A DISTRIBUTED MMSE RELAY STRATEGY FOR WIRELESS SENSOR NETWORKS

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ABSTRACT

In this paper we propose a multi-sensor relay strategy that achieves path-loss saving and improved power efficiency. In the proposed distributed scheme, the relay sensors do not need to share information about the received signals. An mean-square error design is pursued and the performance is shown to improve as the number of relay sensors (N) increases. Specificially, it is shown that the average power usage per sensor and the total average power drop as $O(\frac{1}{N^2})$ and $O(\frac{1}{N})$, respectively.

1. INTRODUCTION

A wireless sensor network is a distributed communication network containing geographically separated sensor nodes [1, 2]. A fundamental task in a wireless sensor network is to broadcast some measured data from an origin sensor to a destination sensor. Since the sensors are typically small, power limited and low cost, they are only able to broadcast low-power signals. This means that the propagation loss from the origin to the destination sensor can attenuate the signals beyond detection. One way to deal with this problem is to pass the transmitted signal through one or more relay sensors [1].

We may categorize relay schemes into three general groups: amplify-forward, compress-forward and decode-forward. In the amplify-forward scheme, the relay nodes amplify the received signal and rebroadcast the amplified signals toward the destination node [3],[4],[5]. In the compress-forward method, the relay nodes compress the received signals by exploiting the statistical dependencies between the signals at the nodes [6],[8],[9]. In the decode-forward scheme, the relay nodes first decode the received signals and then forward the decoded signals toward the destination node [10],[11],[12]. In this paper we propose an amplify-forward scheme.

Usually, in the conventional amplify-forward relay schemes, the relay nodes compensate for the phase of the incoming signal in order to result in coherent signal combination at the receiver. In such schemes, each node utilizes its maximum available power. In contrast, the scheme proposed in this paper allows the relay nodes to adjust their power. Specifically, we propose a two-hop multi-sensor MSE relay strategy that achieves path-loss saving, diversity gain and power efficiency. In the proposed distributed scheme, the relay sensors do not need to share information about the received signals and the relay strategy is based on minimizing the mean square error between the transmitted signal and the received signal at the destination relay node. It is shown that as the number of relay sensors (N) increases, the average power usage per sensor node and the total average power drop as $O(1/N^2)$ and $O(1/N)$, respectively.

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2. SYSTEM MODEL

Consider a sensor network with N relay sensor nodes between a source sensor and a destination sensor. The relay nodes are labelled from 1 to N – see Fig. 1. Let $\boldsymbol{h}_s$ denote the $N \times 1$ (column) channel vector between the source sensor and the relay nodes, and let $\boldsymbol{h}_t$ denote the $1 \times N$ (row) channel vector between the relay sensors and the destination sensor. A quasi-static fading condition is assumed for each channel gain so that the channel realizations stay fixed for the duration of a single frame. Let $h_{s,i}$ denote the ith element of $\boldsymbol{h}_s$, which stands for the channel coefficient from the source sensor to the ith relay node. Likewise, let $h_{t,i}$ denote the ith element of $\boldsymbol{h}_t$, which stands for the channel coefficient from the ith relay node to the destination sensor. A simple two-phase protocol is used to transmit data from the source sensor to the receiver. The first phase is the broadcasting phase, during which the source sensor broadcasts a signal s toward the relay sensors. The second phase is the relaying phase, during which the relay sensors transmit their signals to the destination sensor. We assume synchronous transmission and reception at relays nodes, so that the relay nodes relay their data at the same time instant.

