Adaptive Angle of Arrival Estimation for Multiuser Wireless Location Systems[1]

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Abstract — **Reliable positioning of CDMA cellular users in a mobile environment relies on accurate knowledge of the time-of-arrival (TOA) and angle-of-arrival (AOA) of the first arriving ray. In this paper, an adaptive procedure is developed for AOA estimation under multipath and multiuser conditions. Simulations illustrate how the adaptive solution improves location accuracy.**

I. Introduction

Antenna array beamforming helps improve the signal-to-interference-noise ratio in CDMA cellular systems [1], [2]. An array of antennas can filter the signal in the direction of the desired user while suppressing interferences from other users. In addition, multiple antennas help increase the inherent capacity of the transmission link [4]. Generally, AOA estimation using subspace methods (e.g., MUSIC) [5] for CDMA systems is not practical when the number of users exceeds the number of antennas. Moreover, antenna array estimation in multiuser environments can be demanding depending on how the estimation is performed [5], [6]. Our proposed adaptive AOA estimation method uses knowledge of the users' CDMA spreading sequences and integrates this knowledge into the CDMA RAKE receiver in order to mitigate co-channel interference and fading.

II. Problem Formulation

Consider an uplink DS-CDMA system with an M-element antenna array at the base station. We assume N_u active users in the system so that the received signal at time n is an $M \times 1$ vector:

$$r(n) = \sum_{k=1}^{N_u} x_k(n) + v(n) \tag{1}$$

where $x_k(n)$ is the $M \times 1$ received signal vector from the kth user, and $v(n)$ is $M \times 1$ additive white Gaussian noise with covariance matrix equal to $\sigma_v^2 I_M$. Each user has a unique spreading sequence of length L_c so that (see Fig. 1):

$$x_k(n) = \sum_{i=0}^{N_s-1} \sum_{l=0}^{L-1} s_{k,i} c_k(n - iL_c - \tau_{k,l}) h_{k,l} \tag{2}$$

where N_s is the number of transmitted symbols, L is the number of multipaths, $s_{k,i}$ is the ith symbol transmitted by the kth user, $c_k(n)$ is the kth user unique spreading sequence, and $\tau_{k,l}$ is the channel delay from the kth user to the base station due to the lth multipath. Moreover, $h_{k,l}$ is an $M \times 1$ vector that contains

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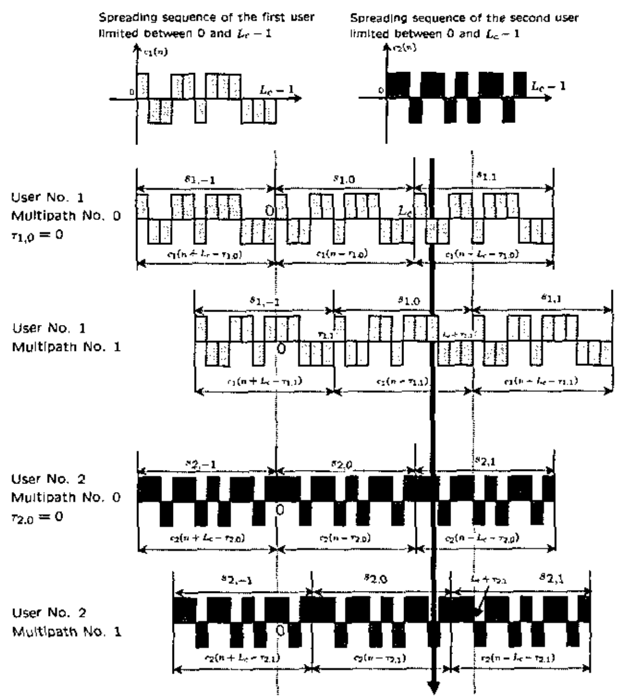

