_R. \tag{27}$$

Directly evaluating $\boldsymbol{F}$ and q from (25)–(27) is formidable due to the Lagrange multiplier λ. To avoid this problem, we follow the optimization procedure in [8]. Let

$$\boldsymbol{F}=q\bar{\boldsymbol{F}}(\xi) \tag{28}$$

in (25), where

$$\bar{\boldsymbol{F}}(\xi)=\sigma_d^2\boldsymbol{\Psi}^{-1}(\xi)\boldsymbol{\Phi}\boldsymbol{\Omega}^{-1} \tag{29}$$

$$\boldsymbol{\Psi}(\xi)=(\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{B}^*\boldsymbol{H}^T+\xi\boldsymbol{I}_M \tag{30}$$

$$\arg\min_{\{\boldsymbol{F},\lambda,q\}}\underbrace{\mathrm{E}\left\|\boldsymbol{P}\boldsymbol{d}-q^{-1}\left(\boldsymbol{B}^*\boldsymbol{H}^T\boldsymbol{F}\boldsymbol{H}\boldsymbol{A}\boldsymbol{d}+\boldsymbol{B}^*\boldsymbol{H}^T\boldsymbol{F}\boldsymbol{n}_s+\boldsymbol{B}^*\boldsymbol{n}_x\right)\right\|^2+\lambda\left(\mathrm{E}\left\|\boldsymbol{F}\left(\boldsymbol{H}\boldsymbol{A}\boldsymbol{d}+\boldsymbol{n}_s\right)\right\|^2-P_R\right)}_{J} \tag{22}$$

and

$$\boldsymbol{\Phi} = (\boldsymbol{H}^*)^T \boldsymbol{B}\boldsymbol{P}\boldsymbol{A}^*\boldsymbol{H}^* \tag{31}$$

Here,

$$\xi = \lambda q^2. \tag{32}$$

Substituting (28) into (27), q is determined by

$$\boxed{q = \sqrt{\frac{P_R}{\mathrm{tr}\left(\bar{\boldsymbol{F}}(\xi)\boldsymbol{\Omega}\bar{\boldsymbol{F}}^*(\xi)\right)}}} \tag{33}$$

Continuing from (28) and (33), the problem in (22) can be rewritten as (34) at the bottom of this page. Here, we note that the second term multiplied by λ in (22) disappears due to (33) satisfying the power constraint (27). Substituting (29) into (34), we can write

$$\begin{aligned} J(\xi) &= 2\sigma_d^2 K - 2\sigma_d^4 \,\mathrm{tr}\left(\boldsymbol{\Psi}^{-1}(\xi)\boldsymbol{\Phi}\boldsymbol{\Omega}^{-1}\boldsymbol{\Phi}^*\right) \\ &\quad + \sigma_d^4\sigma_{n_s}^2 \,\mathrm{tr}\left(\boldsymbol{\Psi}^{-2}(\xi)\boldsymbol{\Phi}\boldsymbol{\Omega}^{-2}\boldsymbol{\Phi}^*(\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{B}^*\boldsymbol{H}^T\right) \\ &\quad + \sigma_d^6 \,\mathrm{tr}\Big(\boldsymbol{\Psi}^{-2}(\xi)\boldsymbol{\Phi}\boldsymbol{\Omega}^{-1}\boldsymbol{H}\boldsymbol{A}\boldsymbol{A}^*\boldsymbol{H}^*\boldsymbol{\Omega}^{-1}\boldsymbol{\Phi}^*(\boldsymbol{H}^*)^T \\ &\quad \times \boldsymbol{B}\boldsymbol{B}^*\boldsymbol{H}^T\Big) + \frac{\sigma_d^4\sigma_{n_x}^2 \,\mathrm{tr}(\boldsymbol{B}\boldsymbol{B}^*)}{P_R} \\ &\quad \times \mathrm{tr}\left(\boldsymbol{\Psi}^{-2}(\xi)\boldsymbol{\Phi}\boldsymbol{\Omega}^{-1}\boldsymbol{\Phi}^*\right). \end{aligned} \tag{35}$$

The derivative of (35) with respect to ξ is

$$\begin{aligned} \frac{\partial J(\xi)}{\partial \xi} &= 2\sigma_d^4 \,\mathrm{tr}\left(\boldsymbol{\Psi}^{-2}(\xi)\boldsymbol{\Phi}\boldsymbol{\Omega}^{-1}\boldsymbol{\Phi}^*\right) \\ &\quad - 2\sigma_d^4\sigma_{n_s}^2 \,\mathrm{tr}\left(\boldsymbol{\Psi}^{-3}(\xi)\boldsymbol{\Phi}\boldsymbol{\Omega}^{-2}\boldsymbol{\Phi}^*(\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{B}^*\boldsymbol{H}^T\right) \\ &\quad - 2\sigma_d^6 \,\mathrm{tr}\Big(\boldsymbol{\Psi}^{-3}(\xi)\boldsymbol{\Phi}\boldsymbol{\Omega}^{-1}\boldsymbol{H}\boldsymbol{A}\boldsymbol{A}^*\boldsymbol{H}^*\boldsymbol{\Omega}^{-1}\boldsymbol{\Phi}^* \\ &\quad \times (\boldsymbol{H}^*)^T\boldsymbol{B}\boldsymbol{B}^*\boldsymbol{H}^T\Big) - 2\sigma_d^4\sigma_{n_x}^2 P_R^{-1} \,\mathrm{tr}\left(\boldsymbol{B}\boldsymbol{B}^*\right) \\ &\quad \times \mathrm{tr}\left(\boldsymbol{\Psi}^{-3}(\xi)\boldsymbol{\Phi}\boldsymbol{\Omega}^{-1}\boldsymbol{\Phi}^*\right). \end{aligned}$$

