Proc. MTNS, Leuven, Belgium, July 2004.

A Power and Rate Control Algorithm for Wireless Networks with State-Delayed Dynamics

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Abstract—**A robust algorithm is developed for jointly controlling the power and rate of flow in a distributed wireless network. The dynamics of the network is modelled as a discrete-time state-delayed system with uncertainties and the proposed algorithm achieves better SIR performance than a conventional power control scheme.**

keywords: Robust estimation, congestion control, power control, rate control, wireless network, convex optimization.

I. Introduction

Power consumption is a key limiting factor in the performance of wireless networks. This limitation is further compounded by the fact that nodes in a network need to cater to desired data rates, which in turn require the SNR level, and consequently the power level, to be above certain values. As such, a fundamental tradeoff exists between power levels, data rates, and congestion rates in a network. There have been a handful of power control algorithms that have been investigated in the literature [1]–[4]. Most of the available solutions do not combine in a cohesive manner the requirements of power, data rate, and congestion. For this reason, such solutions may not perform well when the rates in a network need to vary due to the use of rate adaptation or congestion control algorithms. In recent work [6], the authors proposed algorithms that allow for the *joint* control of rate and power in a network. However, these algorithms do not account for feedback delays that arise from round trip time propagation in the network. In this paper, we show how to develop algorithms for the more demanding situation when there are delayed measurements in the network. From a system-theoretic perspective, the problem requires that we now deal with state-delayed models. As a result of the analysis, we will end up with a joint rate and power control algorithm that minimizes a bound on the error variance between the desired and actual signal-to-interference ratios (SIR).

Notation. For a column vector z, we write $\|z\|^2$ to denote its Euclidean norm.

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This work was partially supported by NSF grants CCR-0208573 and ECS-0401188.

II. Power and Rate Control Strategy

Consider a wireless network with nodes organized into local clusters or cells with one node acting as the master node in each cell. Any node that wishes to communicate is allowed to do so only with the master node and using a time slot. Nodes communicating during the same time-slot in other cells cause interference in this cell. Figure 1 shows a schematic representation with three cells, three master nodes, and active and interfering nodes.

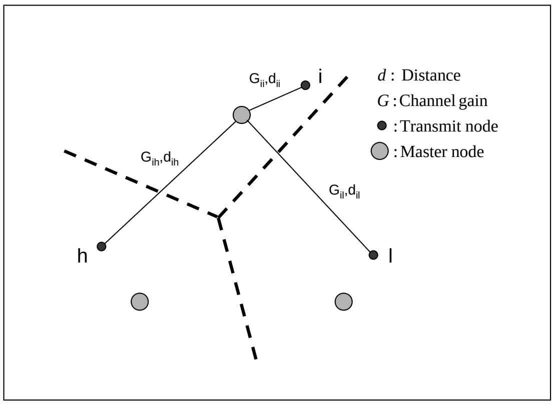

Fig. 1. *A schematic representation with three cells, three master nodes, and active and interfering nodes. The active node is node i and the interfering nodes are nodes h and l.*

The Signal-to-Interference-plus-Noise-Ratio (SIR) for node i at time k on an uplink channel is defined by

$$\gamma_i(k) = \frac{G_{ii}(k)p_i(k)}{\sum_{j\in\mathcal{A}} G_{ij}(k)p_j(k) + \sigma^2} \tag{1}$$

where G_{ij} is the channel gain from the j-th node to the intended master node of the i-th cell, p_i is the transmitted power from the i-th node, σ^2 is the power of the white Gaussian noise at the receiver of the master node, and $\mathcal{A}$ is the set of nodes that are interfering with node i.

Let $f_i(k)$ denote the flow rate at node i at time k. We shall assume that each node in the network employs the following flow-rate control algorithm:

$$f_i(k+1) = f_i(k) + \mu[d(k) - c_1(k)f_i(k) - c_2(k)f_i(k-\tau)] \tag{2}$$

where $\mu > 0$ is a step-size parameter, and $c_1(k)$ and $c_2(k)$ are measures of the amount of congestion in the network. Moreover, $d(k)$ controls the rate increase per iteration and

