TRACKING OF LINEAR TIME-VARIANT SYSTEMS

S. HAYKIN[1], A. H. SAYED[2], J. ZEIDLER[3], P. YEE[1], *and* P. WEI[3]

[1]McMaster University, Canada [2]University of California, Santa Barbara, USA [3]University of California, San Diego, USA

ABSTRACT

In this paper we exploit the one-to-one correspondences between the recursive least-squares (RLS) and Kalman variables to formulate extended forms of the RLS algorithm. Two particular forms are considered, one pertaining to a system identification problem and the other to the tracking of a chirped sinusoid in additive noise. For both applications, experiments are presented that demonstrate the tracking optimality of the extended RLS algorithms, compared with the standard RLS and least-mean-squares (LMS) algorithms.

1. Introduction

The LMS algorithm [1,2] and the RLS algorithm [2] have established themselves as the principal tools for linear adaptive filtering. The convergence behaviors of both of these algorithms are now well understood. Typically, the RLS algorithm has a faster rate of convergence than the LMS algorithm, and is less sensitive to variations in the eigenvalue spread of the correlation matrix of the input vector. However, when operating in a nonstationary environment, the adaptive filter has the additional task of tracking the statistical variations in environmental conditions. In this context, it is well recognized that the convergence behavior of an adaptive filter is a transient phenomenon whereas its tracking behavior is a steady-state phenomenon. In general, a good convergence behavior does not necessarily mean a good tracking behavior.

In recent years, much has been written on a comparative evaluation of the tracking behaviors of the LMS and RLS algorithms [3-6]. The general conclusion drawn from the studies reported in the literature to date is that, typically, the LMS algorithm exhibits a better tracking behavior than the RLS algorithm. This conclusion should not be surprising since the LMS algorithm is model independent, whereas the RLS algorithm is model dependent. Unless the multiparameter regression model assumed in the derivation of the standard RLS algorithm closely matches the underlying model of the environment in which it operates, we would expect a degradation in the tracking performance of the RLS algorithm due to a model mismatch.

In a recent paper, Sayed and Kailath [7] delineated the relationship between the RLS algorithm and the Kalman filter in precise terms. Although work on this relationship may be traced back to the seminal paper by Godard [8], and subsequently elaborated on by many other investigators, the exact nature of the relationship was put on a firm footing for the first time in [7]. Thus recognizing that the RLS algorithm is a special case of the Kalman filter, and recognizing that the Kalman filter is the optimum linear tracking device on the basis of second-order statistics, how then is it that the exponentially weighted RLS algorithm has *not* inherited the good tracking behavior of the Kalman filter? The answer to this fundamental question lies in the fact that in formulating the standard form of the RLS algorithm by incorporating an exponential weighting factor into the cost function, the transition matrix of the RLS algorithm (using the language of Kalman filter theory) is in reality a constant, which is clearly not the way to solve the tracking problem for a nonstationary environment.

The purpose of this paper is two-fold. First, we describe two different methods for the design of an RLS-type algorithm so as to cope with corresponding forms of nonstationary environmental conditions. Second, we present computer experiments, one on system identification assuming a first-order Markov model and the other on tracking a chirped sinusoid in additive noise. The experiments demonstrate the tracking superiority of the extended RLS algorithm(s) over the LMS algorithm, when the right model for the RLS algorithm is chosen to suit the particular problem of interest.

0-7803-2489-7/95 $4.00 © 1995 IEEE

2. The Standard RLS Algorithm and Extensions

According to Sayed and Kailath [7], a state-space model for the exponentially weighted RLS algorithm may be described as follows:

$$\begin{aligned} \mathbf{x}(n+1) &= \lambda^{-1/2}\mathbf{x}(n), \\ y(n) &= \mathbf{u}^H(n)\mathbf{x}(n) + v(n), \end{aligned} \qquad (1)$$

where $\mathbf{x}(n)$ is the state vector of the model at time (iteration) n, $y(n)$ is the observation (reference) signal, $\mathbf{u}(n)$ is the input signal vector, $v(n)$ is the measurement noise modeled as white noise with unit variance, and λ is a forgetting factor. Two observations from the state equation are immediately apparent for the standard RLS algorithm:

- The transition matrix is a constant multiple of the identity matrix, namely $\lambda^{-1/2}\mathbf{I}$.
- The process (state) noise vector is zero.

Now, both of these conditions are synonymous with a stationary environment. Thus, although it is widely believed in the literature that by introducing the forgetting factor λ into the design of the RLS algorithm the algorithm is enabled to track statistical variations of the environment, in reality this is not exactly so. It is therefore not surprising that the RLS algorithm, in its standard form, does not always measure up to the LMS algorithm when it comes to tracking considerations.

