Adaptive Channel Estimation for MIMO Space-Time Coded Communications

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Abstract— **We develop adaptive channel estimation techniques for multiple–input multiple–output (MIMO) channels with space–time block–coded (STBC) communications. We describe a unified framework for adaptive channel estimation for multi-user Alamouti STBC transmissions over flat fading channels and over frequency selective fading channels using Single–Carrier Frequency Domain (SC–FD) STBC or STBC–OFDM. The structure of the space–time code is exploited to reduce the complexity of block NLMS and RLS receivers.**

I. Introduction

Wireless communications systems with multiple transmit and receive antennas provide large capacity gains, especially in rich scattering environments [1]. In order to achieve high transmission rates with high performance and reliability, the receiver needs to have accurate information about the MIMO channel between the transmitter and the receiver. In multi–user multi–antenna scenarios, the channel estimation problem is challenging due to the increasing number of subchannels.

Various space-time block codes (STBC) have been developed to provide high performance and transmission rate over both flat fading and frequency–selective fading channels (e.g., Alamouti–STBC [2], STBC–OFDM [3], and single–carrier frequency domain STBC (SC–FD STBC) [4]). Most of the receivers for these schemes require explicit knowledge of the impulse response of all sub-channels.

In this paper, we develop adaptive channel estimation techniques for three multi–user STBC transmission schemes over both flat and frequency selective fading channels. We show how to exploit the STBC structure to reduce the computational complexity of the receivers. We also develop a unified framework for channel estimation for Alamouti-STBC, SC-FD STBC, and STBC–OFDM transmissions.

II. Problem Formulation

Consider a system consisting of M users, each equipped with two antennas. Each user transmits STBC data from its two antennas. The receiver is equipped with M receive antennas. The block diagram of the system is shown in Fig. 1. The coding structure depends on whether the channel is flat fading or frequency selective.

A. Flat Fading Channels

For flat fading channels, data from each user are transmitted from its two antennas according to the Alamouti space–time block coding scheme of [2]– see Fig. 2. At times $k = 0, 2, 4, \cdots$, the data symbols of the i–th user, denoted by $x_{k,1}^{(i)}$ and $x_{k,2}^{(i)}$, are generated by an information source according to the rule:

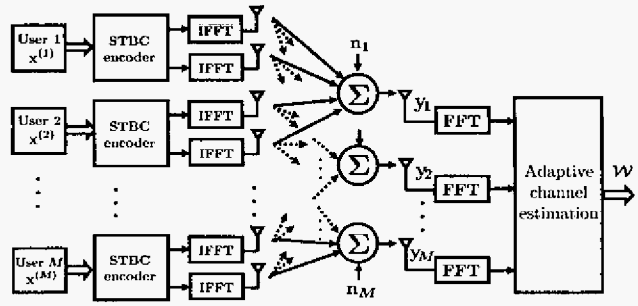

Fig. 1. Block diagram of an M-user communication system with 2–transmit 1–receive antennas per user.

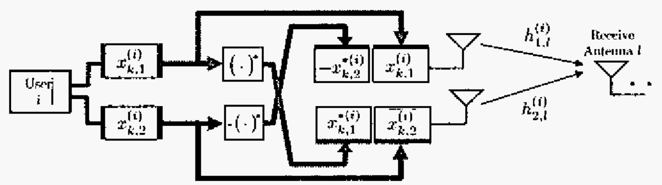

Fig. 2. Transmission scheme from the i–th user to the l–th receive antenna for flat fading channels.

$$\begin{array}{cc|cc} & & \multicolumn{2}{c}{Time} \\ & & k & k+1 \\ \hline Antenna & 1 & x_{k,1}^{(i)} & -x_{k,2}^{*(i)} \\ & 2 & x_{k,2}^{(i)} & x_{k,1}^{*(i)} \end{array} \tag{1}$$

The received signals at the l–th receive antenna at times k and $k+1$ are collected into a 2×1 vector $\mathbf{y}_{k,l}$, which is given by

$$\mathbf{y}_{k,l} = \begin{pmatrix} y_{k,l} \\ y_{k+1,l} \end{pmatrix} = \sum_{i=1}^{M} \begin{pmatrix} x_{k,1}^{(i)} & x_{k,2}^{(i)} \\ -x_{k,2}^{*(i)} & x_{k,1}^{*(i)} \end{pmatrix} \begin{pmatrix} h_{1,l}^{(i)} \\ h_{2,l}^{(i)} \end{pmatrix} + \begin{pmatrix} n_{k,l} \\ n_{k+1,l} \end{pmatrix}$$
$$\triangleq \sum_{i=1}^{M} \mathbf{X}_k^{(i)} \mathbf{h}_l^{(i)} + \mathbf{n}_l \tag{2}$$

Here, $\mathbf{y}_{k,l}$ is the 2×1 received k-th block at the l-th receive antenna, $\mathbf{n}_{k,l}$ is the 2×1 noise vector at the l-th receive antenna from the k-th block, and $\mathbf{X}_k^{(i)}$ is the 2×2 Alamouti matrix of the i-th user at the k-th block. The structure of $\mathbf{X}_k^{(i)}$ will appear frequently throughout the paper, and we shall use the terminology *Alamouti matrix* to refer to it. The coefficients of the flat fading channels, $h_{1,l}^{(i)}$ and $h_{2,l}^{(i)}$, are modelled as iid complex Gaussian random variables with variance equal to 0.5 per dimension, i.e., the covariance matrix is $\mathbf{R}_h = \mathbf{I}$. The noise is modelled as AWGN with zero mean and covariance matrix $\mathbf{R}_n = \sigma_n^2 \mathbf{I}$, and the transmitted symbols have variance σ_x^2. By collecting the received vectors $\mathbf{y}_{k,l}$ form all antennas, we may

This work was partially supported by NSF grants CCR-0208573 and ECS-0401188.