τ is non-zero for any controller that incorporates round trip time. Now, in view of Shannon's capacity formula, the flow rate $f_i(k)$ demands an SIR level $\gamma_i'(k)$ that is given by

$$f_i(k) = \frac{1}{2}\log_2[1+\gamma_i'(k)] \tag{3}$$

Usually, during normal network operation, $\gamma_i'(k) \gg 1$ and, hence, $f_i(k)$ in (3) is proportional to $\log\gamma_i'(k)$. Substituting this fact into (2) we find that the desired SIR, in dB scale, should vary according to the rule

$$\bar{\gamma}_i'(k+1) = [1-\mu c_1(k)]\bar{\gamma}_i'(k) - \mu c_2(k)\bar{\gamma}_i'(k-\tau) + \mu' d(k) \tag{4}$$

where $\mu' = 20\mu/\log_2(10)$ and $\bar{\gamma}_i(k) = 10\log\gamma_i(k)$.

We shall initially assume that each node in the network adjusts its power according to the power control algorithm:

$$\bar{p}_i(k+1) = \bar{p}_i(k) + \alpha_i[\bar{\gamma}_i'(k) - \bar{\gamma}_i(k)] \tag{5}$$

where α_i is a step-size parameter that is allowed to vary from one node to another, and $\gamma_i(k)$ is the actual SIR that is achieved by $p_i(k)$ as given by (1). Now let

$$\beta_i(k) = \frac{G_{ii}(k)}{\sum_{j\in\mathcal{A}} G_{ij}(k)p_j(k) \; + \; \sigma^2}$$

denote the scaling factor that determines how $p_i(k)$ affects the achieved $\gamma_i(k)$ in (1), i.e.,

$$\gamma_i(k) = \beta_i(k)p_i(k)$$

or, equivalently, in dB scale,

$$\bar{\gamma}_i(k) = \bar{\beta}_i(k) \; + \; \bar{p}_i(k) \tag{6}$$

We refer to $\bar{\beta}_i(k)$ as the effective channel gain. We shall model the time variation in $\bar{\beta}_i(k)$ according to the random walk model

$$\bar{\beta}_i(k+1) = \bar{\beta}_i(k) \; + \; n_i(k)$$

where $n_i(k)$ is a zero-mean disturbance of variance σ_n^2 and is independent of $\bar{p}_i(k)$. Substituting this model for $\bar{\beta}_i(k)$ into (6), and using (5), we find that the achieved $\bar{\gamma}_i(k)$ varies according to the rule:

$$\bar{\gamma}_i(k+1) = (1-\alpha_i)\bar{\gamma}_i(k) + \alpha_i\bar{\gamma}_i'(k) + n_i(k) \tag{7}$$

Our objective is to design the power control sequence $\{p_i(k)\}$ such that the actual SIR levels $\{\bar{\gamma}_i(k)\}$ from (7) will tend to the desired SIR levels $\{\bar{\gamma}_i'(k)\}$ from (4). To do so, we shall formulate a robust quadratic control problem as follows. First, we drop the node index i for simplicity of notation (it is to be understood that the resulting control mechanism is implemented at each node). Second, we introduce the two-dimensional state vector:

$$x_k = \begin{bmatrix} \bar{\gamma}_i(k) \\ \bar{\gamma}_i'(k) \end{bmatrix}$$

Then combining (4) and (7) we arrive at the state-space model:

$$\begin{aligned} x_{k+1} \;\; = \;\; & \begin{bmatrix} 1-\alpha & \alpha \\ 0 & 1-\mu c_1(k) \end{bmatrix} x_k \\ & + \begin{bmatrix} 0 & 0 \\ 0 & -\mu c_2(k) \end{bmatrix} x_{k-\tau} + \begin{bmatrix} n(k) \\ \mu' d(k) \end{bmatrix} \end{aligned}$$

or, more compactly,

$$x_{k+1} = A_k x_k + A_{d,k}x_{k-\tau} + w_k \tag{8}$$

where the 2×2 coefficient matrices A_k and $A_{d,k}$ are given by

$$A_k = \begin{bmatrix} 1-\alpha & \alpha \\ 0 & 1-\mu c_1(k) \end{bmatrix}, \; A_{d,k} = \begin{bmatrix} 0 & 0 \\ 0 & -\mu c_2(k) \end{bmatrix} \tag{9}$$

