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Robust Multipath Resolving in Fading Conditions for Mobile-Positioning Systems

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Abstract

Infrastructure-based mobile-positioning is receiving increasing attention in the field of wireless communications. This is is due to a recent order issued by the Federal Communications Commission (FCC) that mandates all wireless service providers to locate an emergency 911 caller within a high accuracy. Overlapping multipath propagation is one of the main sources of mobile-positioning errors, especially in fast channel fading situations. In this paper we present a mathematical framework for overlapping multipath propagation over fading channels, which leads to developing a block least squares method for overlapping multipath resolving.

1 Introduction

Mobile-positioning is an essential feature of future cellular systems [1]. In infrastructure-based mobile-positioning systems, the accurate estimation of the time and amplitude of arrival of the first arriving ray at the receiver(s) is vital (see, e.g, [2]-[5]). Such estimates are used to obtain an estimate of the distance between the transmitter and the receiver(s). However, wireless propagation usually suffers from severe multipath conditions. In many of these cases, the prompt ray is succeeded by a multipath component that arrives at the receiver(s) within a short delay. If this delay is smaller than the duration of the pulse-shape used in the wireless system (the chip duration T_c in CDMA systems), these two rays overlap causing significant errors in the prompt ray time and amplitude of arrival estimation (see, e.g., [5]).

Several earlier works in the literature have addressed the problem of resolving overlaped multipath components by using constrained least-squares methods (see, e.g., [6, 7]). However, these methods lack the required robustness in fast fading environments. This is due to the use of a conventional matched filtering stage that preceeds the application of such algorithms.

In this paper we present a framework for overlaped multipath propagation over fading channels. This framework leads to a block least squares method for multipath resolving in such situations.

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2 Problem Formulation

Consider L_r measurements of the form

$$r(n) = c(n) \star p(n) \star h(n) + v(n), \tag{1}$$

where $\{c(n)\}_{n=1}^{K}$ is a known binary sequence, $\{p(n)\}_{n=1}^{P}$ is a known pulse shape impulse response sequence, $v(n)$ is additive white Gaussian noise of variance σ_v^2, and $h(n)$ is a multipath channel that has the model

$$h(n) = \sum_{l=1}^{L} \alpha_l x_l(n) \delta(n - \tau_l^o), \tag{2}$$

where α_l, $\{x_l(n)\}$, and τ_l^o are respectively the unknown gain, the normalized amplitude sequence, and the time of arrival of the l^{th} multipath component (ray).

In order to obtain a more compact representation of the received signal $\{r(n)\}$, we use the following assumption:

A.1 The variations in the channel amplitudes $\{x_l(n)\}$ within the duration of the pulse-shaping waveform, PT_s, are negligible. Here T_s denotes the sampling period.

This assumption is feasible for cellular systems even for fast fading channels. Using assumption A.1 and (2), we can rewrite (1) in matrix form as

$$\boldsymbol{r} = \boldsymbol{A}\boldsymbol{C}_x\boldsymbol{h} + \boldsymbol{v},$$

where $\boldsymbol{C}_x$ is a prewindowed Toeplitz matrix, whose first column is given by

$$\text{col}[x_1(1) \cdot c(1) \ \ 0 \ \ldots \ 0 \ \ x_1(2) \cdot c(2) \ \ 0 \ \ldots \ x_1(K) \cdot c(K) \ 0 \ \ldots \ 0],$$

$\boldsymbol{h}$ is defined by

$$\boldsymbol{h} \triangleq \text{col}[A_1, A_2, ..., A_L],$$

$\boldsymbol{r}$ and $\boldsymbol{v}$ are the received signal and noise sample vectors, respectively, and $\boldsymbol{A}$ is a pulse-shaping waveform convolution matrix whose columns are shifted versions of the pulse-shape sample vector. Resolving multipath components refers to the problem of estimating the nonzero elements of the unknown gain vector $\boldsymbol{h}$. An optimal estimate for $\boldsymbol{h}$ in the least-squares sense can be found by solving a least-squares problem of the form

$$\hat{\boldsymbol{h}} = \arg\min_{\boldsymbol{h}} \|\boldsymbol{r} - \boldsymbol{A}\boldsymbol{C}_x\boldsymbol{h}\|^2.$$