and it can be expressed as

$$2\sigma_d^4\left(\xi - \sigma_{n_x}^2 P_R^{-1}\,\mathrm{tr}\left(\boldsymbol{B}\boldsymbol{B}^*\right)\right)\mathrm{tr}\left(\boldsymbol{\Psi}^{-3}(\xi)\boldsymbol{\Phi}\boldsymbol{\Omega}^{-1}\boldsymbol{\Phi}^*\right). \tag{36}$$

Since the cost in (34) is convex or strictly quasi-convex[1] with respect to ξ, equating the derivative (36) to zero yields the optimal ξ_o as

$$\xi_o = 2\sigma_{n_x}^2 P_R^{-1} K. \tag{37}$$

Consequently, it is not required to explicitly obtain the Lagrange multiplier λ by evaluating ξ in (37) and the closed formed MMSE solution of $\boldsymbol{F}$ can be obtained from (29)–(31) and (37) as follows:

$$\boxed{\boldsymbol{F}_{MMSE} = \sqrt{\frac{P_R}{\mathrm{tr}\left(\bar{\boldsymbol{F}}(\xi_o)\boldsymbol{\Omega}\bar{\boldsymbol{F}}^*(\xi_o)\right)}}\,\bar{\boldsymbol{F}}(\xi_o)} \tag{38}$$

Note that the results in (33) and (38) also fulfill the condition in (26) as can be proven easily from the fact that q is a real value.

[1] From the second derivative of $J(\xi)$ with respect to ξ, convexity or strictly quasi-convexity of $J(\xi)$ can be easily shown.

3.4. Equalization at Users

The equalization q is determined by (20) and (33) for ZF and MMSE systems, respectively. Note that q's in (20) and (33) are generated from the multiuser channels with a relay noise variance, the users cannot compute them, thereby causing they to be fed back from the relay.

3.5. Transmit and Receive-Beamforming Vectors

For the transmit and receive-beamforming vectors $\{\boldsymbol{a}_k, \boldsymbol{b}_k\}$, we employ two methods: eigen beamforming and equal gain beamforming. Since the two-way communication channel is assumed reciprocal, we naturally assume that the transmit and receive beamforming vectors are also reciprocal, i.e., $\boldsymbol{a}_k$ for $\boldsymbol{H}_k$ and $\boldsymbol{a}^T$ for $\boldsymbol{H}_k^T$, and then the following relation holds:

$$\boxed{\boldsymbol{b}_k^* = \boldsymbol{a}_k^T}$$

3.5.1. Eigen beamforming

If the users know their own channel matrices, the eigen beamformer, which is optimal in the sense of maximizing the received SNR on single-hop systems [9], can be employed as an obvious candidate for the beamforming vectors. The eigen beamforming vector of the kth user is defined as

$$\boldsymbol{a}_k = \boldsymbol{u}_k$$

where $\boldsymbol{u}_k \in \mathbb{C}^{N_k \times 1}$ is the right singular vector corresponding to the largest singular value σ_k of $\boldsymbol{H}_k$. Since a singular vector is a unit norm vector, the eigen beamforming vectors naturally satisfy the source power constraints in (13) and (14). Besides, the relay can easily generate the beamforming vectors from the multiuser CSI to compute the relay transceiver processing matrix.

3.5.2. Equal gain beamforming

Equal gain beamforming provides diversity gain and is obtained as

$$\boldsymbol{a}_k = \frac{1}{\sqrt{N_k}}\left[e^{j\theta_{k,1}} \quad e^{j\theta_{k,2}} \quad \ldots \quad e^{j\theta_{k,N_k}}\right]^T$$

where $\theta_{k,n}$ can be designed from the CSI [10]. The equal gain beamformer transmits the information symbol via multiple antennas by using the equally divided power N_k^{-1} for each transmit antenna. For comparison with the spatial multiplexing system later, we assume that $\theta_{k,n} = 0$, whereby the CSI is not required for the users similarly to the spatial multiplexing systems. Furthermore, the relay can easily generate the beamforming vectors without any additional information.

