

# SPACE-TIME CODING IN MISO-OFDM SYSTEMS WITH IMPLEMENTATION IMPAIRMENTS

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## ABSTRACT

Implementation of OFDM-based systems suffers from In-phase and Quadrature-phase (IQ) imbalances in the front-end analog processing. The resulting distortion due to IQ imbalances can limit the achievable operating SNR at the receiver, especially when space-time coding is used for enhanced received SNR. In this paper, the effect of IQ imbalances on an OFDM-based system with Alamouti coding is studied and an efficient receiver with compensation for IQ imbalances is then derived. Significant performance improvement is achieved by using the proposed receiver compared to a standard receiver with no compensation for IQ imbalances.

## 1. INTRODUCTION

One source of distortion that limits the performance of OFDM receivers is the impairment associated with analog signal processing. One such impairment arises as a result of imbalances in the In-phase (I) and Quadrature-phase (Q) branches.

Perfect IQ matching is not possible in the analog domain, especially when low cost fabrication technologies are used. Moreover, as carrier frequencies increase, IQ imbalances become more severe and more challenging to eliminate. These impairments also tend to increase as integrated circuit technologies such as complementary metal-oxide semiconductor (CMOS) are more widely adopted for radio frequency and analog processing due to their cost advantage and ease of integration with digital baseband processing. In addition, as higher data rates are targeted, higher constellation sizes are needed, and higher operating SNR are to be guaranteed in order to support high density constellations. Higher SNR requirements translate to tougher IQ matching requirements.

The effect of IQ imbalances on SISO OFDM systems and the resulting performance degradation have been investigated in [3]-[4]. Several compensation schemes have been proposed in [5]-[8]. The contribution of this paper is to model and study the effect of IQ imbalances on a MISO OFDM system that uses space-time coding. It will be shown that while IQ imbalances destroy some of the properties of space-time block codes, efficient receive algorithms are still possible by exploiting the structure of the codes.

The paper is organized as follows. The next section briefly describes the model used in [8] for IQ imbalances and formulates the effect of IQ imbalances on a SISO OFDM receiver. In Sec. 3, this formulation is used to develop a framework for an Alamouti coded system and study the effect of IQ on such systems. The

section also derives a receiver structure that is robust to IQ imbalances. Simulation results are included in Sec. 4 and conclusions are given in Sec. 5.

## 2. PROBLEM FORMULATION

Let  $b(t)$  represent the received complex signal before being distorted by IQ imbalances caused by the analog processing of the received signal. The distorted signal in the time domain can be written as [3],[4]:

$$b'(t) = \mu b(t) + \nu b^*(t) \tag{1}$$

where the distortion parameters,  $\mu$  and  $\nu$ , are related to the amplitude and phase imbalances between the I and Q branches in the radio frequency (RF) and analog demodulation process. A simplified physical model for this distortion can be found in [4]. A derivation of the OFDM signals in the presence of IQ imbalances is briefly presented below following [8].

In OFDM systems, a block of data is transmitted as an OFDM symbol. Assuming a block-size equal to  $N$ , the transmitted block of data is given by

$$\mathbf{s} \triangleq [s(1) \ s(2) \ \dots \ s(N)]^T \tag{2}$$

Each block is passed through the IDFT operation:

$$\bar{\mathbf{s}} = \mathbf{F}^* \mathbf{s} \tag{3}$$

where  $\mathbf{F}$  is the unitary discrete Fourier transform (DFT) matrix. A cyclic prefix of length  $P$  is added to each transformed block of data and then transmitted through the channel—see Figure 1. An FIR model with  $L + 1$  taps is assumed for the channel, i.e.,

$$\mathbf{h} = [h_0 \ h_1 \ \dots \ h_L]^T \tag{4}$$

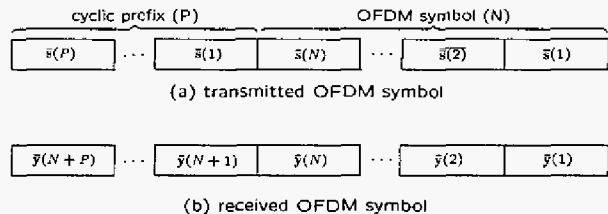


Fig. 1. Transmitted and received OFDM symbols.