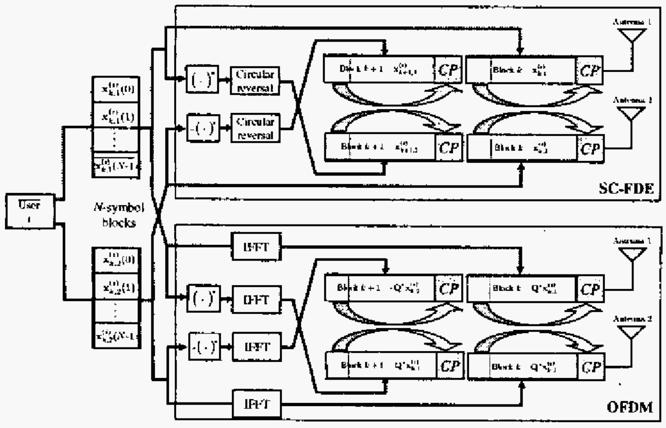

Fig. 3. Two block STBC transmission schemes for frequency-selective channels.

write in matrix form:

$$\mathcal{Y}_k = \mathcal{X}_k \mathcal{H} + \mathcal{N}_k \tag{3}$$

where

$$\mathcal{Y}_k = \underbrace{\begin{pmatrix} \mathbf{y}_{1,1} \ldots \mathbf{y}_{1,M} \end{pmatrix}}_{2\times M}, \quad \mathcal{X}_k = \underbrace{\begin{pmatrix} \mathbf{X}_1^{(1)} \ldots \mathbf{X}_1^{(M)} \end{pmatrix}}_{2\times 2M}$$

$$\mathcal{H} = \underbrace{\begin{pmatrix} \mathbf{h}_1^{(1)} & \ldots & \mathbf{h}_M^{(1)} \\ \vdots & \ddots & \vdots \\ \mathbf{h}_1^{(M)} & \ldots & \mathbf{h}_M^{(M)} \end{pmatrix}}_{2M\times M}, \quad \mathcal{N}_k = \underbrace{\begin{pmatrix} \mathbf{n}_{1,1} \ldots \mathbf{n}_{1,M} \end{pmatrix}}_{2\times M}$$

B. Frequency-Selective Channels

For frequency-selective fading channels, the Alamouti scheme is instead implemented on a block level to achieve multipath diversity. Different schemes have been proposed for the block level implementation. In this section we describe two such schemes, namely, single-carrier frequency domain STBC (SC-FD STBC) and STBC orthogonal frequency division multiplexing (STBC-OFDM). Fig. 3 shows the code structure in both cases. For each scheme, we shall show how to exploit the code structure to reduce the complexity of the problem to that of parallel flat channels.

1) SC-FD STBC: For each user, data are transmitted from its two antennas in blocks of length N according to the following space-time coding scheme. Denote the n–th symbol of the k–th transmitted block from antenna j of user i by $\mathbf{x}_{k,j}^{(i)}(n)$. At times $k = 0, 2, 4, \cdots$, the blocks $\mathbf{x}_{k,1}^{(i)}(n)$ and $\mathbf{x}_{k,2}^{(i)}(n)$ $(0 \le n \le N-1)$ are generated by an information source according to the rule [4], [5]:

$$\begin{array}{cc|cc} & & \multicolumn{2}{l}{\textit{Block}} \\ & & k & k+1 \\ \hline \textit{Antenna} & 1 & \mathbf{x}_{k,1}^{(i)}(n) & -\mathbf{x}_{k,2}^{*(i)}\left((-n)_N\right) \\ & 2 & \mathbf{x}_{k,2}^{(i)}(n) & \mathbf{x}_{k,1}^{*(i)}\left((-n)_N\right) \end{array} \tag{4}$$

where each data vector $\mathbf{x}_{k,j}^{(i)}$ has a covariance matrix equal to $\sigma_x^2 \mathbf{I}_N$, and where $(\cdot)^*$ and $(\cdot)_N$ denote complex conjugation and modulo–N operations, respectively. In addition, a cyclic prefix (CP) of length ν is added to each transmitted block to eliminate inter–block interference (IBI) and to make all channel matrices *circulant*. Here, ν denotes the longest channel memory between the transmit antennas and the receive antennas. With two transmit antennas per user and M receive antennas, and assuming all channels are fixed over two consecutive blocks, the received blocks k and $k+1$ at the l–th antenna, in the presence of additive white noise, are described by

$$\begin{aligned} \mathbf{y}_{k,l} &= \sum_{i=1}^{M} \left(\mathbf{H}_{1,l}^{(i)} \mathbf{x}_{k,1}^{(i)} + \mathbf{H}_{2,l}^{(i)} \mathbf{x}_{k,2}^{(i)} \right) + \mathbf{n}_{k,l} \\ \mathbf{y}_{k+1,l} &= \sum_{i=1}^{M} \left(\mathbf{H}_{1,l}^{(i)} \mathbf{x}_{k+1,1}^{(i)} + \mathbf{H}_{2,l}^{(i)} \mathbf{x}_{k+1,2}^{(i)} \right) + \mathbf{n}_{k+1,l} \end{aligned} \tag{5}$$

where $\mathbf{n}_{k,l}$ and $\mathbf{n}_{k+1,l}$ are the noise vectors for the received blocks k and $k+1$, respectively, at the l-th receive antenna with covariance matrix $\sigma_n^2 \mathbf{I}_N$, and $\mathbf{H}_{1,l}^{(i)}$ and $\mathbf{H}_{2,l}^{(i)}$ have circulant structures. Applying the DFT matrix $\mathbf{Q}$ to $\mathbf{y}_{k,l}$ and $\mathbf{y}_{k+1,l}$ in (5), we get a relation in terms of frequency–transformed variables:

$$\begin{aligned} \mathbf{Y}_{k,l} &= \sum_{i=1}^{M} \left(\mathbf{\Lambda}_{1,l}^{(i)} \mathbf{X}_{k,1}^{(i)} + \mathbf{\Lambda}_{2,l}^{(i)} \mathbf{X}_{k,2}^{(i)} \right) + \mathbf{N}_{k,l} \\ \mathbf{Y}_{k+1,l} &= \sum_{i=1}^{M} \left(\mathbf{\Lambda}_{1,l}^{(i)} \mathbf{X}_{k+1,1}^{(i)} + \mathbf{\Lambda}_{2,l}^{(i)} \mathbf{X}_{k+1,2}^{(i)} \right) + \mathbf{N}_{k+1,l} \end{aligned} \tag{6}$$