Minimizing this cost function leads to

$$\hat{\boldsymbol{h}} = (\boldsymbol{C}_x^* \boldsymbol{A}^* \boldsymbol{A} \boldsymbol{C}_x)^{-1} \boldsymbol{C}_x^* \boldsymbol{A}^* \boldsymbol{r}.$$

This optimal solution requires complete knowledge of the channel gain sequences $\{x_l(n)\}$, which are typically unknown. Such channel gains are usually considered constant for online bit decoding applications, as they do not have significant variation during the relatively short bit estimation period. Such an assumption cannot be made for wireless location applications since the estimation period is typically in the order of a fraction of a second.

3 Conventional Matched Filtering

Now we investigate the use of a conventional matched filtering stage (despreading in the case of CDMA) along with a least-squares technique. The output of a conventional despreading operation is given by $\frac{1}{K}\boldsymbol{C}^*\boldsymbol{r}$, where the matrix $\boldsymbol{C}$ is a Toepliz spreading code matrix $\boldsymbol{C}_m$, whose first column is given by

$$\text{col}[c(1)\ 0 \ldots 0\ c(2)\ 0 \ldots c(K)\ 0 \ldots 0]\ .$$

When K is sufficiently large, it can be shown that the following identity holds:

$$\frac{1}{K}\boldsymbol{C}^*\boldsymbol{A}\boldsymbol{C}_x\boldsymbol{h} = \boldsymbol{A}_L\boldsymbol{X}_K\boldsymbol{h}\ , \tag{3}$$

where $\boldsymbol{A}_L$ is a $(L+P-1)\times L$ pulse-shaping convolution matrix and $\boldsymbol{X}_K$ is an $L\times L$ square diagonal matrix defined by

$$\boldsymbol{X}_K \triangleq \ \tfrac{1}{K}\ \text{diag}\left[\textstyle\sum_{n=1}^{K} x_1(n), ..., \sum_{n=1}^{K} x_L(n)\right]. \tag{4}$$

Taking the limit as $K \to \infty$ of both sides of (4), we obtain

$$\lim_{K\to\infty} \boldsymbol{X}_K = \text{E}\ \ \text{diag}\left[x_1(n), x_2(n), ..., x_L(n)\right].$$

Thus, unless the channel gains have static components, $\lim_{K\to\infty} \boldsymbol{X}_K = \boldsymbol{0}$. This causes the output of the averaging process given in (3) to approach zero as $K \to \infty$, i.e.,

$$\lim_{K\to\infty} \frac{1}{K}\boldsymbol{C}^*\boldsymbol{A}\boldsymbol{C}_x\boldsymbol{h} = \boldsymbol{0}\ , \tag{5}$$

which makes all estimation techniques based on a simple infinite correlation process (*matched filtering*) *unrobust* when used to estimate fading channels. This fact prohibits obtaining an accurate location estimate using these techniques. After understanding the effects of channel fading on multipath resolving, we now introduce an estimation technique for $\boldsymbol{h}$ that does not require knowledge of the $\{x_l(n)\}$ and that is *robust* to fast channel fading. The algorithm is based on exploiting the following two properties of fading channels

$$\lim_{K\to\infty} \frac{1}{K}\sum_{k=1}^{K} x_l(k) = 0, \quad \lim_{K\to\infty} \frac{1}{K}\sum_{k=1}^{K} |x_l(k)|^2 = 1.$$

4 The Proposed Algorithm

The proposed algorithm is summarized as follows:

1. The received signal vector is divided into M smaller vectors with NN_u samples in each vector, $N_u = T_c/T_s$, such that the m^{th} vector is given by

$$\boldsymbol{r}_m = \text{col}\left[\boldsymbol{r}\left((m-1)NN_u + 1\right), ..., \boldsymbol{r}\left(mNN_u\right)\right].$$

