OFDM SYSTEMS WITH BOTH TRANSMITTER AND RECEIVER IQ IMBALANCES

Alireza Tarighat and Ali H. Sayed

Electrical Engineering Department
University of California, Los Angeles, CA 90095
Email: {tarighat,sayed}@ee.ucla.edu

ABSTRACT

The implementation of OFDM-based systems suffers from impairments such as in-phase and quadrature-phase (IQ) imbalances in the front-end analog processing. Such imbalances are caused by the analog processing of the radio frequency (RF) signal and can be present at both the transmitter and receiver. The resulting IQ distortion limits the achievable operating SNR at the receiver and the achievable data rates. In this paper, the effect of both the transmitter and receiver IQ imbalances in an OFDM system is studied and algorithms are developed to compensate for such distortions.

1. INTRODUCTION

A major source of impairment in high-frequency wireless system implementations is the imbalance between the In-phase (I) and Quadrature-phase (Q) branches; or equivalently, the real and imaginary parts of the complex signal [1]. This imbalance can be introduced at both the transmitter (during frequency up-conversion) and the receiver (during frequency down-conversion). Usually, the transmitted baseband signal is first up-converted to a radio frequency before transmission over the antenna. The signal received by the antenna is then down converted from the radio frequency (RF) to baseband before it is processed in the digital domain. Both the up-conversion and down-conversion are implemented in the analog domain by what is known as complex up-conversion and complex down-conversion (for more information see [1, 2]). A complex up/down converter basically multiplies the signal by the complex waveform $e^{\pm j2\pi f_{LO}t}$ and the spectrum of the signal is shifted by $\pm 2\pi f_{LO}$. To perform the complex frequency conversion, both the *sine* and *cosine* oscillating waveforms are required. The IQ imbalance results from any mismatch between the I and Q branches from the ideal case, i.e., from the exact 90^o phase difference and equal amplitudes. The performance of a receiver can be severely limited by such IQ imbalances at the transmitter and receiver.

The effect of *receiver* IQ imbalances on OFDM systems and the resulting performance degradation have been investigated in [3, 4]. Several compensation algorithms have been proposed in [5, 6, 7, 8, 9]. In the works [5, 9], compensation algorithms for OFDM receivers with IQ imbalances have been developed for both cases of SISO and MIMO communications. All these previous studies have focused on the problem of IQ imbalances at the receiver.

The contribution of this paper is to model the effect of IQ imbalances at *both* the transmitter and receiver and to develop algorithms that *jointly* compensate for these distortions.

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2. FORMULATION OF IQ IMBALANCES

Let $y(t)$ represent the received baseband complex signal before being distorted by the IQ imbalance at the receiver. The distorted signal in the time domain can be modeled as [3, 4]:

$$y_d(t) = \mu_r y(t) + \nu_r y^*(t) \tag{1}$$

where the distortion parameters, μ_r and ν_r, are related to the amplitude and phase imbalances between the I and Q branches in the RF/Analog demodulation process at the receiver. A simplified model for the distortion parameters can be written as [4, 5]:

$$\begin{aligned} \mu_r &= \cos(\theta_r/2) + j\alpha_r \sin(\theta_r/2) \\ \nu_r &= \alpha_r \cos(\theta_r/2) - j\sin(\theta_r/2) \end{aligned} \tag{2}$$

where θ_r and α_r are respectively the phase and amplitude imbalance between the I and Q branches at the receiver. The phase imbalance is any phase deviation from the ideal 90^o between the I and Q branches. The amplitude imbalance is defined as:

$$\alpha_r = \frac{a_I - a_Q}{a_I + a_Q}$$

where a_I and a_Q are the gain amplitudes on the I and Q branches. When stated in dB, the amplitude imbalance is $10\log(1+\alpha_r)$. The values of θ_r and α_r are not known at the receiver since they are caused by manufacturing inaccuracies in the analog components.

A similar approach can be used to model IQ imbalances at the transmitter. Let $s(t)$ represent the transmitted baseband complex signal before being distorted by IQ imbalances. Then the distorted baseband signal in the time domain will be given by

$$s_d(t) = \mu_t s(t) + \nu_t s^*(t) \tag{3}$$

where the distortion parameters μ_t and ν_t are defined as in (2). The design of OFDM receivers in the presence of both transmitter and receiver IQ imbalances is discussed next.