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$$\tilde{\Gamma}_k^{(n)} = \begin{bmatrix} \mu\lambda_2(k) & \nu\lambda_2^*(N-k+2) & \mu\lambda_1(k) & \nu\lambda_1^*(N-k+2) \\ \nu^*\lambda_2(k) & \mu^*\lambda_2^*(N-k+2) & \nu^*\lambda_1(k) & \mu^*\lambda_1^*(N-k+2) \end{bmatrix} \quad (23)$$

where

$$\tilde{\mathbf{z}}_k = \begin{bmatrix} \mathbf{z}(k) \\ \mathbf{z}^*(N-k+2) \end{bmatrix} \quad (22)$$

$$\tilde{\mathbf{s}}_k = \begin{bmatrix} \mathbf{s}(k) \\ \mathbf{s}^*(N-k+2) \end{bmatrix}$$

$$\tilde{\Gamma}_k = \begin{bmatrix} \mu\lambda(k) & \nu\lambda^*(N-k+2) \\ \nu^*\lambda(k) & \mu^*\lambda^*(N-k+2) \end{bmatrix} \quad (24)$$

We can then recover  $\tilde{\mathbf{s}}_k$  as

$$\hat{\tilde{\mathbf{s}}}_k = \left( \tilde{\Gamma}_k^* \tilde{\Gamma}_k \right)^{-1} \tilde{\Gamma}_k^* \tilde{\mathbf{z}}_k$$

In [8], we described several methods for estimating the components of matrix  $\tilde{\Gamma}_k$  and for recovering  $\tilde{\mathbf{s}}_k$ .

### 3. ALAMOUTI SCHEME WITH IQ IMBALANCES

Let us now study how IQ imbalances affect data recovery in a multi-antenna system. In this paper, we focus on a 2-transmit 1-receive antenna system that employs the Alamouti scheme. In Figure 2, it is seen that two blocks of data

$$\mathbf{s}_1 \triangleq \begin{bmatrix} s_1(1) \\ \vdots \\ s_1(N) \end{bmatrix}, \quad \mathbf{s}_2 \triangleq \begin{bmatrix} s_2(1) \\ \vdots \\ s_2(N) \end{bmatrix}$$

are transmitted from both antennas before the IFFT operation. These blocks are then followed by the data

$$\begin{bmatrix} -s_2^*(1) \\ \vdots \\ -s_2^*(N) \end{bmatrix}, \quad \begin{bmatrix} s_1^*(1) \\ \vdots \\ s_1^*(N) \end{bmatrix}$$

At the receiver end, two blocks of data are received

$$\begin{bmatrix} \mathbf{z}_1(1) \\ \vdots \\ \mathbf{z}_1(N) \end{bmatrix}, \quad \begin{bmatrix} \mathbf{z}_2(1) \\ \vdots \\ \mathbf{z}_2(N) \end{bmatrix}$$

In this case, equation (5) becomes

$$\tilde{\mathbf{y}} = \mathbf{H}_1^c \tilde{\mathbf{s}}_1 + \mathbf{H}_2^c \tilde{\mathbf{s}}_2 + \tilde{\mathbf{v}} \quad (25)$$

where  $\tilde{\mathbf{s}}_1 = \mathbf{F}^* \mathbf{s}_1$  and  $\tilde{\mathbf{s}}_2 = \mathbf{F}^* \mathbf{s}_2$  are the transmitted blocks of data on antennas 1 and 2, respectively, and  $\mathbf{H}_1^c$  and  $\mathbf{H}_2^c$  are the channel matrices from the transmit antennas 1 and 2 to the receiver. Applying the same arguments as before to the above equation instead of (5) results in the following sets of equations (which correspond to equations (21)-(24) in the single antenna case):

$$\begin{bmatrix} \mathbf{z}_1(k) \\ \mathbf{z}_1^*(N-k+2) \end{bmatrix} = \underbrace{\begin{bmatrix} \tilde{\Gamma}_{2,k} & \tilde{\Gamma}_{1,k} \end{bmatrix}}_{\tilde{\Gamma}_k^{(n)}} \begin{bmatrix} s_2(k) \\ s_2^*(N-k+2) \\ s_1(k) \\ s_1^*(N-k+2) \end{bmatrix} + \tilde{\mathbf{v}}_{1,k} \quad (26)$$