Using the above formulation, the received vector at the relay sensors is given by

$$\boxed{\boldsymbol{r} = \boldsymbol{h}_s s + \boldsymbol{v}_s} \tag{1}$$

where

$$\boldsymbol{h}_s = [h_{s,1}, h_{s,2}, ..., h_{s,N}]^T \quad \text{(column)} \tag{2}$$

and $\boldsymbol{v}_s$ is $N \times 1$ zero-mean complex noise with covariance matrix $\sigma_{v_s}^2 \boldsymbol{I}$. At the second phase of the relaying protocol, the relay sensors rebroadcast a transformed signal vector that is given by

$$\boxed{\boldsymbol{x} = \boldsymbol{F}\boldsymbol{r}} \tag{3}$$

where $\boldsymbol{F}$ is an $N \times N$ linear transformation matrix to be designed. The uncorrupted received scalar signal at the destination sensor is

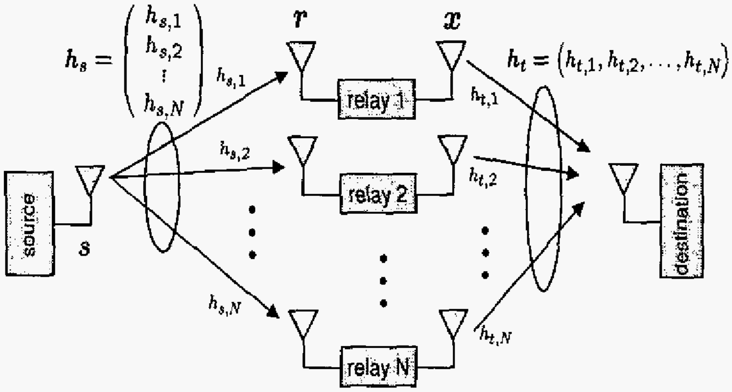

Fig. 1. The relay network scheme.

$\boldsymbol{h}_t\boldsymbol{x}$, where

$$\boldsymbol{h}_t = [h_{t,1}, h_{t,2}, \ldots, h_{t,N}] \quad \text{(row)}$$

We shall choose the relay matrix $\boldsymbol{F}$ such that the signal $\boldsymbol{h}_t\boldsymbol{x}$ is as close to s as possible in the least mean-squares sense. This step helps reduce the effect of the noise disturbance $\boldsymbol{v}_s$.

3. MMSE RELAY STRATEGY

Specifically, we shall select $\boldsymbol{F}$ to solve

$$\hat{\boldsymbol{F}} = \arg\min_{\boldsymbol{F}} J(\boldsymbol{F}) \tag{4}$$

where

$$\begin{aligned} J(\boldsymbol{F}) &= E|\eta s - \boldsymbol{h}_t\boldsymbol{x}|^2 \\ &= E|\eta s - \boldsymbol{h}_t\boldsymbol{F}\boldsymbol{h}_s s - \boldsymbol{h}_t\boldsymbol{F}\boldsymbol{v}_s|^2 \end{aligned} \tag{5}$$

for some positive scalar η chosen by the designer. For example, the choice $\eta = 1$ would minimize the mean-square error (MSE) between $\boldsymbol{h}_t\boldsymbol{x}$ and s itself. More generally, the choice

$$\eta = \sqrt{\mathrm{SNR}_t \frac{\sigma_{v_t}^2}{\sigma_s^2}}$$

where $\sigma_s^2 = E|s|^2$ helps enforce a target signal-to-noise-ratio, SNR_t, at the destination node. Expanding (5) we get

$$\begin{aligned} J &= \sigma_s^2 \boldsymbol{h}_t\boldsymbol{F}\boldsymbol{h}_s\boldsymbol{h}_s^*\boldsymbol{F}^*\boldsymbol{h}_t^* + \sigma_{v_s}^2\boldsymbol{h}_t\boldsymbol{F}\boldsymbol{F}^*\boldsymbol{h}_t^* \\ &\quad -\eta\sigma_s^2\boldsymbol{h}_t\boldsymbol{F}\boldsymbol{h}_s - \eta\sigma_s^2\boldsymbol{h}_s^*\boldsymbol{F}^*\boldsymbol{h}_t^* + \eta^2\sigma_s^2 \end{aligned} \tag{6}$$

Introduce the variable $\boldsymbol{z} = \boldsymbol{h}_t\boldsymbol{F}$. Then (6) becomes

$$J = \sigma_s^2\boldsymbol{z}\boldsymbol{h}_s\boldsymbol{h}_s^*\boldsymbol{z}^* + \sigma_{v_s}^2\boldsymbol{z}\boldsymbol{z}^* - \eta\sigma_s^2\boldsymbol{z}\boldsymbol{h}_s - \eta\sigma_s^2\boldsymbol{h}_s^*\boldsymbol{z}^* + \eta^2\sigma_s^2$$