3. OFDM SIGNALS WITH IQ IMBALANCES

In OFDM systems, a block of data is transmitted as an OFDM symbol. Assuming a block size equal to N (where N is a power of 2), the transmitted block of data is denoted by

$$\mathrm{s} \triangleq \mathrm{col}\{\mathrm{s}(1), \mathrm{s}(2), \ldots, \mathrm{s}(N)\} \tag{4}$$

Each block is passed through the IDFT operation:

$$\bar{\mathrm{s}} = \mathbf{F}^* \mathrm{s} \tag{5}$$

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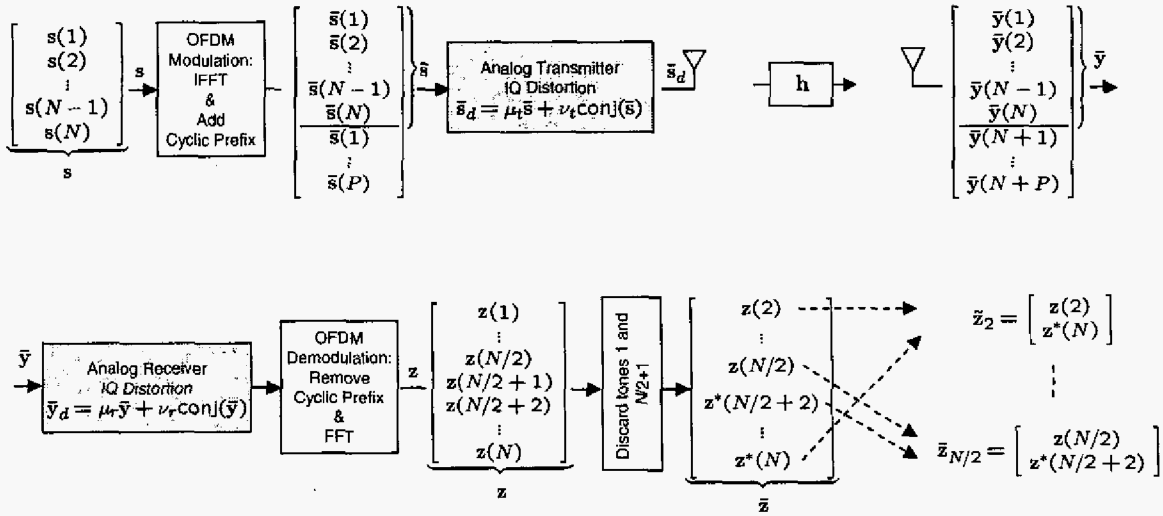


Fig. 1. An OFDM system with both transmit and receive IQ imbalances and the notation used in the derivations.

where $\mathbf{F}$ is the unitary discrete Fourier transform (DFT) matrix of size N defined by

$$[\mathbf{F}]_{ik} \triangleq \frac{1}{\sqrt{N}} \exp \frac{-j2\pi ik}{N}, \; j = \sqrt{-1}$$
$$i, k = \{0, 1, \ldots, N-1\}$$

A cyclic prefix of length P is added to each transformed block of data and then transmitted through the channel–see Figure 1. Due to IQ imbalances at the transmitter, as modeled by (3), the distorted transmitted vector is given by:

$$\boxed{\bar{\mathbf{s}}_d = \mu_t \bar{\mathbf{s}} + \nu_t \text{conj}(\bar{\mathbf{s}})} \tag{6}$$

where the notation conj(.) denotes a column vector (or matrix) whose entries are the complex conjugates of its argument. An FIR model with $L+1$ taps is assumed for the channel, i.e.,

$$\mathbf{h} = \text{col}\{h_0, h_1, \ldots, h_L\} \tag{7}$$

with $L \leq P$ in order to preserve the orthogonality between tones. At the receiver, the received samples corresponding to the transmitted block $\bar{\mathbf{s}}$ are collected into a vector, after discarding the received cyclic prefix samples. The received block of data *before* being distorted by receiver IQ imbalances is given by [5]:

$$\boxed{\bar{\mathbf{y}} = \mathbf{H}^c \bar{\mathbf{s}}_d + \bar{\mathbf{v}}} \tag{8}$$

where

$$\mathbf{H}^c = \begin{bmatrix} h_0 & h_1 & \cdots & h_L & & & & \\ & h_0 & h_1 & \cdots & h_L & & & \\ & & \ddots & & & \ddots & & \\ & & & h_0 & h_1 & \cdots & h_L \\ \vdots & & & & \ddots & & \vdots \\ h_2 & \cdots h_L & & & & h_0 & h_1 \\ h_1 & \cdots & h_L & & & & h_0 \end{bmatrix} \tag{9}$$

is an $N \times N$ circulant matrix and $\bar{\mathbf{v}}$ is additive white noise at the receiver. It is known that $\mathbf{H}^c$ can be diagonalized by the N-point DFT matrix as

$$\mathbf{H}^c = \mathbf{F}^* \mathbf{\Lambda} \mathbf{F} \tag{10}$$

where

$$\mathbf{\Lambda} = \text{diag}\{\lambda\} \tag{11}$$

and the vector λ is related to h via

$$\lambda = \sqrt{N}\, \mathbf{F}^* \begin{bmatrix} \mathbf{h} \\ \mathbf{0}_{(N-(L+1))\times 1} \end{bmatrix} \tag{12}$$

Substituting (5) and (6) into (8) leads to

$$\bar{\mathbf{y}} = \mathbf{H}^c \left[\mu_t \bar{\mathbf{s}} + \nu_t \text{conj}(\bar{\mathbf{s}})\right] + \bar{\mathbf{v}} \tag{13}$$

or

$$\bar{\mathbf{y}} = \mathbf{H}^c \left[\mu_t \mathbf{F}^* \mathbf{s} + \nu_t \text{conj}(\mathbf{F}^* \mathbf{s})\right] + \bar{\mathbf{v}} \tag{14}$$

The received block of data $\bar{\mathbf{y}}$ after being distorted by receiver IQ imbalances will be transformed into (using (1)):

$$\boxed{\bar{\mathbf{z}} = \mu_r \bar{\mathbf{y}} + \nu_r \text{conj}(\bar{\mathbf{y}})} \tag{15}$$

Now remember that the N-point DFT of the complex conjugate of a sequence is related to the DFT of the original sequence through a mirrored relation (assuming $1 \leq n \leq N$ and $1 \leq k \leq N$):

$$\begin{aligned} x(n) &\xrightarrow{\text{DFT}} X(k) \\ x^*(n) &\xrightarrow{\text{DFT}} X^*(N-k+2) \end{aligned} \tag{18}$$

For notational simplicity, we denote the operation which gives the DFT of the complex conjugate of a vector by the superscript #, i.e., for a vector X of size N we write

$$X = \begin{bmatrix} X(1) \\ X(2) \\ \vdots \\ X(N/2) \\ X(N/2+1) \\ X(N/2+2) \\ \vdots \\ X(N) \end{bmatrix} \Longrightarrow X^{\#} = \begin{bmatrix} X^*(1) \\ X^*(N) \\ \vdots \\ X^*(N/2+2) \\ X^*(N/2+1) \\ X^*(N/2) \\ \vdots \\ X^*(2) \end{bmatrix} \tag{19}$$

$$\tilde{\mathbf{\Lambda}}=\begin{bmatrix} \mu_r\mu_t\lambda(2)+\nu_r\nu_t^*\lambda^*(N) & & & & & \mu_r\nu_t\lambda(2)+\nu_r\mu_t^*\lambda^*(N) \\ & \ddots & & & \cdot^{\cdot^{\cdot}} & \\ & & \mu_r\mu_t\lambda(N/2)+\nu_r\nu_t^*\lambda^*(N/2+2) & \mu_r\nu_t\lambda(N/2)+\nu_r\mu_t^*\lambda^*(N/2+2) & & \\ & & \mu_r^*\nu_t^*\lambda^*(N/2+2)+\nu_r^*\mu_t\lambda(N/2) & \mu_r^*\mu_t^*\lambda^*(N/2+2)+\nu_r^*\nu_t\lambda(N/2) & & \\ & \cdot^{\cdot^{\cdot}} & & & \ddots & \\ \mu_r^*\nu_t^*\lambda^*(N)+\nu_r^*\mu_t\lambda(2) & & & & & \mu_r^*\mu_t^*\lambda^*(N)+\nu_r^*\nu_t\lambda(2) \end{bmatrix} \quad (16)$$