Kalman filter theory tells us that a more general form of the state-space model of the RLS algorithm should be as follows:

$$\begin{aligned} \mathbf{x}(n+1) &= \mathbf{F}(n+1,n)\mathbf{x}(n) + \mathbf{r}(n), \\ y(n) &= \mathbf{u}^H(n)\mathbf{x}(n) + v(n). \end{aligned} \qquad (2)$$

The measurement equation (i.e., second line of (2)) is the same as before. However, the state equation (i.e., first line of (2)) differs from that of (1) in two aspects:

- The transition matrix $\mathbf{F}(n+1,n)$ is time varying.
- The process (state) noise vector $\mathbf{r}(n)$ is nonzero.

This, therefore, points to two special ways in which the RLS algorithm may be modified in order to cope with different nonstationary environments, as explained in the next two sections; in one case we assume $\mathbf{F}(n+1,n)$ is *known* and present the proper extension of the RLS solution (refered to here as ERLS-1); this algorithm is applicable to a system identification problem assuming a Markov model. In the other case, we assume $\mathbf{F}(n+1,n)$ is *not known* and proceed to suggest a second extension of the RLS solution by invoking connections with extended Kalman filtering (the extension is refered to as ERLS-2); this second algorithm is applicable to tracking a chirped sinusoid in additive noise.

The fundamental point to stress here is that in both cases, prior knowledge about the original dynamical system model is explicitly built into the formulation of the extended forms of the RLS algorithm, thereby improving the tracking performance of the resulting adaptive filter.

3. A System Identification Problem

Consider a linear time-variant system described by a first-order Markov model. Specifically, we have the following pair of equations as the system description:

$$\begin{aligned} \mathbf{w}_o(n+1) &= \mathbf{F}(n+1,n)\mathbf{w}_o(n) + \mathbf{r}(n), \\ y(n) &= \mathbf{u}^H(n)\mathbf{w}_o(n) + v(n), \end{aligned} \qquad (3)$$

where $\mathbf{F}(n+1,n)$ is a known transition matrix, $\mathbf{w}_o(n)$ is the optimum tap-weight vector of the model at time n, $y(n)$ is the desired response, and $\mathbf{u}(n)$ is the input vector. A special case of interest is when $\mathbf{F}(n+1,n) = a\mathbf{I}$, where a is a positive constant assumed to be less than one to assure the stability of the model. Furthermore, the following assumptions are made:

- $\{\mathbf{r}(n)\}$ is a zero-mean white noise sequence with covariance matrix $\mathbf{Q}(n)$.
- $\{v(n)\}$ is a zero-mean white noise sequence with variance $\sigma^2(n)$.
- The initial state-vector $\mathbf{w}_o(0)$ is random with mean $\bar{\mathbf{w}}_o$ and covariance matrix Π_0.
- The random variables $\{\mathbf{r}(n), v(n), (\mathbf{w}_o(0) - \bar{\mathbf{w}}_o)\}$ are uncorrelated.

Then, building on the classical Kalman filter theory and exploiting the one-to-one correspondences that exist between Kalman variables and RLS variables [7], the RLS algorithm appropriate for the task at hand is the so-called ERLS-1 solution [9]:

Algorithm 1 (Extended RLS Solution I) *The estimates of the weight-vector $\mathbf{w}_o(n)$ in (3), computed in the process of solving the optimization criterion given above, can be recursively evaluated as follows: Start with $\hat{\mathbf{w}}(0|-1) = \bar{\mathbf{w}}_o$, $\mathbf{P}(0,-1) = \Pi_0$, and repeat for $n \geq 0$:*

$$\begin{aligned} \mathbf{k}(n) &= \mathbf{F}(n+1,n)\mathbf{P}(n,n-1)\mathbf{u}(n) \\ &\quad \cdot \left[\mathbf{u}^H(n)\mathbf{P}(n,n-1)\mathbf{u}(n) + \sigma^2(n)\right]^{-1}, \\ \xi(n) &= y(n) - \mathbf{u}^H(n)\hat{\mathbf{w}}(n|n-1), \end{aligned}$$

$$\begin{aligned}\hat{\mathbf{w}}(n+1|n) &= \mathbf{F}(n+1,n)\hat{\mathbf{w}}(n|n-1)+\mathbf{k}(n)\xi(n),\\ \mathbf{P}(n) &= \mathbf{P}(n,n-1)-\frac{\mathbf{P}(n,n-1)\mathbf{u}(n)\mathbf{u}^H(n)\mathbf{P}(n,n-1)}{\mathbf{u}^H(n)\mathbf{P}(n,n-1)\mathbf{u}(n)\ +\ \sigma^2(n)},\\ \mathbf{P}(n+1,n) &= \mathbf{F}(n+1,n)\mathbf{P}(n)\mathbf{F}^H(n+1,n)+\mathbf{Q}(n),\end{aligned}$$

where $\mathbf{k}(n)$ *is the gain vector,* $\xi(n)$ *is the apriori estimation error, and* $\hat{\mathbf{w}}(n|n-1)$ *is the estimate of the unknown* $\mathbf{w}_o(n)$ *given the input data up to time* $(n-1)$.