Figure 1: *Received signals from two multipaths and two users over a single antenna.*

the lth channel tap from user k to the base station antenna array and it has the form (Fig. 2):

$$h_{k,l} = \alpha_{k,l} a_{k,l} \tag{3}$$

where $\alpha_{k,l}$ is the lth tap channel attenuation from the kth user to the base station, and $a_{k,l}$ is the $M \times 1$ array response as a function of the AOA of the lth multipath of the kth user and it is given by

$$a_{k,l} = \left[1, e^{j2\pi \frac{d}{\lambda} \cos(\theta_{k,l})}, \ldots, e^{j2\pi \frac{(M-1)d}{\lambda} \cos(\theta_{k,l})}\right]^T$$

Here, $\theta_{k,l}$ is the AOA of the kth user measured over the lth path, d is the antenna spacing, and λ is the wavelength corresponding to the carrier frequency. Substituting (2) and (3) into (1) we find that the expression for the received signal from N_u users over M antennas with an L-multipath channel is

$$r(n) = \sum_{k=1}^{N_u}\sum_{i=0}^{N_s-1}\sum_{l=0}^{L-1} s_{k,i}c_k(n-iL_c-\tau_{k,l})\alpha_{k,l}\boldsymbol{a}_{k,l} + \boldsymbol{v}(n) \quad (4)$$

The expression for $\boldsymbol{r}(n)$ can be rewritten in matrix form as follows:

$$\boldsymbol{r}(n) = \sum_{i=0}^{N_s-1} \boldsymbol{A}_\theta \boldsymbol{H}\boldsymbol{C}(n,i)\boldsymbol{s}_i + \boldsymbol{v}(n) \quad (5)$$

where $\boldsymbol{A}_\theta$ is an $M \times N_u L$ block diagonal matrix defined as

$$\boldsymbol{A}_\theta = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 & \cdots & \boldsymbol{A}_{N_u} \end{bmatrix} \ \text{(all users)}$$

and each $\boldsymbol{A}_k$ is an $M \times L$ matrix given by

$$\boldsymbol{A}_k = \begin{bmatrix} \boldsymbol{a}_{k,0} & \boldsymbol{a}_{k,1} & \cdots & \boldsymbol{a}_{k,L-1} \end{bmatrix} \ \text{(user } k\text{)}$$

Moreover, $\boldsymbol{H}$ is the $N_uL \times N_uL$ diagonal matrix

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{P}_1 & & & \\ & \boldsymbol{P}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{P}_{N_u} \end{bmatrix} \ \text{(all users)}$$

where each $\boldsymbol{P}_k$ is an $L \times L$ diagonal matrix given by

$$\boldsymbol{P}_k = \begin{bmatrix} \alpha_{k,0} & & \\ & \ddots & \\ & & \alpha_{k,L-1} \end{bmatrix} \ \text{(user } k\text{)}$$

Also, $\boldsymbol{C}(n,i)$ is an $N_uL \times N_uL$ block diagonal matrix given by

$$\boldsymbol{C}(n,i) = \begin{bmatrix} \boldsymbol{J}_1(n,i) & & & \\ & \boldsymbol{J}_2(n,i) & & \\ & & \ddots & \\ & & & \boldsymbol{J}_{N_u}(n,i) \end{bmatrix}$$

where each $\boldsymbol{J}_k(n,i)$ is the $L \times L$ diagonal matrix

$$\boldsymbol{J}_k(n,i) = \begin{bmatrix} c_k(n-iL_c-\tau_{k,0}) & & \\ & \ddots & \\ & & c_k(n-iL_c-\tau_{k,L-1}) \end{bmatrix}$$

Finally, $\boldsymbol{s}_i$ is the $N_uL \times 1$ vector that contains the transmitted symbols from all users to the base station:

$$\boldsymbol{s}_i = \begin{bmatrix} \boldsymbol{s}_{1,i}^T & \cdots & \boldsymbol{s}_{N_u,i}^T \end{bmatrix}^T$$

where

$$\boldsymbol{s}_{k,i} = [\underbrace{s_{k,i} \quad s_{k,i} \quad \cdots}_{L}]^T$$