where $\mathbf{Y} = \mathbf{Q}\mathbf{y}$, $\mathbf{X} = \mathbf{Q}\mathbf{x}$, $\mathbf{N} = \mathbf{Q}\mathbf{n}$, and $\mathbf{\Lambda}_{1,l}^{(i)}$ and $\mathbf{\Lambda}_{2,l}^{(i)}$ are diagonal matrices given by $\mathbf{\Lambda}_{1,l}^{(i)} = \mathbf{Q}\mathbf{H}_{1,l}^{(i)}\mathbf{Q}^*$ and $\mathbf{\Lambda}_{2,l}^{(i)} = \mathbf{Q}\mathbf{H}_{1,l}^{(i)}\mathbf{Q}^*$, respectively. Using the encoding rule (4) and properties of the DFT [6], we have that

$$\mathbf{X}_{k+1,1}^{(i)}(m) = -\mathbf{X}_{k,2}^{*(i)}(m), \quad \mathbf{X}_{k+1,2}^{(i)}(m) = \mathbf{X}_{k,1}^{*(i)}(m) \tag{7}$$

for $m = 0, 1, \cdots, N-1$ and $k = 0, 2, 4, \cdots$. Combining (6) and (7), we arrive at the linear relation (8). Let

$$\mathcal{Y}_{k,l}(m) = \begin{pmatrix} \mathbf{Y}_{k,l}(m) \\ \mathbf{Y}_{k+1,l}(m) \end{pmatrix}$$

Then the m–th entry of received vectors at the l–th receive antenna can be written as

$$\begin{aligned} \mathcal{Y}_{k,l}(m) &= \sum_{i=1}^{M} \begin{pmatrix} \mathbf{X}_{k,1}^{(i)}(m) & \mathbf{X}_{k,2}^{(i)}(m) \\ -\mathbf{X}_{k,2}^{*(i)}(m) & \mathbf{X}_{k,1}^{*(i)}(m) \end{pmatrix} \begin{pmatrix} \mathbf{\Lambda}_{1,l}^{(i)}(m) \\ \mathbf{\Lambda}_{2,l}^{(i)}(m) \end{pmatrix} + \begin{pmatrix} \mathbf{N}_{k,l}(m) \\ \mathbf{N}_{k+1,l}(m) \end{pmatrix} \\ &\triangleq \sum_{i=1}^{M} \mathcal{X}_k^{(i)}(m) \mathbf{\Lambda}_l^{(i)}(m) + \mathcal{N}_{k,l}(m) \end{aligned} \tag{9}$$

By inspecting the structure of (9), we find that it has the same form as (2). The main distinction is that the variables in (2) are time–domain variables, whereas those in (9) are frequency (or DFT) transformed variables. Again, by collecting the m–th entries of the received vectors $\mathcal{Y}_{k,l}(m)$ from all antennas, we may write in matrix form:

$$\mathcal{Y}_k(m) = \mathcal{X}_k(m)\mathbf{\Lambda}(m) + \mathcal{N}_k(m) \tag{10}$$

where

$$\begin{aligned} \mathcal{Y}_k(m) &= \underbrace{\begin{pmatrix} \mathcal{Y}_{k,1}(m) & \ldots & \mathcal{Y}_{k,M}(m) \end{pmatrix}}_{2\times M}, \\ \mathcal{X}_k(m) &= \underbrace{\begin{pmatrix} \mathcal{X}_k^{(1)}(m) & \ldots & \mathcal{X}_k^{(M)}(m) \end{pmatrix}}_{2\times 2M}, \\ \mathbf{\Lambda}(m) &= \underbrace{\begin{pmatrix} \mathbf{\Lambda}_1^{(1)}(m) & \ldots & \mathbf{\Lambda}_M^{(1)}(m) \\ \vdots & \ddots & \vdots \\ \mathbf{\Lambda}_1^{(M)}(m) & \ldots & \mathbf{\Lambda}_M^{(M)}(m) \end{pmatrix}}_{2M\times M}, \\ \mathcal{N}_k(m) &= \underbrace{\begin{pmatrix} \mathcal{N}_{k,1}(m) \ldots \mathcal{N}_{k,M}(m) \end{pmatrix}}_{2\times M} \end{aligned} \tag{11}$$

2) STBC–OFDM: From Fig. 3, we see that STBC–OFDM is similar to SC-FD STBC except for transmitting the IDFT of the data blocks rather than the data blocks themselves. At times $k =$

$$\mathcal{Y}_{k,l} \triangleq \begin{pmatrix} \mathbf{Y}_{k,l}(0) \\ \mathbf{Y}_{k+1,l}(0) \\ \hline \vdots \\ \hline \mathbf{Y}_{k,l}(N-1) \\ \mathbf{Y}_{k+1,l}(N-1) \end{pmatrix} = \sum_{i=1}^{M} \begin{pmatrix} \mathbf{X}_{k,1}^{(i)}(0) & \mathbf{X}_{k,2}^{(i)}(0) & & \\ -\mathbf{X}_{k,2}^{*(i)}(0) & \mathbf{X}_{k,1}^{*(i)}(0) & & \\ & \ddots & & \\ & & \mathbf{X}_{k,1}^{(i)}(N-1) & \mathbf{X}_{k,2}^{(i)}(N-1) \\ & & -\mathbf{X}_{k,2}^{*(i)}(N-1) & \mathbf{X}_{k,1}^{*(i)}(N-1) \end{pmatrix} \begin{pmatrix} \mathbf{\Lambda}_{1,l}^{(i)}(0) \\ \mathbf{\Lambda}_{2,l}^{(i)}(0) \\ \hline \vdots \\ \hline \mathbf{\Lambda}_{1,l}^{(i)}(N-1) \\ \mathbf{\Lambda}_{2,l}^{(i)}(N-1) \end{pmatrix} + \begin{pmatrix} \mathbf{N}_{k,l}(0) \\ \mathbf{N}_{k+1,l}(0) \\ \hline \vdots \\ \hline \mathbf{N}_{k,l}(N-1) \\ \mathbf{N}_{k+1,l}(N-1) \end{pmatrix} \quad (8)$$

$0, 2, 4, \cdots$, N-symbol data blocks $\mathbf{x}_{k,1}^{(i)}$ and $\mathbf{x}_{k,2}^{(i)}$ are generated by an information source. The data blocks are then transmitted from the antennas of the i–th user according to the following rule:

		Block k	$k+1$
Antenna	1	$\mathbf{Q}^*\mathbf{x}_{k,1}^{(i)}$	$-\mathbf{Q}^*\mathbf{x}_{k,2}^{*(i)}$
	2	$\mathbf{Q}^*\mathbf{x}_{k,2}^{(i)}$	$\mathbf{Q}^*\mathbf{x}_{k,1}^{*(i)}$

(12)

where $\mathbf{Q}^*$ is the IDFT matrix of size $N \times N$. A cyclic prefix (CP) is also added to each transmitted block to eliminate inter–block interference (IBI) and to make all channel matrices *circulant*. The received blocks k and $k+1$ at the l–th antenna, in the presence of additive white noise, are described by

$$\mathbf{y}_{k,l} = \sum_{i=1}^{M} \left(\mathbf{H}_{1,l}^{(i)}\mathbf{Q}^*\mathbf{x}_{k,1}^{(i)} + \mathbf{H}_{2,l}^{(i)}\mathbf{Q}^*\mathbf{x}_{k,2}^{(i)} \right) + \mathbf{n}_{k,l}$$
$$\mathbf{y}_{k+1,l} = \sum_{i=1}^{M} \left(-\mathbf{H}_{1,l}^{(i)}\mathbf{Q}^*\mathbf{x}_{k,2}^{*(i)} + \mathbf{H}_{2,l}^{(i)}\mathbf{Q}^*\mathbf{x}_{k,1}^{(i)} \right) + \mathbf{n}_{k+1,l} \quad (13)$$

Applying the DFT matrix $\mathbf{Q}$ to $\mathbf{y}_{k,l}$ and $\mathbf{y}_{k+1,l}$ in (5), we get a relation in terms of frequency–transformed variables:

$$\mathbf{Y}_{k,l} = \sum_{i=1}^{M} \left(\mathbf{\Lambda}_{1,l}^{(i)}\mathbf{x}_{k,1}^{(i)} + \mathbf{\Lambda}_{2,l}^{(i)}\mathbf{x}_{k,2}^{(i)} \right) + \mathbf{N}_{k,l}$$
$$\mathbf{Y}_{k+1,l} = \sum_{i=1}^{M} \left(\mathbf{\Lambda}_{1,l}^{(i)}\mathbf{x}_{k+1,1}^{(i)} + \mathbf{\Lambda}_{2,l}^{(i)}\mathbf{x}_{k+1,2}^{(i)} \right) + \mathbf{N}_{k+1,l} \quad (14)$$

By examining the structure of (14), we find that it is similar to (6). The only difference is that the DFTs of the data blocks have been replaced by the data blocks themselves. We then conclude that the expressions derived in Equations (8)–(10) are also valid for STBC–OFDM by replacing the FFT of the data blocks by the data blocks themselves. This means that for STBC–OFDM, $\mathcal{X}_k^{(i)}(m)$ is replaced by

$$\mathcal{X}_k^{(i)}(m) = \begin{pmatrix} \mathbf{x}_{k,1}^{(i)}(m) & \mathbf{x}_{k,2}^{(i)}(m) \\ -\mathbf{x}_{k,2}^{*(i)}(m) & \mathbf{x}_{k,1}^{*(i)}(m) \end{pmatrix} \quad (15)$$

We thus conclude that the transmissions of N–symbol blocks of SC FD–STBC and STBC–OFDM can be decomposed into N parallel symbol level transmissions with each one similar to transmitting Alamouti–STBC over flat fading channels. This decomposition allows us to decouple the frequency–selective channel estimation problem into N independent flat channel estimation problems. The unknown channel parameters are the different frequency bins of the FFT of the channel impulse response.

III. Adaptive Channel Estimation Schemes

The analysis in Section II shows that the channel estimation problem for flat fading or frequency–selective channels reduces to estimating the $2M \times M$ channel matrix $\mathcal{H}$ or $\mathbf{\Lambda}(m)$ from (3) or (10). Since the models (3) and (10) are similar, it suffices to explain how to estimate either (3) or (10); the same arguments would apply in the other case. In this section we develop an adaptive channel estimator and we show how the complexity of the receiver can be reduced by exploiting the code structure. We consider both block normalized least mean squares (NLMS) and block recursive least squares (RLS) receivers. Denote the coefficients of the adaptive channel estimator for $\mathcal{H}$ or $\mathbf{\Lambda}(m)$ at time k by

$$\mathcal{W}_k = \underbrace{\begin{pmatrix} \mathcal{W}_{k,1}^{(1)} & \cdots & \mathcal{W}_{k,M}^{(1)} \\ \vdots & \ddots & \vdots \\ \mathcal{W}_{k,1}^{(M)} & \cdots & \mathcal{W}_{k,M}^{(M)} \end{pmatrix}}_{2M \times M} \quad (16)$$

The channel estimation procedure can be summarized as follows:

- The received signals from all antennas are transformed to the frequency domain using DFT (ignore this step for the flat-fading case).
- The signals from all receive antennas are combined together to generate the desired response matrix $\mathcal{D}_{k+2}$ ($\mathcal{Y}_{k+2}(m)$ for the m–th frequency bin in the frequency–selective case and $\mathcal{Y}_{k+2}$ for the flat fading case).
- The input training sequence is used to construct $\mathcal{U}_{k+2}$ ($\mathcal{X}_{k+2}$ for the flat case and $\mathcal{X}_{k+2}(m)$ for the frequency–selective case).
- The difference between $\mathcal{Y}_{k+2}$ and the adaptive filter output is used as an error signal to update the filter coefficients.
- Repeat the previous steps for available training blocks or until convergence of the adaptive algorithm.