and

$$\begin{bmatrix} \mathbf{z}_2(k) \\ \mathbf{z}_2^*(N-k+2) \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma}_{2,k} & \tilde{\Gamma}_{1,k} \end{bmatrix} \begin{bmatrix} s_1^*(k) \\ s_1(N-k+2) \\ -s_2^*(k) \\ -s_2(N-k+2) \end{bmatrix} + \tilde{\mathbf{v}}_{2,k} \quad (27)$$

where

$$\tilde{\Gamma}_{1,k} = \begin{bmatrix} \mu\lambda_1(k) & \nu\lambda_1^*(N-k+2) \\ \nu^*\lambda_1(k) & \mu^*\lambda_1^*(N-k+2) \end{bmatrix} \quad (28)$$

$$\tilde{\Gamma}_{2,k} = \begin{bmatrix} \mu\lambda_2(k) & \nu\lambda_2^*(N-k+2) \\ \nu^*\lambda_2(k) & \mu^*\lambda_2^*(N-k+2) \end{bmatrix}$$

The matrix  $\tilde{\Gamma}_k^{(n)}$  is now  $2 \times 4$  and the  $\lambda_i(k)$  in (28) denote the channel taps in the frequency domain corresponding to tone  $k$  from transmit antenna  $i$  to the receive antenna.

Compactly, we write

$$\tilde{\mathbf{z}}_k^{(n)} = \tilde{\Gamma}_k^{(n)} \tilde{\mathbf{s}}_k^{(n)} + \tilde{\mathbf{v}}_k^{(n)} \quad (29)$$

where the matrix  $\tilde{\Gamma}_k^{(n)}$  is given by (23), and

$$\tilde{\mathbf{s}}_k^{(n)} = \begin{bmatrix} s_1^*(k) & s_2(k) \\ s_1(N-k+2) & s_2^*(N-k+2) \\ -s_2^*(k) & s_1(k) \\ -s_2(N-k+2) & s_1^*(N-k+2) \end{bmatrix} \quad (30)$$

$$\tilde{\mathbf{z}}_k^{(n)} = \begin{bmatrix} \mathbf{z}_2(k) & \mathbf{z}_1(k) \\ \mathbf{z}_2^*(N-k+2) & \mathbf{z}_1^*(N-k+2) \end{bmatrix} \quad (31)$$

In order to recover the data in  $\tilde{\mathbf{s}}_k^{(n)}$  from  $\tilde{\mathbf{z}}_k^{(n)}$  in (29), the space-time code structure is exploited as follows. Let

$$\tilde{\mathbf{z}}_k = \begin{bmatrix} \mathbf{z}_1(k) \\ \mathbf{z}_2^*(k) \\ \mathbf{z}_1^*(N-k+2) \\ \mathbf{z}_2(N-k+2) \end{bmatrix}, \quad \tilde{\mathbf{s}}_k = \begin{bmatrix} s_1(k) \\ s_2(k) \\ s_1^*(N-k+2) \\ s_2^*(N-k+2) \end{bmatrix} \quad (32)$$

Then (29) gives

$$\tilde{\mathbf{z}}_k = \tilde{\Gamma}_k \tilde{\mathbf{s}}_k + \tilde{\mathbf{v}}_k \quad (33)$$

with

$$\tilde{\Gamma}_k = \begin{bmatrix} \Delta_{1,k} & \Delta_{2,k} \\ \Delta_{3,k} & \Delta_{4,k} \end{bmatrix} \quad (34)$$

where, interestingly, all the sub-blocks have an Alamouti structure given by

$$\Delta_{1,k} = \begin{bmatrix} \mu\lambda_1(k) & \mu\lambda_2(k) \\ \mu^*\lambda_2^*(k) & -\mu^*\lambda_1^*(k) \end{bmatrix}$$

$$\Delta_{2,k} = \begin{bmatrix} \nu\lambda_1^*(N-k+2) & \nu\lambda_2^*(N-k+2) \\ \nu^*\lambda_2(N-k+2) & -\nu^*\lambda_1(N-k+2) \end{bmatrix} \quad (35)$$

$$\Delta_{3,k} = \begin{bmatrix} \nu^*\lambda_1(k) & \nu^*\lambda_2(k) \\ \nu\lambda_2^*(k) & -\nu\lambda_1^*(k) \end{bmatrix}$$

$$\Delta_{4,k} = \begin{bmatrix} \mu^*\lambda_1^*(N-k+2) & \mu^*\lambda_2^*(N-k+2) \\ \mu\lambda_2(N-k+2) & -\mu\lambda_1(N-k+2) \end{bmatrix}$$