Minimizing J over $\boldsymbol{z}$ gives

$$\hat{\boldsymbol{z}}^* = \eta\left(\boldsymbol{h}_s\boldsymbol{h}_s^* + \frac{\sigma_{v_s}^2}{\sigma_s^2}\boldsymbol{I}\right)^{-1}\boldsymbol{h}_s \tag{7}$$

and using the matrix inversion equality [14]:

$$\hat{\boldsymbol{z}}^* = \eta\left(\frac{\sigma_s^2}{\sigma_{v_s}^2 + \sigma_s^2\|\boldsymbol{h}_s\|^2}\right)\boldsymbol{h}_s$$

Recalling that $\boldsymbol{z} = \boldsymbol{h}_t\boldsymbol{F}$, we are reduced to choosing a relay matrix $\boldsymbol{F}$ such that

$$\boxed{\hat{\boldsymbol{F}}^*\boldsymbol{h}_t^* = \eta\left(\frac{\sigma_s^2}{\sigma_{v_s}^2 + \sigma_s^2\|\boldsymbol{h}_s\|^2}\right)\boldsymbol{h}_s} \tag{8}$$

Expression (8) provides N independent equalities for N^2 unknown elements in $\hat{\boldsymbol{F}}$. In other words, the relation provides several degrees of freedom that can be exploited advantageously as we now explain.

4. RELAY MATRIX SELECTION

Note first that a wireless sensor network is fundamentally a distributed communications network. As a result, we shall assume that each node only has access to local channel information. Specifically, every node i will only have access to the channel gains $h_{s,i}$ and $h_{t,i}$ that connect it to the source and the destination. This structure motivates us to seek a diagonal $\hat{F}$ that satisfies (8). Thus we shall select diagonal entries $\{\hat{f}_i\}$ such that

$$\hat{f}_i^* h_{t,i}^* = \eta\left(\frac{\sigma_s^2}{\sigma_{v_s}^2 + \sigma_s^2\|\boldsymbol{h}_s\|^2}\right)h_{s,i} \tag{9}$$

i.e.,

$$\boxed{\hat{f}_i = \eta\left(\frac{\sigma_s^2}{\sigma_s^2\|\boldsymbol{h}_s\|^2 + \sigma_{v_s}^2}\right)\cdot\frac{h_{s,i}^* h_{t,i}^*}{|h_{t,i}|^2}} \tag{10}$$

It is assumed that the source sensor node provides the relay nodes with the value of $\|\boldsymbol{h}_s\|^2$. Alternatively, $\|\boldsymbol{h}_s\|^2$ could be approximated by $N\sigma_{h_s}^2$.

It is worth noting that conventional relay schemes employ instead [3, 4, 5, 6, 7]:

$$\hat{f}_i = \frac{\sigma_r}{\sqrt{|h_{s,i}|^2\sigma_s^2 + \sigma_{v_s}^2}}\frac{h_{s,i}^*}{|h_{s,i}|}\frac{h_{t,i}^*}{|h_{t,i}|} \tag{11}$$

where σ_r^2 denotes the available power for each relay node. Since the relay factor in (10) is scaled down by the number of relay sensors (i.e., by $\approx N\sigma_{h_s}^2$), unlike (11), the average power consumption per node drops as the number of relay sensor increases.