A. A Block NLMS Receiver

The matrix estimate $\mathcal{W}_k$ can be updated using the block NLMS algorithm as follows [7]:

$$\mathcal{W}_{k+2} = \mathcal{W}_k + \mu \left(\epsilon \mathbf{I}_{2M} + \mathcal{U}_{k+2}^* \mathcal{U}_{k+2} \right)^{-1} \mathcal{U}_{k+2}^* \times \left[\mathcal{D}_{k+2} - \mathcal{U}_{k+2}\mathcal{W}_k \right] \quad (17)$$

Although an inversion of a $2M \times 2M$ matrix is apparently needed at each iteration, we now show that it can be avoided as a result of the code structure. Using the matrix inversion lemma [7] we get:

$$\left(\epsilon \mathbf{I}_{2M} + \mathcal{U}_{k+2}^* \mathcal{U}_{k+2} \right)^{-1} = \epsilon^{-1}\mathbf{I}_{2M} - \epsilon^{-1}\mathcal{U}_{k+2}^* \times \left(\mathbf{I}_2 + \mathcal{U}_{k+2}\epsilon^{-1}\mathcal{U}_{k+2}^* \right)^{-1} \mathcal{U}_{k+2}\epsilon^{-1} \quad (18)$$

so that

$$\left(\epsilon \mathbf{I}_{2M} + \mathcal{U}_{k+2}^* \mathcal{U}_{k+2} \right)^{-1} \mathcal{U}_{k+2}^* = \mathcal{U}_{k+2}^* \left(\epsilon^{-1}\mathbf{I}_{2M} - \epsilon^{-2} \left(\mathbf{I}_2 + \epsilon^{-1}\mathcal{U}_{k+2}\mathcal{U}_{k+2}^* \right)^{-1} \mathcal{U}_{k+2}\mathcal{U}_{k+2}^* \right) \quad (19)$$

By inspecting the structure of $\mathcal{U}_{k+2}$, we find that it is a $2 \times 2M$ matrix with M (2×2) Alamouti blocks. That is,

$$\mathcal{U}_{k+2} = \left(\mathcal{U}_{k+2}^{(1)}, \cdots, \mathcal{U}_{k+2}^{(M)} \right), \quad \mathcal{U}_{k+2}^{(i)} = \begin{pmatrix} \mathcal{U}_{k+2,1}^{(i)} & \mathcal{U}_{k+2,2}^{(i)} \\ -\mathcal{U}_{k+2,2}^{*(i)} & \mathcal{U}_{k+2,1}^{*(i)} \end{pmatrix}$$

Then

$$\mathcal{U}_{k+2}\mathcal{U}_{k+2}^* = \sum_{i=1}^{M} \mathcal{U}_{k+2}^{(i)}\mathcal{U}_{k+2}^{*(i)} = \sum_{i=1}^{M} \left(|\mathcal{U}_{k+2,1}^{(i)}|^2 + |\mathcal{U}_{k+2,2}^{(i)}|^2 \right) \mathbf{I}_2 \triangleq \sigma_{k+2}\mathbf{I}_2$$

is a scaled multiple of the 2×2identity matrix $\mathbf{I}_2$. The NLMS update of (17) then reduces to an LMS update:

$$\mathcal{W}_{k+2} = \mathcal{W}_k + \frac{\mu}{\epsilon + \sigma_{k+2}} \mathcal{U}^*_{k+2} \left[\mathcal{D}_{k+2} - \mathcal{U}_{k+2}\mathcal{W}_k\right] \quad (20)$$

B. A Block RLS Receiver

The coefficient matrix $\mathcal{W}_k$ could also be updated using the block RLS algorithm [7] according to the following recursion:

$$\mathcal{W}_{k+2} = \mathcal{W}_k + \lambda^{-1}\mathcal{P}_k\mathcal{U}^*_{k+2}\mathbf{\Pi}_{k+2}\left[\mathcal{D}_{k+2} - \mathcal{U}_{k+2}\mathcal{W}_k\right] \quad (21)$$

where $\mathcal{P}_k$ is updated as follows:

$$\mathcal{P}_{k+2} = \lambda^{-1}\left[\mathcal{P}_k - \lambda^{-1}\mathcal{P}_k\mathcal{U}^*_{k+2}\mathbf{\Pi}^*_{k+2}\mathcal{U}_{k+2}\mathcal{P}^*_k\right] \quad (22)$$

and

$$\mathbf{\Pi}_{k+2} = \left(\mathbf{I}_2 + \lambda^{-1}\mathcal{U}_{k+2}\mathcal{P}_k\mathcal{U}^*_{k+2}\right)^{-1} \quad (23)$$

where $\mathcal{D}_k = \mathcal{Y}_k$ for the flat case and $\mathcal{D}_k = \mathcal{Y}_k(m)$ for the frequency–selective case. Similarly, $\mathcal{U}_k = \mathcal{X}_k$ for the flat case and $\mathcal{U}_k = \mathcal{X}_k(m)$ for the frequency–selective case. The quantities $\{\mathcal{W}_k, \mathcal{P}_k, \mathbf{\Pi}_k\}$ are updated over k. The initial conditions are $\mathcal{W}_0 = \mathbf{0}$ and $\mathcal{P}_0 = \delta\mathbf{I}_{2M}$, where δ is a large number. The following result is a consequence of the code structure.

Lemma 1: $\mathcal{P}_k$ has a Hermitian block structure with 2×2 subblocks where the off diagonal subblocks, $\mathbf{P}^{(i)}_{k,l}, i \neq l$, are 2×2 Alamouti matrices while the diagonal blocks are scaled multiples of $\mathbf{I}_2$. Moreover, $\mathbf{\Pi}_k(m)$ is also a scaled multiple of $\mathbf{I}_2$.

Proof: Let $\mathbf{A}$ and $\mathbf{B}$ be two Alamouti matrices with entries

$$\mathbf{A} = \begin{pmatrix} a_1 & a_2 \\ -a_2^* & a_1^* \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_1 & b_2 \\ -b_2^* & b_1^* \end{pmatrix}$$

Then it holds that

1) $\mathbf{A}+\mathbf{B}$, $\mathbf{A}-\mathbf{B}$, $\mathbf{AB}$, and $\mathbf{A}^{-1}$ are also Alamouti matrices.
2) $\mathbf{A}+\mathbf{A}^* = 2\text{Re}\{a_1\}\mathbf{I}_2$.
3) $\mathbf{AA}^* = \mathbf{A}^*\mathbf{A} = (|a_1|^2 + |a_2|^2)\mathbf{I}_2$.