and where w_k is a 2×1 zero-mean random vector with covariance matrix

$$Q = Ew_kw_k^T = \begin{bmatrix} \sigma_n^2 & \\ & \mu'^2\sigma_d^2 \end{bmatrix} \le \rho_u I \tag{10}$$

assumed bounded for some known $\rho_u > 0$ assumed bounded. In order to drive $\gamma_i(k)$ towards $\gamma_i'(k)$ we shall employ a control sequence u_k in (8) as follows:

$$\boxed{x_{k+1} = A_k x_k + A_{d,k}x_{k-\tau} + Bu_k + w_k} \tag{11}$$

for some given 2×2 matrix B and 2×1 control sequence u_k. For example, let

$$Bu_k = \begin{bmatrix} u_p(k) \\ u_f(k) \end{bmatrix}$$

denote the individual entries of Bu_k to be designed. Then the inclusion of the term Bu_k in (11) amounts to adding the control signal $u_p(k)$ to the power update (5), and the control signal $u_f(k)$ to the desired SIR update (4). We shall also assume that we have access to output measurements that are related to the state vector as follows:

$$\boxed{y_k = Cx_k + v_k} \tag{12}$$

for some known matrix C and where v_k denotes measurement noise with bounded covariance matrix R,

$$\boxed{R = Ev_kv_k^T \; \le \rho_v I}$$

for some known $\rho_v > 0$. Usually, $C = I$ so that the entries of y_k correspond to noisy measurements of the actual and desired SIR levels $\{\bar{\gamma}(k), \bar{\gamma}'(k)\}$. We now propose a design procedure that takes into account uncertainties that arise due to the lack of perfect knowledge about the network dynamics. For example, the congestion control functions $c_1(k)$ and $c_2(k)$ are usually not known exactly and have to be estimated; the estimation process introduces errors in the assumed state-space model. Let us model the uncertainty in $c_1(k)$ as

$$c_1(k) = \bar{c}_1(k) + g\delta(k)\bar{d} \tag{13}$$

where $\delta(k)$ is a zero mean random noise with variance σ_δ^2, g and $\bar{d}$ are known scalars, and $\bar{c}_1(k)$ is unknown but bounded as

$$c_{1,l} \leq \bar{c}_1(k) \leq c_{1,u} \tag{14}$$

for some known positive scalars $\{c_{1,l}, c_{1,u}\}$. In other words, we allow for both deterministic and stochastic uncertainties in $c_1(k)$. In this way, the matrices A_k themselves are not known exactly but they are modelled as $A_k = \bar{A}_k + \delta A_k$ where

$$\boxed{\bar{A}_k = \begin{bmatrix} 1-\alpha & \alpha \\ 0 & 1-\mu\bar{c}(k) \end{bmatrix}} \tag{15}$$

and

$$\delta A_k = g\delta(k)D \tag{16}$$

where

$$D = \begin{pmatrix} 0 & 0 \\ 0 & -\mu\bar{d} \end{pmatrix} \tag{17}$$

Likewise, let $c_2(k)$ be bounded as $c_{2,l} \leq c_2(k) \leq c_{2,u}$. In this way, the matrices $A_{d,k}$ are also not known exactly but they are now modelled as belonging to a convex polytope. We shall design the control sequence $\{u_k\}$ as follows. First, we use the robust algorithm of [7] to estimate the state of perturbed state-space models as in (15)–(17). Then, the control sequence $\{u_k\}$ will be designed such that an upper bound on the following stochastic quadratic cost function is minimized:

$$\mathcal{J} = E\left\{\sum_{k=0}^{\infty} \|Lx_k\|^2\right\}$$

with $L = [\ 1 \quad -1\ ]$, and where E denotes the expectation operator. This choice of L results in

$$Lx_k = \bar{\gamma}(k) - \bar{\gamma}'(k)$$

so that $\|Lx_k\|^2$ is a measure of the difference between $\{\bar{\gamma}(k), \bar{\gamma}'(k)\}$. The resulting control will guarantee the following performance over all models $\{\bar{A}_k + \delta A_k\}$. Let $\tilde{x}_k = x_k - \hat{x}_k$ denote the state estimation error. Then the construction will determine state estimates $\{\hat{x}_k\}$, and a control sequence $\{u_k\}$ as a function of these state estimates, such that an upper bound on $E\|Lx_k\|^2$ and hence $\mathcal{J}$ is minimized. Specifically, it will hold that

$$\mathcal{J} < \nu^2 E\left\{\sum_{k=0}^{\infty}\left(\|w_k\|^2 + \|v_k\|^2\right)\right\} + b \tag{18}$$

for some constant $b > 0$ and for the smallest possible ν^2, and over all zero-mean noise sequences $\{w_k, v_k\}$ satisfying

$$E\left(\sum_{k=0}^{\infty}\|w_k\|^2\right) < \infty, \quad E\left(\sum_{k=0}^{\infty}\|v_k\|^2\right) < \infty$$

The following statement is specialized to $B = I$.