4. SIMULATION RESULTS

The performance of the proposed multiuser two-way relay system is evaluated in terms of system bit-error-rate (BER). The system BER is defined as the BER averaged over the multiusers. The modulated

$$\arg\min_{\xi} \underbrace{\mathrm{E}\left\|\boldsymbol{P}\boldsymbol{d} - \boldsymbol{B}^*\boldsymbol{H}^T\bar{\boldsymbol{F}}(\xi)\boldsymbol{H}\boldsymbol{A}\boldsymbol{d} - \boldsymbol{B}^*\boldsymbol{H}^T\bar{\boldsymbol{F}}(\xi)\boldsymbol{n}_s - P_R^{-\frac{1}{2}}\sqrt{\mathrm{tr}\left(\bar{\boldsymbol{F}}(\xi)\boldsymbol{\Omega}\bar{\boldsymbol{F}}^*(\xi)\right)}\boldsymbol{B}^*\boldsymbol{n}_x\right\|^2}_{J(\xi)} \tag{34}$$

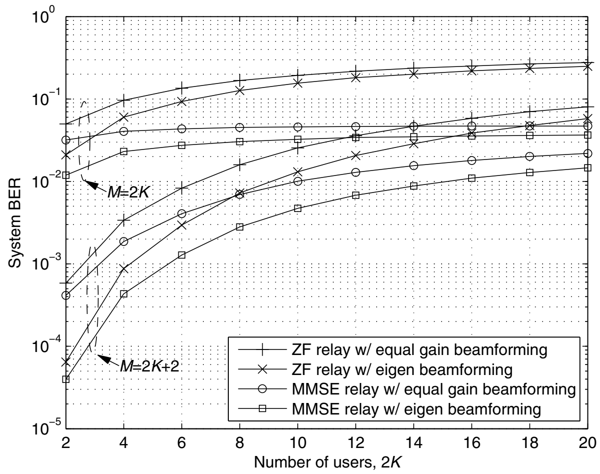

Fig. 3. The BER performance over the number of users when $P_U/\sigma^2_{n_s} = 18$ dB and $P_R/\sigma^2_{n_x} = 15$ dB.

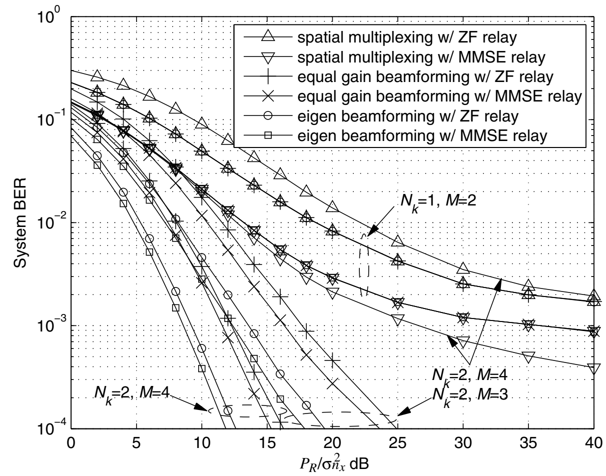

Fig. 4. The BER comparison with the proposed beamforming and the spatial multiplexing systems, when $2K = 2$ and $P_U/\sigma^2_{n_s} = 25$ dB.

symbols are grouped into frames consisting of 100 symbols. The results shown below are the average over 10^5 frames.

In Fig. 3, the system BER performance of the proposed system is evaluated over the number of users. As expected, the MMSE relay and the eigen beamforming system yield better BER performance than the ZF relay and the equal gain beamforming system, respectively. Besides, the BER performance decreases as the number of users increases due to the CCIs.

In Fig. 4, the BER performance of the proposed beamforming system and the spatial multiplexing system are compared. For comparing with the conventional two-user two-way multiplexing systems [3], we fix the total number of users at two. When $N_k = 1$, the proposed systems become identical to spatial multiplexing systems. Here, every user transmits BPSK symbols without spatial multiplexing as well as beamforming. On the other hand, when $N_k = 2$, considerable difference between the spatial multiplexing and the beamforming systems is observed. Here, since the spatial multiplexing with two transmit antennas can transmit two different data symbols simultaneously, the BPSK and QPSK modulations are used for the spatial multiplexing and beamforming systems, respectively, for a fair comparison. The proposed beamforming systems show better BER performance than the spatial multiplexing system. The poor performance of spatial multiplexing in this simulation comes from the fact that the spatially multiplexed signals interfere with each other through the relay processing.

5. CONCLUSION

In this paper, we proposed relay transceiver processing for multiuser two-way relay systems. The optimal transceiver processing matrix was designed under the assumption that every user employs spatial beamforming. The system BER of the proposed system was evaluated for two beamformers: eigen beamformers and equal gain beamformers. The BER performance was compared to that of the conventional two-way relay system with two users performing spatially multiplexed transmission. As a result, it can be verified that beamforming is a promising technique in two-way relay systems. Further work in this area will include investigation into the optimal power control for the multiuser two-way relay system considering fairness among users and sum rate.

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