Using these properties, we now verify that $\mathcal{P}_k$ and $\mathbf{\Pi}_k$ have the desired structures by induction. Without loss of generality, assume $M = 3$ and let $\mathcal{U}_{k+2} = \left(\mathcal{U}^{(1)}_{k+2}, \cdots, \mathcal{U}^{(M)}_{k+2}\right)$ with Alamouti subblocks $\mathcal{U}^{(i)}_k$,

$$\mathcal{U}^{(i)}_{k+2} = \begin{pmatrix} \mathcal{U}^{(i)}_{k+2,1} & \mathcal{U}^{(i)}_{k+2,2} \\ -\mathcal{U}^{*(i)}_{k+2,2} & \mathcal{U}^{*(i)}_{k+2,1} \end{pmatrix}$$

- At $k=0$, we have $\mathcal{P}_0 = \delta\mathbf{I}_{2M}$, which has the desired structure (with off diagonal subblocks = 0). Then

$$\mathcal{U}_2\mathcal{P}_0\mathcal{U}^*_2 = \delta\left(\mathcal{U}^{(1)}_2\mathcal{U}^{*(1)}_2 + \mathcal{U}^{(2)}_2\mathcal{U}^{*(2)}_2 + \mathcal{U}^{(3)}_2\mathcal{U}^{*(3)}_2\right) = \zeta_2\mathbf{I}_2$$

where $\zeta_2 = \delta\left(\sum_{i=1}^3(|\mathcal{U}^{(i)}_{2,1}|^2 + |\mathcal{U}^{(i)}_{2,2}|^2)\right)$ and $\mathbf{\Pi}_2 = (1+\zeta_2)^{-1}\mathbf{I}_2 = \alpha_2^{-1}\mathbf{I}_2$. Using this result, we evaluate $\mathcal{P}_2$ as

$$\begin{aligned}\mathcal{P}_2 &= \lambda^{-1}\mathcal{P}_0 - \lambda^{-2}\delta^2\alpha_2^{-1}\mathcal{U}^*_2\mathcal{U}_2 \\ &= \lambda^{-1}\delta\mathbf{I}_{2M} - \lambda^{-2}\delta^2\alpha_2^{-1} \\ &\quad\times\begin{pmatrix} \mathcal{U}^{*(1)}_2\mathcal{U}^{(1)}_2 & \mathcal{U}^{*(1)}_2\mathcal{U}^{(2)}_2 & \mathcal{U}^{*(1)}_2\mathcal{U}^{(3)}_2 \\ \mathcal{U}^{*(2)}_2\mathcal{U}^{(1)}_2 & \mathcal{U}^{*(2)}_2\mathcal{U}^{(2)}_2 & \mathcal{U}^{*(2)}_2\mathcal{U}^{(3)}_2 \\ \mathcal{U}^{*(3)}_2\mathcal{U}^{(1)}_2 & \mathcal{U}^{*(3)}_2\mathcal{U}^{(2)}_2 & \mathcal{U}^{*(3)}_2\mathcal{U}^{(3)}_2 \end{pmatrix}\end{aligned}$$

It is obvious that the diagonal blocks are multiples of $\mathbf{I}_2$ and the off diagonal blocks are Alamouti.

- At time k, assume that $\mathbf{\Pi}_k$ and $\mathcal{P}_k$ have the desired structures, i.e.,

$$\mathbf{\Pi}_k = \alpha_k^{-1}\mathbf{I}_2, \quad \mathcal{P}_k = \begin{pmatrix} \gamma_{k,1}\mathbf{I}_2 & \mathbf{P}^{(1)}_{k,2} & \mathbf{P}^{(1)}_{k,3} \\ \mathbf{P}^{*(1)}_{k,2} & \gamma_{k,2}\mathbf{I}_2 & \mathbf{P}^{(2)}_{k,3} \\ \mathbf{P}^{*(1)}_{k,3} & \mathbf{P}^{*(2)}_{k,3} & \gamma_{k,3}\mathbf{I}_2 \end{pmatrix}$$

where $\mathbf{P}^{(i)}_{k,l}$ are Alamouti. We need to show that these structures are preserved at $k+2$:

$$\begin{aligned}\mathcal{U}_{k+2}\mathcal{P}_k\mathcal{U}^*_{k+2} &= \sum_{i=1}^{3}\gamma_{k,i}\mathcal{U}^{(i)}_{k+2}\mathcal{U}^{*(i)}_{k+2} \\ &+ \sum_{i=2}^{3}\sum_{j=1}^{i-1}\left(\mathcal{U}^{(i)}_{k+2}\mathbf{P}^{(j)}_{k,i}\mathcal{U}^{*(j)}_{k+2} + \mathcal{U}^{*(j)}_{k+2}\mathbf{P}^{*(j)}_{k,i}\mathcal{U}^{(i)}_{k+2}\right) = \zeta_{k+2}\mathbf{I}_2\end{aligned}$$

Note that the first term is a multiple of $\mathbf{I}_2$. Moreover, the second term is the sum of an Alamouti matrix and its complex conjugate, which is also a multiple of $\mathbf{I}_2$. Therefore, $\mathbf{\Pi}_{k+2} = (1+\zeta_{k+2})^{-1}\mathbf{I}_2 = \alpha^{-1}_{k+2}\mathbf{I}_2$. Now let

$$\begin{aligned}\mathbf{\Phi}_{k+2} &= \mathcal{P}_k\mathcal{U}^*_{k+2} \\ &= \begin{pmatrix} \gamma_{k,1}\mathcal{U}^{*(1)}_{k+2} + \mathbf{P}^{(1)}_{k,2}\mathcal{U}^{*(2)}_{k+2} + \mathbf{P}^{(1)}_{k,3}\mathcal{U}^{*(3)}_{k+2} \\ \mathbf{P}^{*(1)}_{k,2}\mathcal{U}^{*(3)}_{k+2} + \gamma_{k,2}\mathcal{U}^{*(2)}_{k+2} + \mathbf{P}^{(2)}_{k,3}\mathcal{U}^{*(3)}_{k+2} \\ \mathbf{P}^{*(1)}_{k,3}\mathcal{U}^{*(1)}_{k+2} + \mathbf{P}^{*(2)}_{k,3}\mathcal{U}^{*(2)}_{k+2} + \gamma_{k,3}\mathcal{U}^{*(3)}_{k+2} \end{pmatrix} = \begin{pmatrix} \mathbf{\Phi}^{(1)}_{k+2} \\ \mathbf{\Phi}^{(2)}_{k+2} \\ \mathbf{\Phi}^{(3)}_{k+2} \end{pmatrix}\end{aligned}$$