A Robust Power and Rate Control Algorithm. Let

$$A_l = \begin{bmatrix} 1-\alpha & \alpha \\ 0 & 1-\mu c_{1,l} \end{bmatrix}, \ A_u = \begin{bmatrix} 1-\alpha & \alpha \\ 0 & 1-\mu c_{1,u} \end{bmatrix}$$

and

$$A_{d,l} = \begin{bmatrix} 0 & 0 \\ 0 & 1-\mu c_{2,l} \end{bmatrix}, \quad A_{d,u} = \begin{bmatrix} 0 & 0 \\ 0 & 1-\mu c_{2,u} \end{bmatrix}$$

Given a 1×2 vector L, the following is a robust joint power and rate-flow control strategy:

1. Introduce a 2×2 matrix A_f and a 2×1 vector B_f to be determined. Let

$$\bar{A}_l = \begin{bmatrix} A_l & 0 \\ A_l - A_f - B_f C & A_f \end{bmatrix}, \ \bar{A}_u = \begin{bmatrix} A_u & 0 \\ A_u - A_f - B_f C & A_f \end{bmatrix}$$

and define

$$\bar{A}_{d,l} = \begin{bmatrix} A_{d,l} & 0 \\ A_{d,l} & 0 \end{bmatrix}, \quad \bar{A}_{d,u} = \begin{bmatrix} A_{d,u} & 0 \\ A_{d,u} & 0 \end{bmatrix}$$

The quantities A_f and B_f are determined in the following manner [7]. Given a scalar $0 < \alpha < 1$, solve the following convex optimization problem over the variables $\{P = \text{diag}\{P_1, P_2\}, R, A_f, B_f\}$:

$$\min \ \text{Tr}(\rho_u(P_1 + P_2) + \rho_v B_f^T P_2 B_f) \tag{19}$$

subject to the conditions

$$\left(\begin{array}{cc|cc} \bar{H} & -\bar{A}_m^T P \bar{A}_{d,m} & \bar{A}_m^T P & 0 \\ -\bar{A}_{d,m}^T P \bar{A}_m & R - \bar{A}_{d,m}^T P \bar{A}_{d,m} & 0 & 0 \\ \hline P\bar{A}_m & 0 & P & 0 \\ 0 & 0 & 0 & I \end{array}\right) > \alpha I \tag{20}$$

for $m = l, u$, and with $P > I$, $R > I$,

$$\bar{H} \ \triangleq \ P - R - \sigma_\delta^2 \bar{D}^T \bar{G}^T P \bar{G} \bar{D}$$

and

$$\bar{G} = g, \quad \bar{D} = \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix} \tag{21}$$

With A_f and B_f found in this fashion, we minimize a bound on the state error variance in the absence of control [7]. In addition, the construction below ensures asymptotic stability in the presence of a control signal (as shown in Appendix B).

2. Using the just found $\{A_f, B_f\}$, define

$$\check{A}_1 = \begin{bmatrix} A_l - K_c & K_c \\ A_l - A_f - B_f C & A_f \end{bmatrix}$$

$$\check{A}_2 = \begin{bmatrix} A_u - K_c & K_c \\ A_u - A_f - B_f C & A_f \end{bmatrix}$$

$$\check{B} = \begin{bmatrix} I & 0 \\ I & -B_f \end{bmatrix}$$

for some 2×2 matrix K_c to be determined. Determine K_c, X, Y, and the smallest positive ν^2 that guarantee