2. Using a Toepliz despreading matrix $\boldsymbol{C}_m$, whose first column is given by

$$\mathrm{col}[c(n_o)\ 0\ \ldots\ 0\ c(n_o+1)\ 0\ \ldots\ c(mN)\ 0\ \ldots\ 0]$$

with $n_o = (m-1)N+1$, each vector is multiplied from the left by $\frac{1}{N}\boldsymbol{C}_m^*$, with $m = 1, 2, \ldots M$. The despread output of this operation is denoted by

$$\boldsymbol{y}_m = \frac{1}{N}\boldsymbol{C}_m^*\boldsymbol{r}_m.$$

When N is large enough, and using (3), $\boldsymbol{y}_m$ can be approximated by

$$\boldsymbol{y}_m \approx \boldsymbol{A}_L\boldsymbol{X}_m\boldsymbol{h} + \frac{1}{N}\boldsymbol{C}_m^*\boldsymbol{v}_m, \tag{6}$$

where $\boldsymbol{v}_m$ is defined by

$$\boldsymbol{v}_m \triangleq \mathrm{col}\left[\boldsymbol{v}\left((m-1)NN_u+1\right), \ldots, \boldsymbol{v}\left(mNN_u\right)\right],$$

and $\boldsymbol{X}_m$ is defined by

$$\boldsymbol{X}_m \triangleq \mathrm{diag}[a_1\ a_2\ \ldots\ a_L],$$

with

$$a_i \triangleq \sum_{n=n_o}^{mN} x_i(n).$$

3. The despread vector $\boldsymbol{y}_m$ is used to obtain an estimate for $\boldsymbol{X}_m\boldsymbol{h}$. The optimum estimate of $\boldsymbol{X}_m\boldsymbol{h}$ given $\boldsymbol{y}_m$ in the least-squares sense is thus given, from (6), by

$$\hat{\boldsymbol{h}}_m = \arg\min_{\boldsymbol{X}_m\boldsymbol{h}} \|\boldsymbol{y}_m - \boldsymbol{A}_L\boldsymbol{X}_m\boldsymbol{h}\|^2\ ,$$

which yields

$$\hat{\boldsymbol{h}}_m = \left(\boldsymbol{A}_L^*\boldsymbol{A}_L\right)^{-1}\boldsymbol{A}_L^*\boldsymbol{y}_m.$$

4. The absolute values of the elements of the sequence $\{\hat{\boldsymbol{h}}_m\}$ are obtained, squared, and averaged to obtain a rough estimate for the vector $|\boldsymbol{h}|^2$ (whose enteries are $|h(i)|^2$). This sample average is given by

$$\widehat{|\boldsymbol{h}_s|}^2 = \frac{1}{M}\sum_{m=1}^{M}\left|\frac{1}{N}\left(\boldsymbol{A}_L^*\boldsymbol{A}_L\right)^{-1}\boldsymbol{A}_L^*\boldsymbol{C}_m^*\boldsymbol{r}_m\right|^2\ . \tag{7}$$

5 Parameter Optimization and Bias Equalization

We will consider the case of an infinite received sequence length ($L_r \to \infty$). Thus, expression (7) becomes

$$\widehat{|h_s|}^2 = \lim_{M\to\infty} \frac{1}{M} \sum_{m=1}^{M} \left| \frac{1}{N} \left(A_L^* A_L\right)^{-1} A_L^* C_m^* r_m \right|^2 .$$

As $M \to \infty$, the averaging process is equal to the expectation operation. Thus,

$$\widehat{|h_s|}^2 = \mathrm{E} \left| \frac{1}{N} \left(A_L^* A_L\right)^{-1} A_L^* C_m^* r_m \right|^2 .$$

Using (6), one obtains

$$\widehat{|h_s|}^2 = \mathrm{E} \left| X_m h + \frac{1}{N} \left(A_L^* A_L\right)^{-1} A_L^* C_m^* v_m \right|^2 .$$

We can rewrite the above expression as

$$\widehat{|h_s|}^2 = \mathrm{E} \left| X_m h + \frac{1}{N} A_L^{\dagger} v'_m \right|^2 , \tag{8}$$

where

$$v'_m = C_m^* v_m \ ,$$

and the pseudo-inverse matrix $A_L^{\dagger}$ is given by

$$A_L^{\dagger} = \left(A_L^* A_L\right)^{-1} A_L^* \ .$$

For mathematical tractability of the analysis, we impose the following assumptions:

A.2 The sequence $\{v'(n)\}$ is identically statistically independent (i.i.d), and is independent of each of the fading channel gain sequences $\{x_l^o(n)\}$.