4.1. Equalization

The signal received at the destination sensor is given by

$$t = \boldsymbol{h}_t\boldsymbol{x} + v_t \tag{12}$$

where v_t is zero mean noise with variance $\sigma_{v_t}^2$. Using any $\hat{\boldsymbol{F}}$ from (4), we would get

$$t \approx \eta s + v_t \tag{13}$$

with an SNR level that is equal to SNR_t. Thus we still need to equalize t in order to remove the effect of v_t and recover s itself. To do so, we forced to choose a scalar α so as to minimize

$$\alpha = \arg\min_{\alpha} J(\alpha) \tag{14}$$

where now

$$\begin{aligned} J(\alpha) &= E|s - \alpha t|^2 \\ &= E|s - \alpha\boldsymbol{h}_t\hat{\boldsymbol{F}}\boldsymbol{h}_s s - \alpha\boldsymbol{h}_t\hat{\boldsymbol{F}}\boldsymbol{v}_s - \alpha v_t|^2 \end{aligned} \tag{15}$$

The optimal α would provide the minimum-mean-square-error estimate of s given t. Expanding (15) we get

$$\begin{aligned} J(\alpha) &= |\alpha|^2\sigma_s^2 \boldsymbol{h}_t\hat{\boldsymbol{F}}\boldsymbol{h}_s\boldsymbol{h}_s^*\hat{\boldsymbol{F}}^*\boldsymbol{h}_t^* + |\alpha|^2\sigma_{v_s}^2\boldsymbol{h}_t\hat{\boldsymbol{F}}\hat{\boldsymbol{F}}^*\boldsymbol{h}_t^* \\ &\quad -\alpha\sigma_s^2\boldsymbol{h}_t\hat{\boldsymbol{F}}\boldsymbol{h}_s - \alpha^*\sigma_s^2\boldsymbol{h}_s^*\hat{\boldsymbol{F}}^*\boldsymbol{h}_t^* + |\alpha|^2\sigma_{v_t}^2 + \sigma_s^2 \end{aligned} \tag{16}$$

and the optimal α is given by

$$\hat{\alpha} = \frac{\sigma_s^2\boldsymbol{h}_s^*\hat{\boldsymbol{F}}^*\boldsymbol{h}_t^*}{\sigma_s^2|\boldsymbol{h}_t\hat{\boldsymbol{F}}\boldsymbol{h}_s|^2 + \sigma_{v_s}^2\|\boldsymbol{h}_t\hat{\boldsymbol{F}}\|^2 + \sigma_{v_t}^2} \tag{17}$$

Substituting the diagonal $\hat{\boldsymbol{F}}$ from (10) into (17) leads to

$$\boxed{\hat{\alpha} = \frac{\eta \text{SNR}\sigma_s^2\|\boldsymbol{h}_s\|^2(1+\text{SNR}\|\boldsymbol{h}_s\|^2)}{\eta^2\sigma_s^2\text{SNR}^2\|\boldsymbol{h}_s\|^4+\eta^2\sigma_{v_s}^2\text{SNR}^2\|\boldsymbol{h}_s\|^2+\sigma_{v_t}^2(1+\text{SNR}\|\boldsymbol{h}_s\|^2)}} \tag{18}$$

where $\text{SNR} = \sigma_s^2/\sigma_{v_s}^2$. To get a better insight into the result, let us assume that

$$\text{SNR}.\|\boldsymbol{h}_s\|^2 \gg 1$$

which is a reasonable assumption for large N. Then $1+\text{SNR}\|\boldsymbol{h}_s\|^2 \approx \text{SNR}\|\boldsymbol{h}_s\|^2$, and (18) becomes

$$\boxed{\hat{\alpha} \approx \frac{\eta\sigma_s^2}{\eta^2\sigma_s^2+\eta^2\frac{\sigma_{v_s}^2}{\|\boldsymbol{h}_s\|^2}+\sigma_{v_t}^2}} \tag{19}$$

If we further ignore $\frac{\sigma_{v_s}^2}{\|\boldsymbol{h}_s\|^2}$ in comparison with σ_s^2, we get

$$\boxed{\hat{\alpha} \approx \frac{\eta\sigma_s^2}{\eta^2\sigma_s^2+\sigma_{v_t}^2}} \quad \text{(for large } N\text{)} \tag{20}$$

This expression indicates that when the number of relay sensors increases, the destination node does not need the power of the broadcast channel, $\|\boldsymbol{h}_s\|^2$, in order to perform equalization.