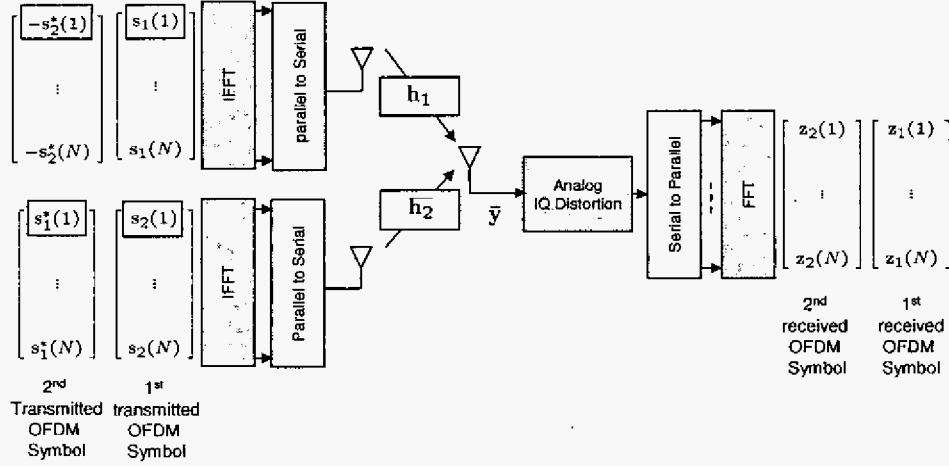


Fig. 2. Alamouti scheme applied to an OFDM system.

$$\tilde{\Gamma}_k^* \tilde{\Gamma}_k = \begin{bmatrix} (|\mu|^2 + |\nu|^2) (|\lambda_1(k)|^2 + |\lambda_2(k)|^2) \mathbf{I}_{2 \times 2} & \Delta_{1,k}^* \Delta_{2,k} + \Delta_{3,k}^* \Delta_{4,k} \\ \Delta_{2,k}^* \Delta_{1,k} + \Delta_{4,k}^* \Delta_{3,k} & (|\mu|^2 + |\nu|^2) (|\lambda_1(N-k+2)|^2 + |\lambda_2(N-k+2)|^2) \mathbf{I}_{2 \times 2} \end{bmatrix} \quad (36)$$

The matrix  $\tilde{\Gamma}_k$  in (34) would be block-diagonal if  $\nu = 0$ , i.e., if there were no IQ imbalances, in which case equation (33) would reduce to two  $2 \times 2$  decoupled systems as in standard Alamouti decoding. The off-diagonal matrices in (34) are a direct result of the IQ imbalances. So now we need to deal with a  $4 \times 4$  linear system of equations as opposed to two  $2 \times 2$  systems of equations in Alamouti coded receivers with ideal IQ branches. However, the Alamouti structure of the sub-matrices (35) allows a computationally efficient implementation of the least-squares estimator:

$$\hat{\tilde{\mathbf{s}}}_k = (\tilde{\Gamma}_k^* \tilde{\Gamma}_k)^{-1} \tilde{\Gamma}_k^* \tilde{\mathbf{z}}_k$$

as follows. First we note that

$$\Delta_{1,k}^* \Delta_{1,k} = |\mu|^2 (|\lambda_1(k)|^2 + |\lambda_2(k)|^2) \mathbf{I}_{2 \times 2}$$

$$\Delta_{2,k}^* \Delta_{2,k} = |\nu|^2 (|\lambda_1(N-k+2)|^2 + |\lambda_2(N-k+2)|^2) \mathbf{I}_{2 \times 2}$$

$$\Delta_{3,k}^* \Delta_{3,k} = |\nu|^2 (|\lambda_1(k)|^2 + |\lambda_2(k)|^2) \mathbf{I}_{2 \times 2}$$

$$\Delta_{4,k}^* \Delta_{4,k} = |\mu|^2 (|\lambda_1(N-k+2)|^2 + |\lambda_2(N-k+2)|^2) \mathbf{I}_{2 \times 2}$$