A.3 The fading channel gain sequences $\{x_l^o(n)\}$ are statistically independent of each other.

Both of these assumptions are typical in the context of wireless channel modeling [8]. Using (8), the elements of the vector $\hat{h}_s$ are individually given by

$$\widehat{|h_s|}^2(l) = \mathrm{E} \left| \alpha_l \sum_{n=1}^{N} (x_l(n)) + \frac{1}{N} \sum_{i=1}^{L} \left(A_L^{\dagger}(l,i) v'(i) \right) \right|^2 .$$

Using A.2 and A.3, it is straightforward to show that

$$\widehat{|h_s|}^2(l) = B_f(l) \alpha_l^2 + B_n(l), \tag{9}$$

where $B_f(l)$ and $B_n(l)$ are respectively defined by

$$B_f(l) = \frac{R_{x_l}(0)}{N} + \sum_{i=1}^{N-1} \frac{2(N-i)R_{x_l}(i)}{N^2},$$

and

$$B_n(l) = \frac{\sigma_v^2}{N} \sum_{i=1}^{L} A_L^{\dagger\,2}(l,i),$$

where $R_{x_l}(i)$ is the autocorrelation function of each of the fading channel complex gain sequences,

$$R_{x_l}(|i|) = \mathrm{E}\, x_l^o(n) x_l^{o*}(n-i).$$

Expression (9) shows that each of the fading channel multipath elements suffers from a multiplicative fading bias $B_f(l)$, which results from the coherent averaging process and an additive noise bias $B_n(l)$ that arises from the non-coherent averaging operation. We will now consider the case of identical autocorrelation functions for all the channel rays ($R_x(i)$). In such a case, the SNR gain at the output of each of the channel rays estimators ($S_G(l)$) is given by

$$S_G(l) = \frac{1}{\sum_{i=1}^{L} A_L^{\dagger\,2}(l,i)} \left(R_x(0) + \sum_{i=1}^{N-1} \frac{2(N-i)R_x(i)}{N} \right).$$

The optimal value of the coherent averaging period (N_{opt}) is obtained by maximizing the SNR gain with respect to N. Thus, N_{opt} is computed by solving the following equation

$$\sum_{i=1}^{N_{opt}-1} i R_x(i) = 0. \tag{10}$$

Therefore, if the channel multipath components have identical fading autocorrelation functions, the coherent integration interval N *should be adapted* according to (10). The more general case will be addressed in future work. Two sets of correction factors are needed to equalize for the noise and fading biases of each channel path. Both of the biases should be estimated and used to correct $\widehat{|h_s|}^2$ to get accurate estimates of the channel gains ($\hat{\alpha}_l$) which are finally given by

$$\hat{\alpha}_l = \sqrt{C_f(l) \left(\widehat{|h_s|}^2(l) - C_n(l) \right)}\ ,$$

where $C_n(l)$ and $C_f(l)$ are the two needed correction factors that are given by

$$C_n(l) = \hat{B}_n(l),$$

and

$$C_f(l) = \frac{1}{\hat{B}_f(l)},$$

where $\hat{B}_n(l)$ and $\hat{B}_f(l)$ are estimates for $B_n(l)$ and $B_f(l)$, respectively. Figure 1 shows a complete multipath searcher.