4.2. Mean Square Error Behavior

We now examine how the minimum mean-square error $J(\hat{\alpha})$ varies as a function of the number of relay sensors. Substituting (8) and (17) into (16) gives

$$\begin{aligned} J_{\min} &= \frac{\eta^2|\hat{\alpha}|^2\sigma_s^2\text{SNR}^2\|\boldsymbol{h}_s\|^4}{(1+\text{SNR}\|\boldsymbol{h}_s\|^2)^2} + \frac{\eta^2|\hat{\alpha}|^2\sigma_s^2\text{SNR}\|\boldsymbol{h}_s\|^2}{(1+\text{SNR}\|\boldsymbol{h}_s\|^2)^2} \\ &\quad - \frac{\eta\hat{\alpha}\sigma_s^2\text{SNR}\|\boldsymbol{h}_s\|^2}{1+\text{SNR}\|\boldsymbol{h}_s\|^2} - \frac{\eta\hat{\alpha}^*\sigma_s^2\text{SNR}\|\boldsymbol{h}_s\|^2}{1+\text{SNR}\|\boldsymbol{h}_s\|^2} \\ &\quad +|\hat{\alpha}|^2\sigma_{v_t}^2 + \sigma_s^2 \end{aligned}$$

Using again $\text{SNR}.\|\boldsymbol{h}_s\|^2 \gg 1$, gives

$$\begin{aligned} J_{\min} &\approx \eta^2|\hat{\alpha}|^2\sigma_s^2 + \frac{\eta^2|\hat{\alpha}|^2\sigma_s^2}{1+\text{SNR}\|\boldsymbol{h}_s\|^2} \\ &\quad -\eta\hat{\alpha}\sigma_s^2 - \eta\hat{\alpha}^*\sigma_s^2 + |\hat{\alpha}|^2\sigma_{v_t}^2 + \sigma_s^2 \end{aligned}$$

Averaging this result over different channel realizations (and using App. A) leads to

$$EJ_{\min} \approx (\eta\hat{\alpha}-1)^2\sigma_s^2 + \hat{\alpha}^2\sigma_{v_t}^2 + \frac{\eta^2|\hat{\alpha}|^2\sigma_s^2}{\text{SNR}.N\sigma_{h_s}^2} \tag{21}$$

Thus increasing the number of relay nodes decreases the MSE and it converges to a non-zero steady-state value given by

$$\boxed{J_{\min} \approx \left(\frac{\sigma_s^2}{\eta^2\sigma_s^2+\sigma_{v_t}^2}\right)\sigma_{v_t}^2} \tag{22}$$

In contrast, for the conventional relaying strategy (11), the MSE will tend to zero as $N \to \infty$; this however occurs at the expense of increased total power consumption (it increases with N).

5. POWER CONSUMPTION

We can also examine the power consumption of the proposed method. The transmitted signal from the ith relay sensor is given by

$$x_i = r_i\hat{f}_i \tag{23}$$

where r_i and $\hat{f}_i$ are the received signal from (1) and the relay factor (10) at the ith sensor, respectively. Then the average consumed power at the ith relay sensor is

$$\begin{aligned} P_i &= E|x_i|^2 \\ &= E\left[\frac{\eta^2\sigma_s^4}{(\sigma_{v_s}^2+\sigma_s^2\|\boldsymbol{h}_s\|^2)^2}\frac{|h_{s,i}|^2}{|h_{t,i}|^2}|h_{s,i}s + v_{s,i}|^2\right] \end{aligned} \tag{24}$$

A simple protocol can be used in order to control the peak power usage of a sensor. A sensor will be allowed to participate in the relay process if $|h_{s,i}|^2/|h_{t,i}|^2 < \gamma$, where γ is a threshold that determines the maximum allowed peak power. Using this protocol we simply ignore relay sensors with weak relay channels. We could deploy a strategy to adjust the gain of $h_{s,i}$ in order to use the weak $h_{t,i}$ as well. However, this power allocation scheme requires inter relay cooperation. If the maximum allowed power consumption per sensor is some value σ_r^2, then (25) could be used to suggest a value for γ. It will follow from (25) that γ varies as $N^2\sigma_r^2$, so that the more sensors we have the larger γ should be.