III. Array Response Estimation Using Conventional Matched Filtering

The problem we are interested in is estimating $\boldsymbol{A}_\theta$ from the received signal $\boldsymbol{r}(n)$ in (5). In conventional matched filtering, we would first correlate the received signal $\boldsymbol{r}(n)$ with the spreading sequence of the desired user. Let $n_{0,i}$ and $n_{1,i}$ denote the

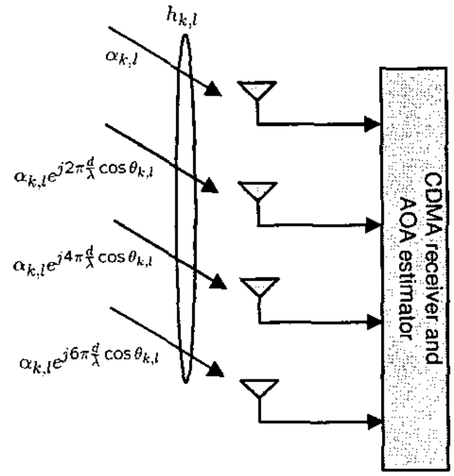

Figure 2: *Transmissions from multiple users to multiple antenna over a multipath channel.*

beginning and the end of the ith symbol correlation interval for the kth user and lth multipath, respectively, i.e.,

$$\begin{aligned} n_{0,i} &\triangleq iL_c + \tau_{k,l} \\ n_{1,i} &\triangleq (i+1)L_c + \tau_{k,l} - 1 \end{aligned} \quad (6)$$

Then the correlation gives

$$\begin{aligned} \boldsymbol{y}_{k,l,i} &= \sum_{n=n_{0,i}}^{n_{1,i}} c_k^*(n-iL_c-\tau_{k,l})\boldsymbol{r}(n) \\ &= \sum_{n=n_{0,i}}^{n_{1,i}}\sum_{j=0}^{N_s-1} c_k^*(n-iL_c-\tau_{k,l}) \\ &\times (\boldsymbol{A}_\theta\boldsymbol{H}\boldsymbol{C}(n,j)\boldsymbol{s}_j + \boldsymbol{v}(n)) \end{aligned} \quad (7)$$

where $\boldsymbol{y}_{k,l,i}$ is the output of the matched filter. Equation (7) can be written as

$$\begin{aligned} \boldsymbol{y}_{k,l,i} &= \sum_{n=n_{0,i}}^{n_{1,i}} c_k^*(n-iL_c-\tau_{k,l})\boldsymbol{A}_\theta\boldsymbol{H}\boldsymbol{C}(n,i)\boldsymbol{s}_i \\ &+ \sum_{n=n_{0,i}}^{n_{1,i}}\sum_{j=0,j\neq i}^{N_s-1} c_k^*(n-iL_c-\tau_{k,l})\boldsymbol{A}_\theta\boldsymbol{H}\boldsymbol{C}(n,j)\boldsymbol{s}_j \\ &+ \sum_{n=n_{0,i}}^{n_{1,i}} c_k^*(n-iL_c-\tau_{k,l})\boldsymbol{v}(n) \end{aligned} \quad (8)$$

When L_c is large enough, we have

$$\sum_{n=n_{0,i}}^{n_{1,i}} c_k^*(n-iL_c-\tau_{k,l})c_{k'}(n-jL_c-\tau_{k',l'}) \approx 0 \quad (9)$$

when $l \neq l'$. Using (9), we can simplify the derivation by treating inter-symbol interference as noise. Thus sum of the second and third terms in (8) define the post correlated noise vector