$$\mathbf{\Gamma}_k=\begin{bmatrix} \mu_r\mu_t\lambda(k)+\nu_r\nu_t^*\lambda^*(N-k+2) & \mu_r\nu_t\lambda(k)+\nu_r\mu_t^*\lambda^*(N-k+2) \\ \nu_r^*\mu_t\lambda(k)+\mu_r^*\nu_t^*\lambda^*(N-k+2) & \nu_r^*\nu_t\lambda(k)+\mu_r^*\mu_t^*\lambda^*(N-k+2) \end{bmatrix} \quad (17)$$

so that

$$X=\mathbf{F}x \Longrightarrow X^{\#}=\mathbf{F}\mathrm{conj}\,(x) \quad (20)$$

It can be verified similarly that

$$x=\mathbf{F}^*X \Longrightarrow x^{\#}=\mathbf{F}^*\mathrm{conj}\,(X) \quad (21)$$

Now substituting (10) into (14) gives

$$\begin{aligned} \bar{\mathbf{y}} &= \mathbf{F}^*\mathbf{\Lambda}\mathbf{F}\left[\mu_t\mathbf{F}^*\mathbf{s}+\nu_t\mathrm{conj}(\mathbf{F}^*\mathbf{s})\right]+\bar{\mathbf{v}} \\ &= \mu_t\mathbf{F}^*\mathbf{\Lambda}\mathbf{s}+\nu_t\mathbf{F}^*\mathbf{\Lambda}\mathbf{F}\mathrm{conj}(\mathbf{F}^*\mathbf{s})+\bar{\mathbf{v}} \\ &= \mu_t\mathbf{F}^*\mathbf{\Lambda}\mathbf{s}+\nu_t\mathbf{F}^*\mathbf{\Lambda}\mathbf{s}^{\#}+\bar{\mathbf{v}} \\ &= \mathbf{F}^*\mathrm{diag}\{\lambda\}\left(\mu_t\mathbf{s}+\nu_t\mathbf{s}^{\#}\right)+\bar{\mathbf{v}} \end{aligned} \quad (22)$$

where we used the fact that $\mathbf{F}\mathrm{conj}(\mathbf{F}^*\mathbf{s})=(\mathbf{F}\mathbf{F}^*\mathbf{s})^{\#}=\mathbf{s}^{\#}$. Moreover, using (13), we can write

$$\mathrm{conj}(\bar{\mathbf{y}})=\mathrm{conj}(\mathbf{H}^c)\left[\mu_t^*\mathrm{conj}(\bar{\mathbf{s}})+\nu_t^*(\bar{\mathbf{s}})\right]+\mathrm{conj}(\bar{\mathbf{v}}) \quad (23)$$

where $\mathrm{conj}(\mathbf{H}^c)$ is again a circulant matrix defined in terms of $\mathrm{conj}(\mathbf{h})$ as in (9) so that

$$\mathrm{conj}(\mathbf{H}^c)=\mathbf{F}^*\mathrm{diag}\left\{\lambda^{\#}\right\}\mathbf{F} \quad (24)$$

where

$$\sqrt{N}\mathbf{F}^*\begin{bmatrix}\mathrm{conj}(\mathbf{h}) \\ \mathbf{0}_{(N-(L+1))\times 1}\end{bmatrix}=\lambda^{\#} \quad (25)$$

Substituting the above into (23) results in

$$\begin{aligned} \mathrm{conj}(\bar{\mathbf{y}}) &= \mathbf{F}^*\mathrm{diag}\left\{\lambda^{\#}\right\}\mathbf{F}\left[\mu_t^*\mathrm{conj}(\bar{\mathbf{s}})+\nu_t^*\bar{\mathbf{s}}\right]+\mathrm{conj}(\bar{\mathbf{v}}) \\ &= \mathbf{F}^*\mathrm{diag}\left\{\lambda^{\#}\right\}\left(\mu_t^*\mathbf{s}^{\#}+\nu_t^*\mathbf{s}\right)+\mathrm{conj}(\bar{\mathbf{v}}) \end{aligned} \quad (26)$$

where we substituted $\mathbf{F}\bar{\mathbf{s}}=\mathbf{s}$ and $\mathbf{F}\mathrm{conj}(\bar{\mathbf{s}})=\mathbf{s}^{\#}$ using (5) and (20).