Using this protocol, the power usage per sensor will be approximately upper bounded by

$$\boxed{P_i < \frac{\eta^2\gamma}{N^2\sigma_{h_s}^4}\left(\sigma_s^2\sigma_{h_s}^2+\sigma_{v_s}^2\right) \approx O\left(\frac{1}{N^2}\right)} \tag{25}$$

where, as argued in appendix A, we have employed the approximation

$$E\left[\frac{\sigma_{v_s}^4}{(\sigma_{v_s}^2+\sigma_s^2\|\boldsymbol{h}_s\|^2)^2}\right] \approx \frac{\sigma_{v_s}^4}{N^2\sigma_s^4\sigma_{h_s}^4}$$

It can be seen that the power usage per sensor drops as $O(\frac{1}{N^2})$. In this way, increasing the number of sensors not only improves the mean-square-error performance, but it also decreases power consumption per sensor. The total power used by the relay network is bounded by

$$\boxed{P_{\text{Total}} < \sum_{i=1}^{N}\frac{\eta^2\gamma}{N^2\sigma_{h_s}^4}\left(\sigma_s^2\sigma_{h_s}^2+\sigma_{v_s}^2\right) \approx O\left(\frac{1}{N}\right)}$$

Recall that relay networks are meant to combat the effect of path loss over long distances. The proposed distributed relay network achieves this property, along with improving the mean-square error performance of the network and its power efficiency.

6. POWER CONSTRAINED RELAY STRATEGIES

The relay strategy (8)-(10) does not constrain the power usage by the relay nodes. One could consider alternative formulations that would constrain the power usage either locally (i.e., by each individual node) or globally (i.e., by the entire relay network). For example, we could replace (5) by

$$\hat{\boldsymbol{F}} = \arg \min_{\substack{\boldsymbol{F}: E|f_i r_i|^2 = p_i \\ \boldsymbol{F} \text{ diagonal} \\ i=1,\ldots,N}} E|\eta s - \boldsymbol{h}_t \boldsymbol{x}|^2 \qquad (26)$$

or

$$\hat{\boldsymbol{F}} = \arg \min_{\substack{\boldsymbol{F}: \sum_{i=1}^{N} E|f_i r_i|^2 = P \\ \boldsymbol{F} \text{ diagonal}}} E|\eta s - \boldsymbol{h}_t \boldsymbol{x}|^2 \qquad (27)$$

where (26) limits the power usage of relay node i to p_i. It turns out that a solution to (26) is

$$\boxed{\hat{f}_i = \frac{\sqrt{p_i}}{\sqrt{\sigma_s^2|h_{s,i}|^2+\sigma_{v_s}^2}} \frac{h_{t,i}^*}{|h_{t,i}|} \frac{h_{s,i}^*}{|h_{s,i}|}} \qquad (28)$$

which reduces to the conventional relay scheme (11) when $p_i = \sigma_r^2 = \text{cte}$ for all i. On the other hand, an approximate solution to (27) is

$$\boxed{\hat{f}_i \approx \frac{\sqrt{P} h_{t,i}^* h_{s,i}^*}{(\sigma_s^2|h_{s,i}|^2+\sigma_{v_s}^2)\sqrt{\sum_{j=1}^{N} \frac{|h_{t,j}|^2|h_{s,j}|^2}{\sigma_s^2|h_{s,j}|^2+\sigma_{v_s}^2}}}} \qquad (29)$$

Details are omitted for brevity.

7. SIMULATION RESULTS

The performance of the scheme (8)-(10) is investigated for a relay network with one source and one destination. We assume that all relay sensors are essentially at the same distance from the source and destination sensors. Using this assumption, the channels from the source sensor to the relay sensors have the same second moment statistics as the channels from the relay sensors to the destination sensor, i.e., $E(\boldsymbol{h}_s \boldsymbol{h}_s^*) = E(\boldsymbol{h}_t^* \boldsymbol{h}_t)$. Moreover, we use zero-mean unit variance complex Gaussian channel models for $\boldsymbol{h}_s$ and $\boldsymbol{h}_t$, and the transmitted signal from the source sensor is assumed to be QPSK with unit power. Fig. 2 shows the BER performance of the scheme (10) when the destination sensor has less noise variance than the relay sensors, i.e., $10 \log \left(\sigma_{v_t}^2/\sigma_{v_s}^2\right) = -8\text{dB}$. Fig. 3 illustrates the MSE performance of the scheme (10). Fig.4 shows the power usage per sensor and the total power usage by the relay network versus the number of relay sensor nodes. Fig.5 compares the BER performance of the power constrained amplify-forward scheme (28) when power is allocated uniformly and when it is optimized and allocated globally (29).