$v_{k,l,i}$:

$$\begin{aligned} v_{k,l,i} &= \sum_{n=n_{0,i}}^{n_{1,i}} \sum_{j=0, j\neq i}^{N_s-1} c_k^*(n - iL_c - \tau_{k,l}) A_\theta H C(n,j) s_j \\ &+ \sum_{n=n_{0,i}}^{n_{1,i}} c_k^*(n - iL_c - \tau_{k,l}) v(n) \end{aligned}$$

so that

$$\begin{aligned} y_{k,l,i} &= \sum_{n=n_{0,i}}^{n_{1,i}} c_k^*(n - iL_c - \tau_{k,l}) A_\theta H C(n,i) s_i + v_{k,l,i} \\ &= A_\theta H \sum_{n=n_{0,i}}^{n_{1,i}} c_k^*(n - iL_c - \tau_{k,l}) C(n,i) s_i + v_{k,l,i} \end{aligned}$$

The above expression can be simplified into matrix form by introducing the following $N_u L \times N_u L$ matrix:

$$G_{k,l} = \begin{bmatrix} T(1) & & \\ & \ddots & \\ & & T(N_u) \end{bmatrix}$$

where

$$\begin{aligned} g_{k,l,k',l'} &= \sum_{n=n_{0,i}}^{n_{1,i}} c_k^*(n - iL_c - \tau_{k,l}) c_{k'}(n - iL_c - \tau_{k',l'}) \\ &= \sum_{n=\tau_{k,l}}^{L_c+\tau_{k,l}-1} c_k^*(n - \tau_{k,l}) c_{k'}(n - \tau_{k',l'}) \\ &= \sum_{n=0}^{L_c-1} c_k^*(n) c_{k'}(n + \tau_{k,l} - \tau_{k',l'}) \\ T(k') &= \begin{bmatrix} g_{k,l,k',0} & & \\ & \ddots & \\ & & g_{k,l,k',L-1} \end{bmatrix} \quad (L \times L) \end{aligned}$$

Then

$$y_{k,l,i} = A_\theta H G_{k,l} s_i + v_{k,l,i} \tag{10}$$

Let $z_{k,l,i} = H G_{k,l} s_i$, where $z_{k,l,i}$ is $N_u L \times 1$, so that

$$y_{k,l,i} = A_\theta z_{k,l,i} + v_{k,l,i} \tag{11}$$

Collecting all post-correlated vectors $y_{k,l,i}$ into a matrix Y_i, and collecting the $\{z_{k,l,i}\}$ into a matrix Z_i, we get

$$\begin{aligned} [y_{1,0,i} \cdots y_{N_u,L-1,i}] &= A_\theta [z_{1,0,i} \cdots z_{N_u,L-1,i}] \\ &+ [v_{1,0,i} \cdots v_{N_u,L-1,i}] \end{aligned}$$

That is,

$$Y_i = A_\theta Z_i + V_i$$

This equation can be vectorized using Kronecker product notation:

$$\underbrace{\mathrm{vec}(Y_i)}_{\bar{y}_i} = \underbrace{(Z_i^T \otimes I_{N_u L})}_{\bar{Z}_i} \underbrace{\mathrm{vec}(A_\theta)}_{a_\theta} + \underbrace{\mathrm{vec}(V_i)}_{\bar{v}_i}$$

In other words,

$$\boxed{\bar{y}_i = \bar{Z}_i a_\theta + \bar{v}_i} \tag{12}$$

where $\mathrm{vec}(X)$ denotes the vector obtained by stacking the columns of X into a vector. Moreover $\otimes$ denotes the Kronecker product and $I_{N_u L}$ is the $N_u L \times N_u L$ identity matrix.