After applying the DFT operation to the received block of data $\bar{\mathbf{z}}$ given by (15) (as is done in an OFDM receiver) and using (22) and (26), we obtain

$$\begin{aligned} \mathbf{z} &\triangleq \mathbf{F}\bar{\mathbf{z}} \\ &= \mu_r\mathrm{diag}\{\lambda\}\left(\mu_t\mathbf{s}+\nu_t\mathbf{s}^{\#}\right)+ \\ &\quad \nu_r\mathrm{diag}\left\{\lambda^{\#}\right\}\left(\mu_t^*\mathbf{s}^{\#}+\nu_t^*\mathbf{s}\right)+\mu_r\mathbf{v}+\nu_r\mathbf{v}^{\#} \end{aligned} \quad (27)$$

or after rearranging terms

$$\boxed{\begin{aligned} \mathbf{z}= &\left(\mu_r\mu_t\mathrm{diag}\{\lambda\}+\nu_r\nu_t^*\mathrm{diag}\left\{\lambda^{\#}\right\}\right)\mathbf{s}+ \\ &\left(\mu_r\nu_t\mathrm{diag}\{\lambda\}+\nu_r\mu_t^*\mathrm{diag}\left\{\lambda^{\#}\right\}\right)\mathbf{s}^{\#}+ \\ &\left(\mu_r\mathbf{v}+\nu_r\mathbf{v}^{\#}\right) \end{aligned}} \quad (28)$$

This result gives the exact input-output relation in an OFDM system with both transmitter and receiver IQ imbalances as a function of the channel taps $\{\lambda\}$ and the distortion parameters μ_r, μ_t, ν_r, and ν_t. Note that (28) collapses to the input-output relation derived in [5] for $\mu_t=1$ and $\nu_t=0$, as a special case where ideal IQ branches were assumed at the transmitter.

As seen from (28), the vector $\mathbf{z}$ is no longer related only to the transmitted block $\mathbf{s}$ through a diagonal matrix, as is the case in an OFDM system with ideal I and Q branches. There is also a contribution from $\mathbf{s}^{\#}$. In the sequel, we show how the system of equations defined by (28) can be reduced to independent 2×2 systems of equations that can be used to estimate $\mathbf{s}$. Discarding the samples corresponding to tones 1 and $N/2+1$, i.e., $\mathbf{z}(1)$ and $\mathbf{z}(N/2+1)$, and defining two new vectors[1]:

$$\tilde{\mathbf{z}}=\begin{bmatrix}\mathbf{z}(2)\\ \vdots \\ \mathbf{z}(N/2) \\ \mathbf{z}^*(N/2+2) \\ \vdots \\ \mathbf{z}^*(N)\end{bmatrix},\quad \tilde{\mathbf{s}}=\begin{bmatrix}\mathbf{s}(2)\\ \vdots \\ \mathbf{s}(N/2) \\ \mathbf{s}^*(N/2+2) \\ \vdots \\ \mathbf{s}^*(N)\end{bmatrix} \quad (29)$$

then equation (28) can be rewritten as

$$\tilde{\mathbf{z}}=\tilde{\mathbf{\Lambda}}\tilde{\mathbf{s}}+\tilde{\mathbf{v}} \quad (30)$$

where $\tilde{\mathbf{\Lambda}}$ is given by (16) (with only diagonal and anti-diagonal non-zero elements) and $\tilde{\mathbf{v}}$ is related to $\mathbf{v}$ in a manner similar to (29). Note that the matrix $\tilde{\mathbf{\Lambda}}$ in the above equation is not diagonal, as is the case for $\mathbf{\Lambda}$ in (11) for an ideal system, although it collapses to a diagonal matrix by setting ν_r and ν_t equal to zero. Equation (30) can be reduced to 2×2 de-coupled sub-equations, for $k=\{2,\ldots,N/2\}$, each written as

$$\mathbf{z}_k=\mathbf{\Gamma}_k\mathbf{s}_k+\mathbf{v}_k \quad (31)$$

[1] The reason for discarding these two samples is that the transformation (19) returns the same indices only for $k=1$ and $k=N/2+1$. Note that in standardized OFDM systems these two tones do not carry information due to implementation issues.