where each $\mathbf{\Phi}^{(i)}_{k+2}$ is Alamouti. The update equation for $\mathcal{P}_{k+2}$ is

$$\begin{aligned}\mathcal{P}_{k+2} &= \lambda^{-1}\mathcal{P}_k - \lambda^{-2}\alpha^{-1}_{k+2}\mathbf{\Phi}_{k+2}\mathbf{\Phi}^*_{k+2} \\ &= \lambda^{-1}\mathcal{P}_k - \lambda^{-2}\alpha^{-1}_{k+2} \\ &\quad\times\begin{pmatrix} \mathbf{\Phi}^{(1)}_{k+2}\mathbf{\Phi}^{*(1)}_{k+2} & \mathbf{\Phi}^{(1)}_{k+2}\mathbf{\Phi}^{*(2)}_{k+2} & \mathbf{\Phi}^{(1)}_{k+2}\mathbf{\Phi}^{*(3)}_{k+2} \\ \mathbf{\Phi}^{(2)}_{k+2}\mathbf{\Phi}^{*(1)}_{k+2} & \mathbf{\Phi}^{(2)}_{k+2}\mathbf{\Phi}^{*(2)}_{k+2} & \mathbf{\Phi}^{(2)}_{k+2}\mathbf{\Phi}^{*(3)}_{k+2} \\ \mathbf{\Phi}^{(3)}_{k+2}\mathbf{\Phi}^{*(1)}_{k+2} & \mathbf{\Phi}^{(3)}_{k+2}\mathbf{\Phi}^{*(2)}_{k+2} & \mathbf{\Phi}^{(3)}_{k+2}\mathbf{\Phi}^{*(3)}_{k+2} \end{pmatrix}\end{aligned}$$

The product $\mathbf{\Phi}_{k+2}\mathbf{\Phi}^*_{k+2}$ has the same structure as $\mathcal{P}_k$. Therefore, $\mathcal{P}_{k+2}$ has the same structure as $\mathcal{P}_k$.

■

It follows from Lemma 1 that the structures of $\mathcal{P}_k$ and $\mathbf{\Pi}_k$ are as follows:

$$\mathcal{P}_k = \begin{pmatrix} \gamma_{k,1}\mathbf{I}_2 & \mathbf{P}^{(1)}_{k,2} & \cdots & \mathbf{P}^{(1)}_{k,M} \\ \mathbf{P}^{*(1)}_{k,2} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{P}^{(M-1)}_{k,M} \\ \mathbf{P}^{*(1)}_{k,M} & \cdots & \mathbf{P}^{*(M-1)}_{k,M} & \gamma_{k,M}\mathbf{I}_2 \end{pmatrix} \quad (24)$$

$$\mathbf{\Pi}_{k+2} \triangleq \alpha^{-1}_{k+2}\mathbf{I}_2 \quad (25)$$

where the $\mathbf{P}^{(i)}_{k,j}$ are 2×2 Alamouti matrices. Table I shows how we may exploit the STBC structure to update the entries of $\mathbf{\Pi}_k$ and $\mathcal{P}_k$. The reduction in computational complexity due to the STBC structure as a function of the number of users is illustrated in Fig. 4

IV. Simulation Results

We simulate three different scenarios; the flat fading case, the frequency selective case with SC-FD STBC, and the frequency selective case with STBC–OFDM. The system has M users, each equipped with two transmit antennas. The number of receive antennas is equal to the number of users. The channels from each transmit antenna to each receive antenna are assumed to be independent. The data bits of each user are mapped into an 8-PSK signal constellation. The processed symbols are transmitted at a symbol rate equal to 271 KSymbols/sec. The signal to noise ratios of all users at the receiver are assumed to be equal. For the flat fading scenario, we assume single tap independent channels between transmit and receive antennas and symbol level Alamouti STBC [2]. For the frequency selective one, a Typical Urban (TU) channel model with overall channel impulse response memory ν equals to 3 is considered for all channels. The transmitted symbols are grouped into blocks of 32 symbols. A cyclic prefix is added to each block by copying the first ν

TABLE I

BLOCK RLS CHANNEL ESTIMATION FOR STBC TRANSMISSIONS FOR BOTH FLAT FADING AND FREQUENCY–SELECTIVE CHANNELS ASSUMING M USERS.

Let the entries of the 2×2 matrices $\mathbf{P}_{k,l}^{(i)}$ and $\mathcal{X}_k^{(i)}$ be

$$\mathbf{P}_{k,l}^{(i)} = \begin{pmatrix} p_{k,l}^{(i)}(1) & p_{k,l}^{(i)}(2) \\ -p_{k,l}^{*(i)}(2) & p_{k,l}^{*(i)}(1) \end{pmatrix} \quad \mathcal{U}_{k+2}^{(i)} = \begin{pmatrix} \mathcal{U}_{k+2,1}^{(i)} & \mathcal{U}_{k+2,2}^{(i)} \\ -\mathcal{U}_{k+2,2}^{*(i)} & \mathcal{U}_{k+2,1}^{*(i)} \end{pmatrix}$$

Starting from $k = 0$, update the entries of $\mathbf{\Pi}_k$, $\mathcal{P}_k$, and $\mathcal{W}_k$ as follows:

1) Compute the entries of $\mathbf{\Pi}_{k+2}$, i.e., α_{k+2} as

$$\begin{aligned} \alpha_{k+2} &= 1 + \sum_{j=1}^{M} \gamma_{k,i} \left(|\mathcal{U}_{k,1}^{(i)}|^2 + |\mathcal{U}_{k,2}^{(i)}|^2 \right) \\ &\quad + 2\mathrm{Re} \left\{ \sum_{i=2}^{M} \sum_{j=1}^{i-1} \begin{pmatrix} \mathcal{U}_{k,1}^{(i)} & \mathcal{U}_{k,2}^{(i)} \end{pmatrix} \mathbf{P}_{k,i}^{(j)} \begin{pmatrix} \mathcal{U}_{k,1}^{*(j)} \\ \mathcal{U}_{k,2}^{*(j)} \end{pmatrix} \right\} \end{aligned}$$