IV. Least-Squares Adaptive Estimation

Least-squares estimation can be used to estimate a_θ, which contains the AOA and array response information for all users and all multipaths. Collecting N_s realizations of the correlation results from N_s symbols into a vector y, the block LS estimation of a_θ from y is defined by

$$\begin{aligned} y &\triangleq \begin{bmatrix} \bar{y}_0 \\ \bar{y}_1 \\ \vdots \\ \bar{y}_{N_s-1} \end{bmatrix}, \quad Z \triangleq \begin{bmatrix} \bar{Z}_0 \\ \bar{Z}_1 \\ \vdots \\ \bar{Z}_{N_s-1} \end{bmatrix}, \quad w \triangleq \begin{bmatrix} \bar{v}_0 \\ \bar{v}_1 \\ \vdots \\ \bar{v}_{N_s-1} \end{bmatrix} \\ y &= Z a_\theta + w \\ \hat{a}_\theta &= (Z^* Z)^{-1} Z^* y \end{aligned} \tag{13}$$

This solution can also be obtained adaptively by using a bank of adaptive filters. We shall denote the entries of the successive $\{\bar{y}_i\}$ by $\{d(m)\}$. The solution (13) can be estimated by training an adaptive filter with reference sequence $\{d(m)\}$ and regressors given by the rows of the $\{\bar{Z}_i\}$. We shall denote these rows by $\{u_m\}$. Since Z has $N_s M N_u L$ rows, the adaptive filter is cycled repeatedly through these regression rows until sufficient convergence is obtained. The adaptive algorithm would be as follows:

1. The received signal $r(n)$ is applied to a bank of matched filters in order to generate the vectors $\{\bar{y}_i\}$.

2. A parallel to serial converter is applied to $\bar{y}_i$ in order to form the reference sequence $\{d(m)\}$.

3. An adaptive filter of weight vector w_m is used to estimate a_θ at the m^{th} iteration (i.e., w_m is the estimate of a_θ at iteration m). The regression vector u_m is obtained from the rows of $\bar{Z}_i$. The adaptive filter is iterated repeatedly in a cyclic manner over the rows of Z until sufficient performance is attained.

   Specifically, the adaptive filter weight vector w_m is updated as follows:

$$w_m = w_{m-1} + \mu(m) u_m^* [d(m) - u_m w_{m-1}] \tag{14}$$

Figure 3 shows the proposed system model for the array response estimation.

V. Joint Estimation of Channel Attenuation and Array Response

In the proposed array response estimation we have used all users' spreading sequences to form the matrix $G_{k,l}$, and we have also used the channel attenuation matrix H. We can use conventional adaptive CDMA channel estimation methods [3] for single antenna systems to estimate H. Equation (10) can be

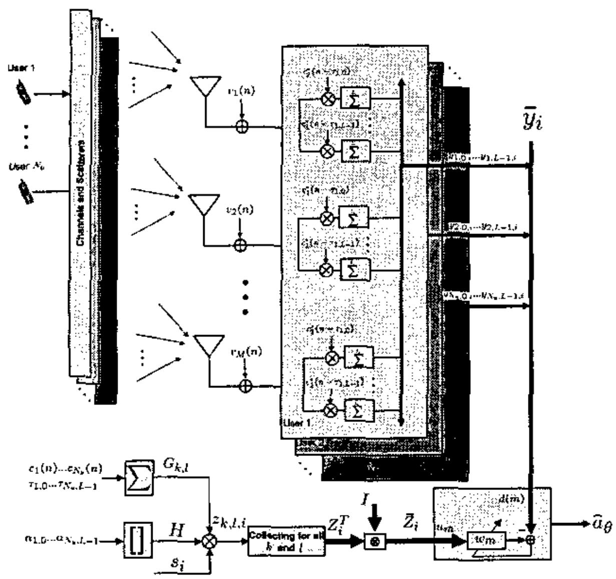

Figure 3: *An adaptive multiuser CDMA RAKE receiver and AOA estimator*

revised to have both A_θ and H as unknown parameters as follows:

$$y_{k,l,i} = \underbrace{A_\theta H}_{B_{\theta,H}} \underbrace{G_{k,l} s_i}_{z_{k,l,i}} + v_{k,l,i} \tag{15}$$

where $z_{k,l,i}$ is an $N_u L \times 1$ vector and $B_{\theta,H}$ is an $M \times N_u L$ matrix. So we have