where

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{z}(k) \\ \mathbf{z}^*(N-k+2) \end{bmatrix}, \; \mathbf{s}_k = \begin{bmatrix} \mathbf{s}(k) \\ \mathbf{s}^*(N-k+2) \end{bmatrix} \tag{32}$$

and the 2×2 matrix $\mathbf{\Gamma}_k$ is given by (17). The objective is to recover $\mathbf{s}_k$ from $\mathbf{z}_k$ in (31) for $k = \{2, \ldots, N/2\}$ or, equivalently, $\tilde{\mathbf{s}}$ from $\tilde{\mathbf{z}}$ in (30). Several algorithms, adaptive and otherwise, for estimating channel/disortion parameters and recovering the $\mathbf{s}_k$ for the special case with ideal transmitter ($\mu_t = 1$ and $\nu_t = 0$) were proposed in [5]. In the sequel, we extend some of these schemes to the more general case of imbalances at both the transmitter and receiver.

4. JOINT TX/RX COMPENSATION AT THE RECEIVER

4.1. Least-Squares Compensation

The least-squares estimate of $\mathbf{s}_k$, $k = \{2, \ldots, N/2\}$, denoted by $\hat{\mathbf{s}}_k$, is given by [10]:

$$\hat{\mathbf{s}}_k = (\mathbf{\Gamma}_k^* \mathbf{\Gamma}_k)^{-1} \mathbf{\Gamma}_k^* \mathbf{z}_k \tag{33}$$

Regularization could be used when it is desired to combat ill-conditioning in the data $\mathbf{\Gamma}_k$. In order to implement the solution (33), the channel information (λ) and the distortion parameters (μ_t,ν_t,μ_r,ν_r) are required. Training symbols are required to enable the receiver to estimate those values. Thus note that we may use equation (31) for channel estimation by rewriting it as:

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{s}(k) & 0 & \mathbf{s}^*(N-k+2) & 0 \\ 0 & \mathbf{s}(k) & 0 & \mathbf{s}^*(N-k+2) \end{bmatrix} \times \begin{bmatrix} \mu_r \mu_t \lambda(k) + \nu_r \nu_t^* \lambda^*(N-k+2) \\ \nu_r^* \mu_t \lambda(k) + \mu_r^* \nu_t^* \lambda^*(N-k+2) \\ \mu_r \nu_t \lambda(k) + \nu_r \mu_t^* \lambda^*(N-k+2) \\ \nu_r^* \nu_t \lambda(k) + \mu_r^* \mu_t^* \lambda^*(N-k+2) \end{bmatrix} + \mathbf{v}_k \tag{34}$$

Assuming n_{Tr} OFDM symbols are transmitted for training, then n_{Tr} realizations of the above equation can be collected to perform the least-squares estimation of the elements forming $\mathbf{\Gamma}_k$. The estimated $\mathbf{\Gamma}_k$ can then be substituted into (33) for data estimation.

4.2. Adaptive Equalization

As in [5], the adaptive estimation of $\mathbf{s}(k)$ and $\mathbf{s}^*(N-k+2)$ in (32) can be attained as follows:

$$\begin{aligned} \hat{\mathbf{s}}(k) &= \mathbf{w}_k \mathbf{z}_k \\ \hat{\mathbf{s}}^*(N-k+2) &= \mathbf{w}_{N-k+2} \mathbf{z}_k \end{aligned} \tag{35}$$