$$\begin{pmatrix} \check{H}_m & \check{A}_m^T X \bar{A}_{d,m} & -\check{A}_m^T X \check{B} \\ -\bar{A}_{d,m}^T X \check{A}_m & Y - \bar{A}_{d,m}^T X \bar{A}_{d,m} & -\bar{A}_{d,m}^T X \check{B} \\ -\check{B}^T X \check{A}_m & -\check{B}^T X \bar{A}_{d,m} & \nu^2 I - \check{B}^T X \check{B} \end{pmatrix} > 0 \tag{22}$$

where

$$\check{H}_m = X - Y - \check{A}_m^T X \check{A}_m - \check{L}^T \check{L} - \sigma_\delta^2 \bar{D}^T \bar{G}^T X \bar{G} \bar{D}$$

for $m = l, u$ and

$$\check{L} = [\ L \quad 0\ ]$$

Then set

$$\begin{aligned} u_k &= -K_c \hat{x}_k \\ \hat{x}_{k+1} &= A_f \hat{x}_k \ + \ B_f y_k \ + \ u_k \end{aligned}$$

3. Partition u_k as

$$u_k = \begin{bmatrix} u_p(k) \\ u_f(k) \end{bmatrix}$$

and update the rate flow and the power at the relevant node as follows. Let $\kappa = (\log_2(10))/20$. Then

$$\begin{aligned} \bar{\gamma}'(k) &= f_i(k)/\kappa \\ \bar{p}_i(k+1) &= \bar{p}_i(k) + \alpha_i[\bar{\gamma}_i'(k) - \bar{\gamma}_i(k)] \ + \ u_p(k) \\ f_i(k+1) &= f_i(k) + \mu[d(k) - c_1(k) f_i(k) - c_2(k) f_i(k-\tau)] \\ & \quad + \kappa u_f(k) \end{aligned}$$

◊

III. Simulations

To illustrate the performance of the algorithm, we simulate the following model from [8]. The space is divided into virtual geographical cells, each containing many nodes with one node acting as a master node. A frequency slot is allocated to each node that wishes to communicate with the master node in a cell. We allow for frequency reuse across cells in a manner similar to that in mobile cellular systems. The nodes are made to take turns as master nodes with equal probability, but with the constraint that there can be only one master node in a cell at any time. Priority to act as a master node is given to the node that has lowest interference from other cells. The nodes communicating in the same frequency slot in other cells cause interference with this cell and this interference is measured in terms of the signal-to-interference ratio (SIR). The channel gain G_{ii} is assumed to have a lognormal distribution, i.e.,

$$G_{ii} = S_0 d_{ii}^{-\beta} 10^{\alpha/10} \qquad (23)$$

where S_0 is a function of the carrier frequency, β is the path loss exponent (PLE), and d_{ii} is the distance of the master node from the node. The value of β depends on the physical environment and varies between 2 and 6 (usually 4), while α is a zero mean Gaussian random variable with variance σ_α^2, which usually ranges between 6 and 12. Data is transmitted from a source to its final destination through intermediary master nodes. Fig. 2 illustrates the performance of the proposed robust power and rate algorithm in comparison to the following power algorithm from [3]:

$$\bar{p}_i(k+1) = \bar{p}_i(k) + \alpha[\bar{\gamma}_i'(k) - \bar{\gamma}_i(k)] \qquad (24)$$

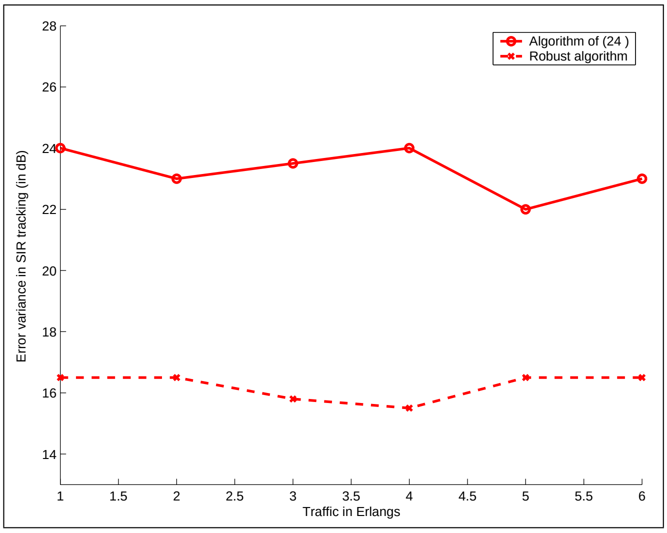

Fig. 2. *Variance in SIR tracking.*

Appendix A: Robust Performance

In this appendix, we show that the proposed algorithm is stable and ensures a robust performance level of ν^2, as in (18). Define

$$o_k \triangleq \begin{pmatrix} w_k \\ v_k \end{pmatrix} \qquad (25)$$

and

$$\phi(k) = [\ \eta_k^T \quad \eta_{k-1}^T \quad \dots \quad \eta_{k-\tau}^T\ ]^T$$