$$y_{k,l,i} = B_{\theta,H} z_{k,l,i} + v_{k,l,i} \tag{16}$$

As in (12), we can write

$$\underbrace{\mathrm{vec}(Y_i)}_{\bar{y}_i} = \underbrace{(Z_i^T \otimes I_{N_u L})}_{\bar{Z}_i} \underbrace{\mathrm{vec}(B_\theta)}_{b_{\theta,H}} + \underbrace{\mathrm{vec}(V_i)}_{\bar{v}_i} \tag{17}$$

$$\boxed{\bar{y}_i = \bar{Z}_i b_{\theta,H} + \bar{v}_i} \tag{18}$$

Let

$$y \triangleq \begin{bmatrix} \bar{y}_0 \\ \bar{y}_1 \\ \vdots \\ \bar{y}_{N_s-1} \end{bmatrix}, \quad Z \triangleq \begin{bmatrix} \bar{Z}_0 \\ \bar{Z}_1 \\ \vdots \\ \bar{Z}_{N_s-1} \end{bmatrix}, \quad w \triangleq \begin{bmatrix} \bar{v}_0 \\ \bar{v}_1 \\ \vdots \\ \bar{v}_{N_s-1} \end{bmatrix}$$

Then

$$\begin{aligned} y &= Z b_{\theta,H} + w \\ \hat{b}_{\theta,H} &= (Z^* Z)^{-1} Z^* y \end{aligned} \tag{19}$$

Let

$$\hat{B}_{\theta,H} = \mathrm{unvec}(\hat{b}_{\theta,H})$$

which we decompose as

$$\hat{B}_{\theta,H} = \hat{A}_\theta \hat{H}$$

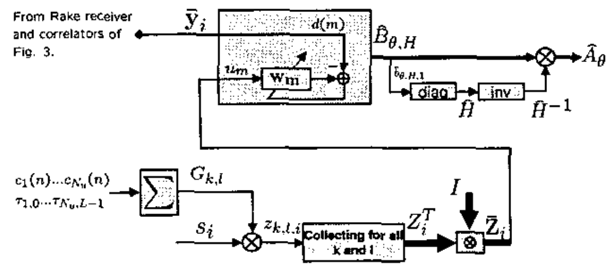

Figure 4: *A model for estimating angle of arrival without prior knowledge of channel attenuations.*

Since H is a diagonal matrix, and all the elements in the first row of A_θ are 1, the first row of $B_{\theta,H}$ contains the diagonal elements of H, so that

$$\begin{aligned} \hat{H} &= \mathrm{diag}(\hat{b}_{\theta,H,1}) \\ \hat{A}_\theta &= \hat{B}_{\theta,H} \hat{H}^{-1} \end{aligned} \tag{20}$$

where $\hat{b}_{\theta,H,1}$ denotes the first row of $\hat{B}_{\theta,H}$. Fig. 4 illustrates the block diagram of the proposed method.

VI. Simulation Results

An uplink DS-CDMA system is used to evaluate the performance of the algorithm. We use the cdma2000 standard [8] to model users within a cell and simulate the link (any DS-CDMA standard can be used). We assume a processing gain of 64 for all users. The known pilot sequence transmitted by each user is used for training purposes at the base station. The CDMA chip-rate in the simulation is 4MHz, and a 4-element antenna array is employed at the base station. The antenna spacing is assumed to be half a wavelength. The mobile users are uniformly distributed in a 120 degrees sector. This is due to the fact that in current cell designs, the cell is divided into 3 sectors of 120 degrees each. We simulate the performance of the proposed AOA estimator in a multiuser multipath scenario. The propagation model used for each user is a 4-multipath channel with independent taps and random AOA. The Outage (or Cumulative Distribution Function) as a function of the error in AOA estimation is plotted in Figure 5. The figure depicts the simulation results for a cell with 2,4,8 and 16 active users and it compares the result of the proposed method to the maximum likelihood LS estimation of [9]. The FCC mandatory requirements [7] of location accuracy (using AOA estimation) are indicated in Figure 5 by the $\star$ symbol, for 67% and 95% outage. A cell size of 1 mile has been considered to translate the location accuracy in meters to AOA accuracy in degrees–see Figure 7. The *circle* and *star* markers are the FCC Network-Based and Handset-Based requirements respectively. Figure 6 demonstrates the standard deviation of AOA estimation error for 2,4,8 and 16 active users and compares the results with the method of [9] when no multiuser interference cancellation is performed. We may add that the algorithm for finding the mobile location in the fading environments in the single antenna case is from [10].