where $\mathbf{w}_k$ and $\mathbf{w}_{N-k+2}$ are 1×2 equalization vectors updated according to an adaptive algorithm (for instance LMS or some other adaptive form) for $k = \{2, \ldots, N/2\}$ [10]. To better illustrate the update equations, we introduce the time (or iteration) index i. As a result, let $\mathbf{w}_k^{(i)}$ and $\mathbf{w}_{N-k+2}^{(i)}$ represent the equalization vectors at time instant i. Furthermore, let $\mathbf{z}_k^{(i)}$ represent the vector $\mathbf{z}_k$ defined in (32) at iteration i. Now, the equalization coefficients for $k = \{2, \ldots, N/2\}$ are updated according to the LMS rules:

$$\mathbf{w}_k^{(i+1)} = \mathbf{w}_k^{(i)} + \mu_{\text{LMS}} \left(\mathbf{z}_k^{(i)}\right)^* e_k^{(i)} \tag{36}$$

$$\mathbf{w}_{N-k+2}^{(i+1)} = \mathbf{w}_{N-k+2}^{(i)} + \mu_{\text{LMS}} \left(\mathbf{z}_k^{(i)}\right)^* e_{N-k+2}^{(i)} \tag{37}$$

where $e_k^{(i)} = d_k^{(i)} - \mathbf{w}_k^{(i)} \mathbf{z}_k^{(i)}$ is the error signal generated at iteration i for the tone index k using a training symbol $d_k^{(i)}$, where the training symbol $d_k^{(i)}$ can be different for different tone indices k. A similar relation holds for $e_{N-k+2}^{(i)}$. Moreover, μ_{LMS} is the LMS step-size parameter.

In LMS, the coefficients in (36) and (37) are usually initiated with zero as their initial value. We use a different initialization in order to enhance the convergence rate of the algorithm. The compensation coefficients are initialized to values calculated as if the receiver assumes ideal I and Q branches. Referring to (34) and setting $\mu_r = \mu_t = 1$ and $\nu_r = \nu_t = 0$, the system of equations for channel estimation becomes

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{s}(k) & 0 \\ 0 & \mathbf{s}^*(N-k+2) \end{bmatrix} \begin{bmatrix} \lambda(k) \\ \lambda^*(N-k+2) \end{bmatrix} + \mathbf{v}_k \tag{38}$$

Due to the diagonal structure of the above system, it can be seen that the least-squares solutions for $\lambda(k)$ and $\lambda^*(N-k+2)$ in (38) are given by (see also the notation defined in (32)):

$$\hat{\lambda}(k) = \frac{\sum_{i=1}^{n_{Tr}} \mathbf{s}_i(k)^* \mathbf{z}_i(k)}{\sum_{i=1}^{n_{Tr}} \mathbf{s}_i(k)^* \mathbf{s}_i(k)} \tag{39}$$

where n_{Tr} is the number of training symbols. A new index i has been added to represent the symbol time instant. In other words, $\mathbf{s}_i(k)$ and $\mathbf{z}_i(k)$ are respectively the transmitted and received kth tones at time instant i. A similar expression holds for $\hat{\lambda}(N-k+2)$. Using the above estimation, which is derived assuming ideal IQ branches, the equalization vectors $\mathbf{w}_k$ and $\mathbf{w}_{N-k+2}$ in (35) are initialized to