so that the product  $\tilde{\Gamma}_k^* \tilde{\Gamma}_k$  reduces to the result given by (36). This matrix would be diagonal if  $\nu = 0$ , i.e., if there were no IQ imbalances. When  $\nu \neq 0$ , the matrix  $\tilde{\Gamma}_k^* \tilde{\Gamma}_k$  is no longer diagonal, however it has a particular structure that is induced by the Alamouti code and the distortion model. Specifically, its  $2 \times 2$  off-diagonal blocks are Alamouti, which means their inverses can be obtained by simple transposition. This is due to the fact that the sum and product of two Alamouti matrices is still an Alamouti matrix. Thus denote the  $2 \times 2$  block entries of  $\tilde{\Gamma}_k^* \tilde{\Gamma}_k$  in (36) by

$$\tilde{\Gamma}_k^* \tilde{\Gamma}_k = \begin{bmatrix} \mathbf{D}_1 & \mathbf{A}_1 \\ \mathbf{A}_2 & \mathbf{D}_2 \end{bmatrix}$$

where  $\mathbf{D}_1$  and  $\mathbf{D}_2$  are diagonal (actually scalar multiples of the identity due to the Alamouti structure) and  $\mathbf{A}_1$  and  $\mathbf{A}_2$  are also

Alamouti, say  $\mathbf{D}_1 = d_1 \mathbf{I}$  and  $\mathbf{D}_2 = d_2 \mathbf{I}$ . Using the block inversion formula

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1} = \begin{bmatrix} \Sigma^{-1} & -\Sigma^{-1} \mathbf{B} \mathbf{D}^{-1} \\ -\mathbf{D}^{-1} \mathbf{C} \Sigma^{-1} & \mathbf{D}^{-1} + \mathbf{D}^{-1} \mathbf{C} \Sigma^{-1} \mathbf{B} \mathbf{D}^{-1} \end{bmatrix}$$

where

$$\Sigma = \mathbf{A} - \mathbf{B} \mathbf{D}^{-1} \mathbf{C}$$

we get

$$(\tilde{\Gamma}_k^* \tilde{\Gamma}_k)^{-1} = \begin{bmatrix} \Sigma^{-1} & -\Sigma^{-1} \mathbf{A}_1 \mathbf{D}_2^{-1} \\ -\mathbf{D}_2^{-1} \mathbf{A}_2 \Sigma^{-1} & \mathbf{D}_2^{-1} + \mathbf{D}_2^{-1} \mathbf{A}_2 \Sigma^{-1} \mathbf{A}_1 \mathbf{D}_2^{-1} \end{bmatrix}$$

where

$$\begin{aligned} \Sigma &= \mathbf{D}_1 - \mathbf{A}_1 \mathbf{D}_2^{-1} \mathbf{A}_2 \\ &= d_1 \mathbf{I} - \frac{1}{d_2} \mathbf{A}_1 \mathbf{A}_2 \end{aligned}$$

We see that all terms in the expression are trivial to compute. The only inversion term is the  $2 \times 2$  matrix  $\Sigma^{-1}$ . The  $2 \times 2$  matrix  $\Sigma$  is the sum of a scaled identity matrix and an Alamouti matrix (since the product of  $\mathbf{A}_1$  and  $\mathbf{A}_2$  is also Alamouti); computing its inverse is trivial.

### 3.1. Joint Channel and Distortion Parameter Estimation

The problem of estimating the channel and distortion parameters  $\{\lambda_1(k), \lambda_2(k), \mu, \nu\}$  needed at the receiver for data recovery is not discussed in this paper. It is useful to realize that in the presence of IQ imbalances, the channel and distortion parameters are required for data recovery in a joint manner—see (35). In other words, the channel and distortion parameters are not required separately at the receiver, but a combination of them is used in the proposed receiver algorithms. For instance, in the Alamouti case, the scalar factor of the diagonal matrix  $\mathbf{D}_1$  in (36) is the sum of the squared values of the first-column elements in  $\Delta_{1,k}$  and  $\Delta_{3,k}$  given by (35). A joint channel and distortion estimation task at the receiver is then