Let

$$V(\phi_k) = \eta_k^T X \eta_k + \sum_{i=k-\tau}^{k-1} \eta_i^T Y \eta_i \qquad (26)$$

for some $X > 0$ and $Y > 0$ to be determined in order to satisfy the inequality

$$EV(\phi_{k+1}) - EV(\phi_k) - \nu^2 E(\|w_k\|^2 + \|v_k\|^2) + E\|\tilde{z}_k\|^2 < 0 \qquad (27)$$

where $\tilde{z}_k = \check{L}\eta_k = \bar{\gamma}(k) - \bar{\gamma}'(k)$. We will show that, for a given A_f and B_f, if X and Y are determined such that the above inequality is satisfied, then (18) is guaranteed. Indeed, if we sum inequality (27) over k, and if we use the fact that the system is asymptotically stable (which is shown in the next appendix), we would get

$$E\left\{\sum_{k=0}^{\infty} |\bar{\gamma}(k) - \bar{\gamma}'(k)|^2\right\} < EV(\eta_0) + \nu^2 E\left\{\sum_{k=0}^{\infty} \|w_k\|^2 + \|v_k\|^2\right\} \qquad (28)$$

as desired. Now assume a control structure of the form

$$\hat{x}_{k+1} = A_f \hat{x}_k + B_f y_k + u_k, \quad u_k = -K_c \hat{x}_k \qquad (29)$$

for some given $\{A_f, B_f\}$ and unknown K_c. Combining this equation with

$$\begin{aligned} x_{k+1} &= (\bar{A} + \delta A_k) x_k + A_d x_{k-\tau} + u_k + w_k \\ y_k &= C x_k + v_k \end{aligned}$$

and assuming, for example, that $\check{A}$ is equal to one of the boundary points, say $\check{A}_1$, and that $\bar{A}_d$ is equal to one of the boundary points, say $\bar{A}_{d,l}$, we find that η_k satisfies the state-space model:

$$\eta_{k+1} = (\check{A}_1 + \delta\check{A}_k)\eta_k + \bar{A}_{d,1}\eta_{k-\tau} + \check{B}o_k \tag{30}$$

where

$$\check{A}_1 = \begin{pmatrix} A_l - K_c & K_c \\ A_l - A_f - B_f C & A_f \end{pmatrix}, \quad \check{B} = \begin{pmatrix} I & 0 \\ I & -B_f \end{pmatrix} \tag{31}$$

Likewise, for the boundary point $\check{A}_2$. Using (30) and expanding (27) gives

$$\begin{aligned} E\{&\eta_k^T \check{A}^T X \check{A}\eta_k - \eta_k^T X \eta_k + \sigma_\delta^2 \eta_k^T \bar{G}^T \bar{D}^T X \bar{D}\bar{G}\eta_k \\ &+ \eta_k^T \check{A}^T X \check{B} o_k + o_k^T \check{B}^T X \check{A}\eta_k + \eta_k^T Y \eta_k \\ &+ \eta_k^T \check{A}^T X \bar{A}_d \eta_{k-\tau} + \eta_{k-\tau}^T \bar{A}_d{}^T X \check{A}\eta_k \\ &+ \eta_{k-\tau}^T \bar{A}_d{}^T X \bar{A}_d \eta_{k-\tau} - \eta_{k-\tau}^T Y \eta_{k-\tau} \\ &+ \eta_{k-\tau}^T \bar{A}_d{}^T X \check{B} o_k + o_k^T \check{B}^T X \bar{A}_d \eta_{k-\tau} \\ &- \nu^2 o_k^T o_k + o_k^T \check{B}^T X \check{B} o_k + \eta_k^T \check{L}^T \check{L}\eta_k\} < 0 \end{aligned} \tag{32}$$