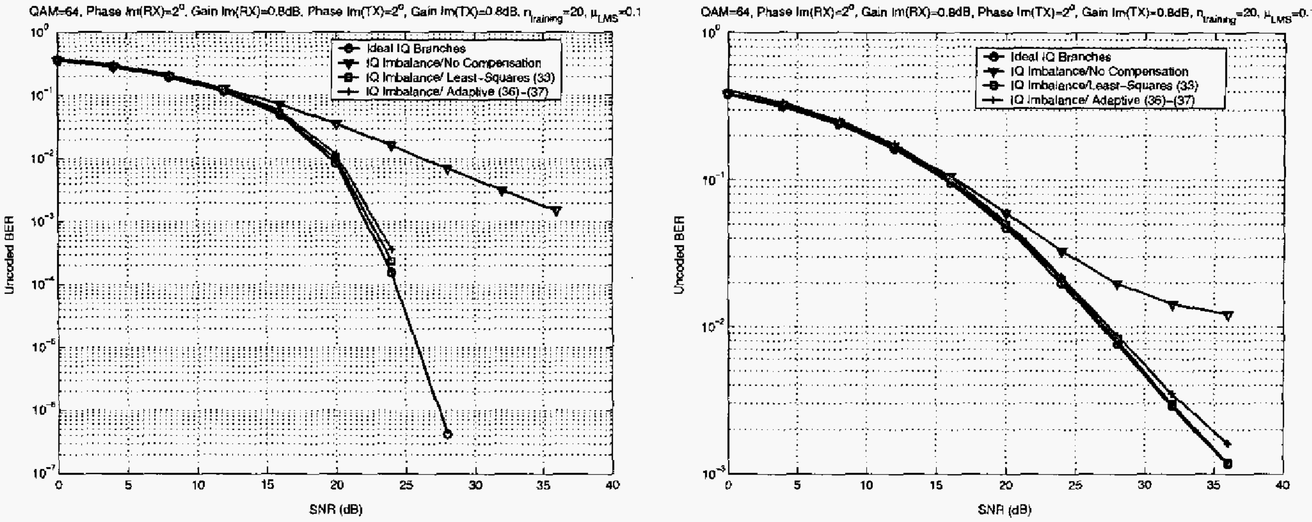
$$\mathbf{w}_k^{(0)} = \begin{bmatrix} \hat{\lambda}(k) & 0 \end{bmatrix} \tag{40}$$

$$\mathbf{w}_{N-k+2}^{(0)} = \begin{bmatrix} 0 & \hat{\lambda}^*(N-k+2) \end{bmatrix} \tag{41}$$

Using these initial values, equations (36) and (37) are then used to calculate the LMS solution. These calculated initial values are closer to the final value when compared to an all zero initialization, since the parameters ν_r and ν_t in (17) are typically much smaller than the parameters μ_r and μ_t.

5. SIMULATIONS

A typical OFDM system (similar to IEEE802.11a) is simulated to evaluate the performance of the proposed schemes in comparison to an ideal OFDM system with no transmit-receive IQ imbalance and a receiver with no compensation scheme. The parameters used in the simulation are: OFDM symbol length of $N = 64$, cyclic prefix of $P = 16$. Each simulation configuration is repeated for two different channel profiles: 1) additive white Gaussian noise (AWGN) channel with a single tap unity gain and 2) a multipath channel with $(L+1) = 4$ taps where the taps are chosen independently with complex Gaussian distribution. Every channel realization is independent of the previous one and the BER results depicted are from averaging the BER curves over independent channels. The uncoded BER versus SNR are shown in Figure 2 for 64QAM constellation. The values used for phase and amplitude imbalances for both the transmitter and the receiver are typical values achievable in practical integrated circuit implementations.



(a) AWGN flat channel (*non-fading*).

(b) 4-tap complex Gaussian channel (*fading*).

Fig. 2. BER vs. SNR simulated for the following configuration: 64QAM constellation, training length of 20 OFDM symbols in least-squares and LMS solutions, LMS step-size of $\mu_{LMS} = 0.1$, transmitter phase imbalance of $\theta_t = 2^o$, transmitter amplitude imbalance of α_t=0.8dB, receiver phase imbalance of $\theta_r = 2^o$, and receiver amplitude imbalance of α_t=0.8dB.

6. CONCLUSIONS

The paper studied the problem of transmitter and receiver IQ imbalances in OFDM systems. An input-output relation is derived as a function of both transmit and receiver distortion parameters. The input-output relation is then used to develop compensation algorithms for the IQ imbalances in the digital domain. An important property of the schemes proposed in this section is that they compensate for both transmit and receive imbalances jointly at the *receiver*. In other words, the transmitter is not necessarily required to achieve good IQ matching. This is an advantage for proprietary systems where the transmitter and the receiver are designed by the same manufacturer, since it can significantly relax the design specification on the transmitter. However, this may be a concern for standardized systems where the transmitters and receivers may be designed and manufactured by different manufacturers. In such systems, the transmitted signal's distortion has to be below a certain level specified by the standard, namely the error vector magnitude (EVM), so that receivers by other manufacturers can correctly decode it. In this case, the transmitter has to meet a certain level of IQ matching. This issue can be addressed by a pre-distortion scheme similar to [5] at the transmitter, such that the final transmitted signal is sufficiently close to an ideal transmitter.

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