VII. Concluding Remarks

In this paper we presented an array response identification algorithm that extracts a priori channel information. The AOA estimator proposed in previous works for single-path single-user receivers has been generalized to multipath multiuser receivers. For AOA estimation, an adaptive estimator has been developed for a multi-user multi-path scenario. The estimator of AOA exploits the users' spreading code orthogonality to cancel interference and to increase the accuracy of the AOA estimation in the presence of a large number of users. The performance of the proposed architecture meets the location accuracy requirements on AOA estimation.

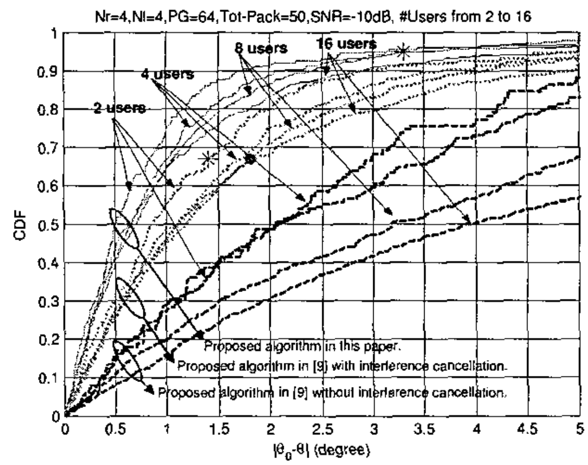

Figure 5: *Outage curves vs. error in AOA for 2,4,8 and 16 active users.*

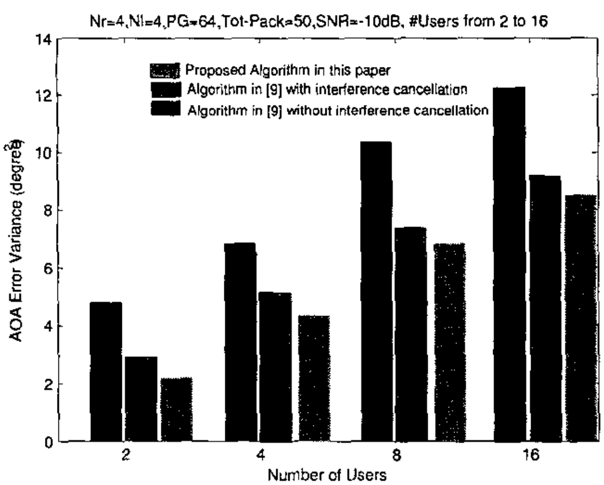

Figure 6: *Standard deviation of AOA estimation error for 2,4,8 and 16 active users.*

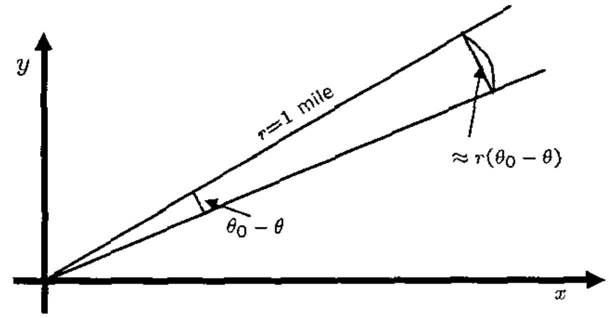

Figure 7: *Relation between the AOA estimation error and the location estimation error.*

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