8. APPENDIX A

Assuming iid Gaussian complex entries $h_{s,i}$, then $\|\boldsymbol{h}_s\|^2$ has a Chi-Square distribution with $2N$ degrees of freedom [13],[14].

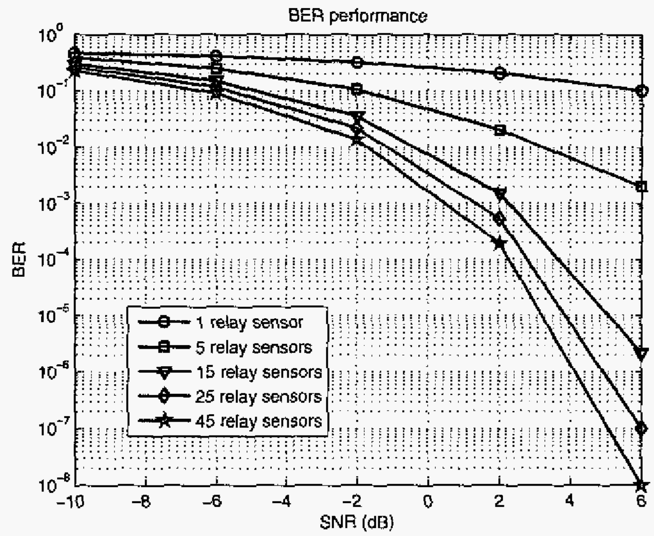

Fig. 2. The BER performance of the proposed scheme (10) when the relay sensors are assumed to have more noise power than the destination sensor. The relay sensors are placed such that they have the same distance from the source and destination sensors.

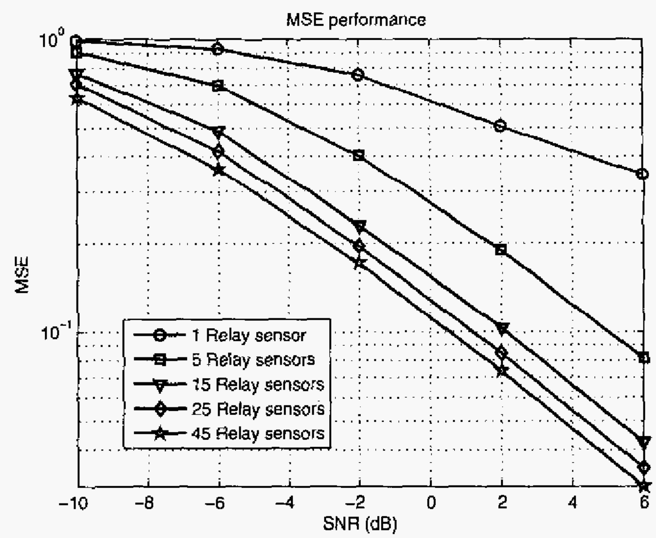

Fig. 3. MSE performance of the proposed scheme (10) when the relay sensors are assumed to have more noise power than the destination sensor. The relay sensors are placed such that they have the same distance from the source and destination sensors.