With $\check{A}$ taking values between $\check{A}_1$ and $\check{A}_2$, condition (32) is satisfied if we require

$$\begin{pmatrix} \check{H}_m & -\check{A}_m^T X \bar{A}_{d,m} & -\check{A}_m^T X \check{B} \\ -\bar{A}_{d,m}^T X \check{A}_m & Y - \bar{A}_{d,m}^T X \bar{A}_{d,m} & -\bar{A}_{d,m}^T X \check{B} \\ -\check{B}^T X \check{A}_m & -\check{B}^T X \bar{A}_{d,m} & \nu^2 I - \check{B}^T X \check{B} \end{pmatrix} > 0 \tag{33}$$

for $m = l, u$ and for some K_c, ν^2, $X > 0$ and $Y > 0$, as desired. Inequality (33) also implies that the system is asymptotically stable as we show next.

Appendix B: Stability of the Network

Let J denote the set of sources (or equivalently nodes) in the network. Let r denote a route. Without loss of generality, we ignore routing choices and identify each source with a route. We consider first a single route and a single source scenario. Consider a particular source, and a route r adopted by the source. The rate catered to by the route is proportionally given in terms of $\gamma'(k)$ (recall that the rate f_i is proportional to $\bar{\gamma}'(k)$). Now the stability of η_k implies the stability of $\bar{\gamma}'(k) = [0\ 1\ 0\ 0]\eta_k$. Hence we will derive conditions for the stability of η_k, which will imply stability of $\gamma'(k)$ and hence that of route r. Consider again equation (30) in the absence of noises with $\check{A}$ taking values in the polytope with vertices $\check{A}_1$ and $\check{A}_2$, and $\bar{A}_d$ taking values in the polytope with vertices $\bar{A}_{d,l}$ and $\bar{A}_{d,u}$:

$$\eta_{k+1} = (\check{A} + \delta\check{A}_k)\eta_k + \bar{A}_d \eta_{k-\tau} \tag{34}$$

where

$$\check{A} = \begin{pmatrix} A_m - K_c & K_c \\ A_m - A_f - B_f C & A_f \end{pmatrix} \tag{35}$$

and A_m takes values in the polytope with vertices A_l and A_u. Now condition [10]

$$E[V(\phi_{k+1})|\phi_k, ..., \phi_0] - V(\phi_k) < 0 \tag{36}$$

is satisfied if

$$\begin{aligned} \{&\eta_k^T \check{A}^T X \check{A}\eta_k - \eta_k^T X \eta_k + \sigma_\delta^2 \eta_k^T \bar{G}^T \bar{D}^T X \bar{D}\bar{G}\eta_k \\ &+ \eta_k^T Y \eta_k + \eta_k^T \check{A}^T X \bar{A}_d \eta_{k-\tau} + \eta_{k-\tau}^T \bar{A}_d{}^T X \check{A}\eta_k \\ &+ \eta_{k-\tau}^T \bar{A}_d{}^T X \bar{A}_d \eta_{k-\tau} - \eta_{k-\tau}^T Y \eta_{k-\tau}\} < 0 \end{aligned}$$

with $\check{A}$ taking values in the polytope with vertices $\check{A}_1$ and $\check{A}_2$. Condition (36) is therefore satisfied if

$$\begin{pmatrix} \tilde{H}_m & -\check{A}_m^T X \bar{A}_{d,m} \\ -\bar{A}_{d,m}^T X \check{A}_m & Y - \bar{A}_{d,m}^T X \bar{A}_{d,m} \end{pmatrix} > 0, \quad m = l, u \tag{37}$$

where

$$\tilde{H}_m = X - Y - \check{A}_m^T X \check{A}_m - \sigma_\delta^2 \bar{G}^T \bar{D}^T X \bar{D}\bar{G}, \quad m = l, u$$

for some K_c, $X > 0$ and $Y > 0$. But it can be seen that K_c, $X > 0$ and $Y > 0$ satisfying (33) also satisfy (37). Condition (36) implies the asymptotic stability of the process $\{\eta_k\}$ and, hence, the stability of route r is guaranteed.

For the scenario of multiple sources, when m sources use a route r, each of the sources being stable will contribute to a bounded arrival of packets in the route r ensuring network stability.

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