2) Let $\Phi_{k+2} = \mathcal{P}_k \mathcal{U}_{k+2}^{*} \triangleq \begin{pmatrix} \Phi_{k+2}^{(1)T} & \cdots & \Phi_{k+2}^{(M)T} \end{pmatrix}^T$, where $(\cdot)^T$ indicates the matrix transposition. Let the entries of the 2×2 subblocks $\Phi_{k+2}^{(i)}$ be

$$\Phi_{k+2}^{(i)} = \begin{pmatrix} \Phi_{k+2,1}^{(i)} & \Phi_{k+2,2}^{(i)} \\ -\Phi_{k+2,2}^{*(i)} & \Phi_{k+2,1}^{*(i)} \end{pmatrix}$$

Then $\Phi_{k+2,1}^{(i)}$ and $\Phi_{k+2,2}^{(i)}$, $i = 1, \cdots, M$, are given by

$$\begin{aligned} \Phi_{k+2,1}^{(i)} &= \gamma_{k,i} \mathcal{U}_{k+2,1}^{(i)} + \sum_{j=1,\ j\neq i}^{M} p_{k,j}^{(i)}(1) \mathcal{U}_{k+2,1}^{*(j)} + p_{k,j}^{(i)}(2) \mathcal{U}_{k+2,2}^{*(j)} \\ \Phi_{k+2,2}^{(i)} &= \gamma_{k,i} \mathcal{U}_{k+2,2}^{(i)} - \sum_{j=1,\ j\neq i}^{M} p_{k,j}^{(i)}(1) \mathcal{U}_{k+2,1}^{(j)} - p_{k,j}^{(i)}(2) \mathcal{U}_{k+2,2}^{(j)} \end{aligned}$$

3) Compute the entries of $\mathbf{P}_{k+2,l}^{(i)}$, $l, i = 1, \cdots, M$, as follows

$$p_{k+2,l}^{(i)}(1) = \begin{cases} \lambda^{-1}\gamma_{k,i} - \frac{\lambda^{-2}}{\alpha_{k+2}} \left(|\Phi_{k+2,1}^{(i)}|^2 + |\Phi_{k+2,2}^{(i)}|^2 \right) & i = l \\ \lambda^{-1} p_{k,l}^{(i)}(1) - \frac{\lambda^{-2}}{\alpha_{k+2}} & \\ \quad \cdot \left(\Phi_{k+2,1}^{(i)} \Phi_{k+2,1}^{*(l)} + \Phi_{k+2,2}^{(i)} \Phi_{k+2,2}^{*(l)} \right) & i \neq l \end{cases}$$

$$p_{k+2,l}^{(i)}(2) = \begin{cases} 0 & i = l \\ \lambda^{-1} p_{k,l}^{(i)}(2) - \frac{\lambda^{-2}}{\alpha_{k+2}} & \\ \quad \cdot \left(\Phi_{k+2,2}^{(i)} \Phi_{k+2,2}^{(l)} - \Phi_{k+2,1}^{(i)} \Phi_{k+2,1}^{(l)} \right) & i \neq l \end{cases}$$

4) Repeat the previous steps for each iteration over k.
5) Repeat the previous procedure for $m = 0, \cdots, N-1$

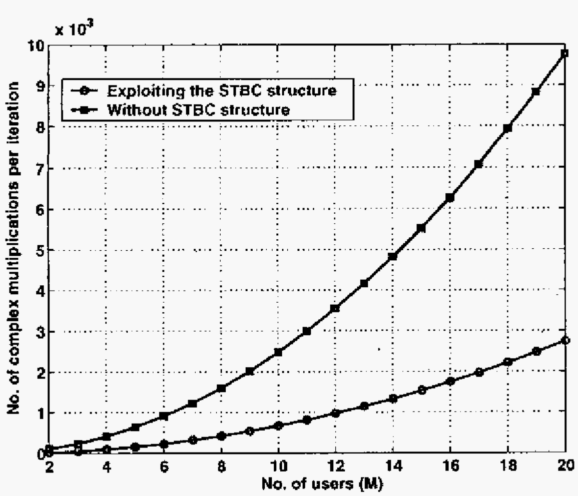

Fig. 4. Number of complex multiplications per iteration needed for updating $\mathcal{P}_k$ in the block RLS receiver.

symbols after the last symbol of the same block. The MSE results are obtained by averaging over 1000 different channel realizations. Fig. 5 shows the learning curves of block NLMS and RLS for different number of users using STBC–OFDM.

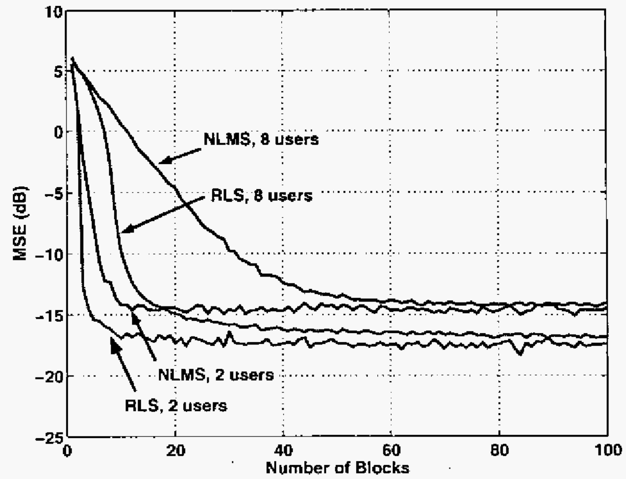

Fig. 5. MSE performance of the adaptive channel estimator.

V. CONCLUSIONS

In this paper, we developed adaptive channel estimation techniques for STBC transmissions over flat-fading and frequency selective fading channels. We first presented a unified framework for Alamouti STBC over flat fading channels and SC-FD STBC and STBC–OFDM over frequency–selective channels. Then we showed that the channel estimation for SC-FD STBC and STBC–OFDM reduces to the problem of estimating N parallel flat fading channels where N is the block size. We also showed how to exploit the STBC structure to reduce the complexity of both block NLMS and RLS receivers.

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