Using the probability density function of a Chi-Square random variable we have

$$E\left[\frac{1}{1+\text{SNR}\|\boldsymbol{h}_s\|^2}\right] = \int_0^{\infty} \left[\frac{1}{1+\text{SNR}.x}\right] p(x)dx$$

where

$$p(x) = \frac{1}{\sigma_{h_s}^{2N}\Gamma(N)} x^{N-1} e^{-x/\sigma_{h_s}^2}$$

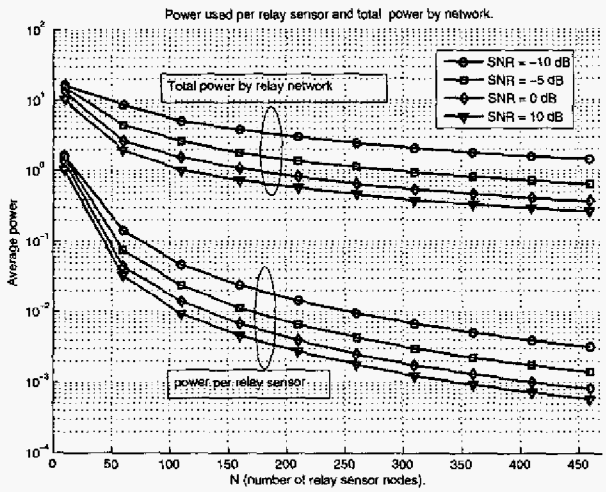

Fig. 4. The average power usage per relay sensor node and the total average power usage for all relay sensors vs. number of relay nodes.

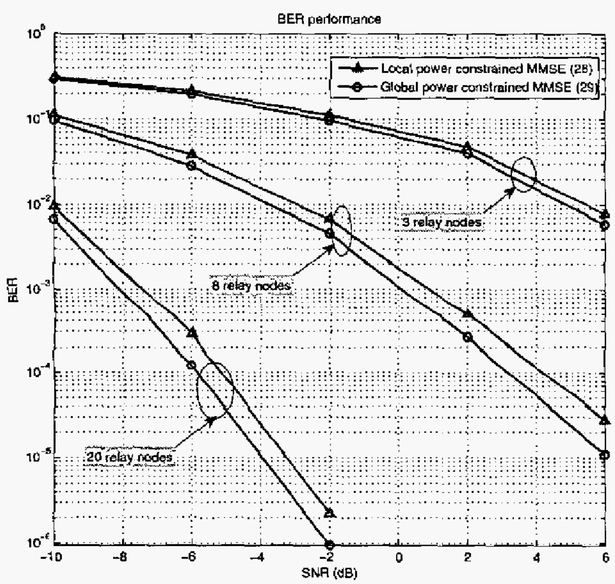

Fig. 5. The BER performance of the proposed scheme in (28) vs. the BER performance of a relay network that uses the optimized global power constraint (29).

and $\Gamma(.)$ denotes the Gamma function. Assume $1 + \text{SNR}.x \approx \text{SNR}.x$. Then[1]

$$E\left[\frac{1}{1+\text{SNR}\|h_s\|^2}\right] \approx \int_0^\infty \frac{1}{\text{SNR}.\sigma_{h_s}^{2N}\Gamma(N)} x^{N-2} e^{-x/\sigma_{h_s}^2} dx$$

$$= \frac{1}{\text{SNR}\sigma_{h_s}^{2N}\Gamma(N)} \int_0^\infty x^{N-2} e^{-x/\sigma_{h_s}^2} dx$$

$$= \frac{1}{\text{SNR}.\sigma_{h_s}^{2N}(N-1)!}(N-2)!\left(\sigma_{h_s}^2\right)^{N-1}$$

$$\approx \frac{1}{\text{SNR}(N-1)\sigma_{h_s}^2} \quad (30)$$

where we assumed $N - 1 \approx N$ (for large N). Then

$$\boxed{E\left[\frac{1}{1+\text{SNR}\|h_s\|^2}\right] \approx \frac{1}{\text{SNR}.N\sigma_{h_s}^2}} \quad (31)$$

In a similar manner,

$$\boxed{E\left[\frac{1}{1+\text{SNR}\|h_s\|^2}\right]^2 \approx \frac{1}{(\text{SNR}.N\sigma_{h_s}^2)^2}} \quad (32)$$

[1] In order to evaluate the expectation, we use the equality $\int_0^\infty t^n e^{-at} dt = \frac{\Gamma(n+1)}{a^{n+1}}$, where $\Gamma(n+1) = n!$ when n is an integer.

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