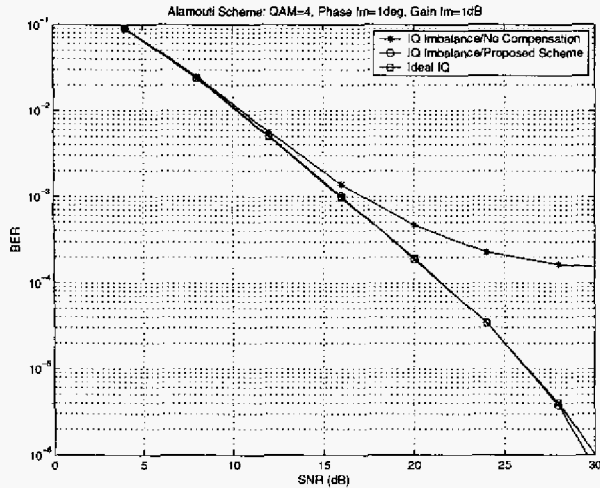


Fig. 3. BER vs. SNR for an OFDM system with Alamouti coding applied. QPSK constellation is used in a receiver with phase imbalance of  $1^\circ$  and gain imbalance of 1dB.

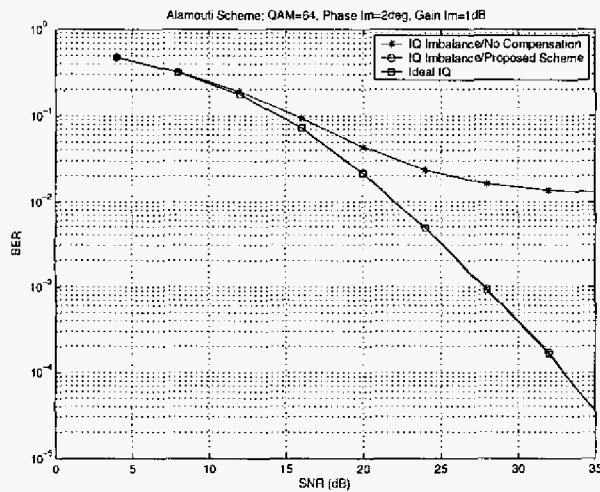


Fig. 4. BER vs. SNR for an OFDM system with Alamouti coding applied. 64QAM constellation is used in a receiver with phase imbalance of  $2^\circ$  and gain imbalance of 1dB.

equivalent to estimating the matrix  $\tilde{\Gamma}_k$  defined by (34). The input-output system given by (33) can be used to estimate  $\tilde{\Gamma}_k$  from the received data  $\tilde{z}_k$ ; possibly by transmitting training data. Similar to data recovery algorithms, the structure of the system given by (34) can be exploited to derive efficient channel estimation algorithms. Addressing the channel/distortion estimation problem is beyond the scope of this paper. However, some efficient channel/distortion estimation schemes for the SISO case can already be found in [8].

#### 4. SIMULATION RESULTS

A typical OFDM system with space-time coding is simulated to evaluate the performance of the proposed schemes in comparison to an ideal OFDM receiver with no IQ imbalance and a receiver

with no IQ compensation scheme. In the simulations, the Alamouti scheme is applied on a  $(2 \times 1)$  system, as was described in Sec. 3. The parameters used in the simulation are: OFDM symbol length of  $N = 64$ , cyclic prefix of  $P = 16$ , and channel length of  $(L + 1) = 4$ . The channel taps corresponding to two transmit antennas are chosen independently with complex Gaussian distribution. The BER versus SNR for the proposed scheme are simulated and shown in Figures 3-4. In all the figures, 'Ideal IQ' legend refers to a receiver with no IQ imbalance and perfect channel knowledge and 'IQ Imbalance/No Compensation' refers to a receiver with IQ imbalance but no compensation scheme. 'IQ Imbalance/Proposed Scheme' refers to the algorithm proposed in Sec. 3. The results are depicted for different constellation sizes (4QAM and 64QAM) and different phase and gain imbalances.

#### 5. CONCLUSIONS

The effect of IQ imbalances on OFDM receivers with space-time coding was studied and a framework for deriving OFDM receivers with IQ imbalance compensation was presented. It was shown how efficient receivers can be designed by exploiting the space-time coding structure and the model for IQ imbalances. Significant improvements in the overall performance of the receiver with compensation for IQ imbalances are achieved compared to receivers that assume ideal IQ branches.

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