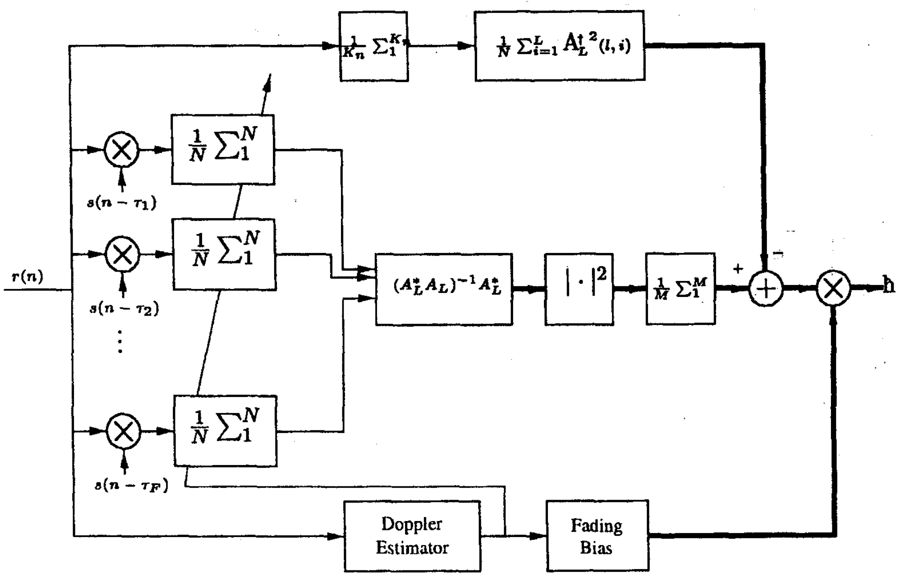

Figure 1: *A least-squares multipath searcher.*

6 Simulation Example

The robustness of the proposed algorithm in overlapped multipath components of a Rayleigh fading channel is tested by simulations. The static gains of the used channel are shown in Figure 2(a). The channel consists of two Rayleigh fading rays with a maximum Doppler frequency of 10 Hz. An IS-95 pulse-shaped CDMA signal is transmitted over this channel. The signal-to-noise ratio at the output of the channel is -10 dB. The delay between the two rays corresponds to $T_c/4$. Figure 2(b) shows the output of a conventional matched filtering stage followed by a conventional least-squares deconvolution stage. It is clear that the amplitude of the signal at the output of such a procedure is significantly degraded leading to significant errors in the estimation of the time and amplitude of arrival of the first arriving ray. Figure 2(c) shows the output of the proposed estimation scheme. It is clear that the proposed algorithm possesses remarkable robustness to channel fading. Here, we may add that the simple least-squares operation used in the proposed scheme can be replaced by a more advanced constrained least-squares operation that exploits more knowledge of the channel (see, e.g., [6, 7]). Such methods would enhance the estimation accuracy and avoid ill-conditioning in the deconvolution matrix.

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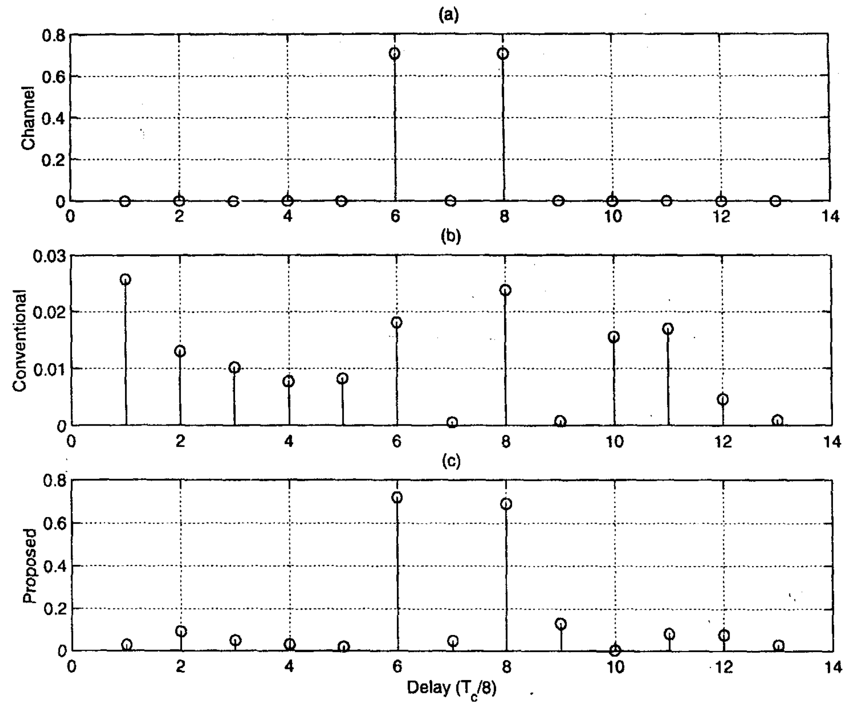

Figure 2: *Simulation results.*

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