In the Kalman filtering context, the matrix $\mathbf{P}(n,n-1)$ is the covariance matrix of the predicted weight-error vector,

$$\tilde{\mathbf{w}}(n,n-1)=\mathbf{w}_o(n)-\hat{\mathbf{w}}(n|n-1).$$

4. Tracking of a Chirped Sinusoid in Noise

For our second example of a nonstationary environment, we consider the tracking of a chirped sinusoid in additive noise. The state-space model of interest in this case takes the following form

$$\begin{aligned}\mathbf{w}_o(n+1) &= \mathbf{F}(\psi)\mathbf{w}_o(n),\\ y(n) &= \mathbf{u}^H(n)\mathbf{w}_o(n)\ +\ v(n),\end{aligned}\qquad (4)$$

$\mathbf{F}(\psi)$ is an *unknown* diagonal matrix that is parameterized fully in terms of a single *unknown* parameter ψ. This parameter is related to the linear shift of the center frequency in the chirped signal and the dependence of $\mathbf{F}$ on it is as follows:

$$\mathbf{F}(\psi)\triangleq \text{diag}\left[e^{j\psi},\ e^{j2\psi},\ldots,e^{jM\psi}\right]$$

Here, M is the size of the tap-weight vector. If the parameter ψ were known, then $\mathbf{F}(\psi)$ will be a known transition matrix and a standard least-squares (RLS) problem results, the solution of which can be written down as a special case of the standard Kalman filter recursions.

Both $\mathbf{w}_o(n)$ and ψ are unknowns that we wish to estimate. Ideally, we may want to determine these estimates so as to meet the optimality criterion

$$\min_{\{\mathbf{w}_o(0),\psi\}}\left[\begin{array}{cc}\mathbf{w}_o(0)-\bar{\mathbf{w}}_o & \psi-\bar{\psi}\end{array}\right]^H\left[\begin{array}{cc}\Pi_0^{-1} & 0\\ 0 & \pi_0^{-1}\end{array}\right]\cdot\left[\begin{array}{c}\mathbf{w}_o(0)-\bar{\mathbf{w}}_o\\ \psi-\bar{\psi}\end{array}\right]+\sum_{n=0}^{N}\frac{|y(n)-\mathbf{u}(n)\mathbf{w}_o(n)|^2}{\sigma^2(n)},$$

subject to $\mathbf{w}_o(n+1)=\mathbf{F}(\psi)\mathbf{w}_o(n)$. For this purpose, we collect the unknowns into an extended (state) vector:

$$\mathbf{x}(n)\triangleq\left[\begin{array}{c}\mathbf{w}_o(n)\\ \psi\end{array}\right],$$

and note that it satisfies the *nonlinear* (state-space) model:

$$\begin{aligned}\mathbf{x}(n+1) &= \left[\begin{array}{c}\mathbf{F}(\psi)\mathbf{w}_o(n)\\ \psi\end{array}\right]=\left[\begin{array}{cc}\mathbf{F}(\psi) & 0\\ \mathbf{0} & 1\end{array}\right]\mathbf{x}(n)\\ &\triangleq f(\mathbf{x}(n)),\\ y(n) &= \left[\begin{array}{cc}\mathbf{u}^H(n) & 0\end{array}\right]\mathbf{x}(n)+v(n),\end{aligned}\qquad (5)$$

where $f(\cdot)$ is a nonlinear functional of the state-vector. Note also that the last entry ψ of the state vector does not change with time. Following a procedure similar to that used to derive the extended Kalman filter, we may derive the ERLS-2 algorithm as summarized here [9].

Algorithm 2 (Extended RLS Solution II) *The estimates of the weight-vector in (4) can be recursively evaluated as follows: start with* $\hat{\mathbf{w}}(0|-1)=\bar{\mathbf{w}}_o$, $\hat{\psi}_{|-1}=\bar{\psi}$,

$$\mathbf{P}(0,-1)=\left[\begin{array}{cc}\Pi_0 & 0\\ 0 & \pi_0\end{array}\right],$$

and repeat for $n\geq 0$:

$$\begin{aligned}\mathbf{k}(n) &= \mathbf{P}(n,n-1)\left[\begin{array}{c}\mathbf{u}(n)\\ 0\end{array}\right]\left(\left[\begin{array}{c}\mathbf{u}(n)\\ 0\end{array}\right]^H\mathbf{P}(n,n-1)\cdot\left[\begin{array}{c}\mathbf{u}(n)\\ 0\end{array}\right]+\sigma^2(n)\right)^{-1},\\ \xi(n) &= y(n)-\mathbf{u}^H(n)\hat{\mathbf{w}}(n|n-1),\\ \left[\begin{array}{c}\hat{\mathbf{w}}(n|n)\\ \hat{\psi}_{|n}\end{array}\right] &= \left[\begin{array}{c}\hat{\mathbf{w}}(n|n-1)\\ \hat{\psi}_{|n-1}\end{array}\right]+\mathbf{k}(n)\xi(n),\\ \hat{\mathbf{w}}(n+1|n) &= \mathbf{F}(\hat{\psi}_{|n})\hat{\mathbf{w}}(n|n),\\ \mathbf{P}(n+1,n) &= \mathbf{F}(n+1,n)\mathbf{P}(n,n)\mathbf{F}^H(n+1,n),\\ \mathbf{P}(n,n) &= \left(\mathbf{I}-\mathbf{k}(n)\left[\begin{array}{cc}\mathbf{u}^H(n) & 0\end{array}\right]\right)\mathbf{P}(n,n-1).\end{aligned}$$

5. Computer Experiments

5.1. System Identification

In this experiment, we consider the system identification of a simplified version of the first-order Markov model described in (3), viz.,

$$\begin{aligned}\mathbf{w}_o(n+1) &= a\mathbf{w}_o(n)+\mathbf{r}(n),\\ y(n) &= \mathbf{u}^H(n)\mathbf{w}_o(n)+v(n),\quad n\geq 0,\end{aligned}\qquad (6)$$

where, for all m and n,

$$\bar{\mathbf{w}}_o(0)=\mathbf{0},\ \mathbf{u}(n)\sim N(\mathbf{0},\mathbf{R}),\ \mathbf{r}(n)\sim N(\mathbf{0},\mathbf{Q}),\ v(n)\sim N(0,\sigma^2),$$

$$\text{E}\left[\mathbf{u}(m)\mathbf{r}^H(n)\right]=\text{E}\left[\mathbf{u}(m)v^*(n)\right]=\text{E}\left[\mathbf{r}(m)v^*(n)\right]=\mathbf{0},$$

and all vectors are $M-$dimensional. We restrict our attention to the case $M=2$ when

$$\mathbf{Q}=\sigma_Q^2\left[\begin{array}{cc}1 & q_1\\ q_1 & q_2\end{array}\right],\quad |q_1|\leq 1,\ q_2>q_1^2,$$

for the two specific cases (1): $\mathbf{R}^{-1}=c\mathbf{Q}$ and (2): $\mathbf{R}=c\mathbf{Q}$, both for $c>0$. Our choices of the basic parameters for the experiments that follow are given in Table 1. Letting D represent the *mean-square deviation* and M the *relative mean-square misadjustment*, it can be shown [2,5] that for each case, the results of Table 2 hold.

σ_Q	q_1	q_2	σ	a	c in case 1	c in case 2
0.01	−0.75	1	0.2	0.9998	6.250×10^4	3.657×10^3

Table 1: *Basic parameters for experiments.*

	Case 1: $\mathbf{R}^{-1} = c\mathbf{Q}$	Case 2: $\mathbf{R} = c\mathbf{Q}$
$\frac{D_{\min}^{\mathrm{RLS}}}{D_{\min}^{\mathrm{LMS}}}$	$\frac{1+q_2}{\sqrt{2(1+2q_1^2+q_2^2)}}$	$\frac{1+q_2}{2\sqrt{q_2-q_1^2}}$
$\frac{M_{\min}^{\mathrm{RLS}}}{M_{\min}^{\mathrm{LMS}}}$	$\frac{2\sqrt{q_2-q_1^2}}{1+q_2}$	$\frac{\sqrt{2(1+2q_1^2+q_2^2)}}{1+q_2}$

Table 2: *Relative theoretical performance of RLS and LMS algorithms.*

The simulation results of Table 3 clearly show reasonable agreement between the experimentally and theoretically evaluated quantities of interest in relative terms. Furthermore, the results demonstrate the superiority of the RLS algorithm over the LMS algorithm in case (1), and vice versa in case (2). This condition depends, of course, on the particular choice of experimental parameters, but what is constant is the reciprocal symmetry between cases (1) and (2) for the ratios $D_{\min}^{\mathrm{RLS}}/D_{\min}^{\mathrm{LMS}}$ and $M_{\min}^{\mathrm{RLS}}/M_{\min}^{\mathrm{LMS}}$ as given in Table 2.

It is also interesting to note that the ERLS1 algorithm performs only marginally better than the optimal RLS/LMS algorithm in each case. Most likely, this situation is an artifact of the choice of experimental parameters; it makes both the relative mean-square weight deviation δ and relative mean-square misadjustment M sufficiently small, so that differences between the performances of the algorithms are not discernable

	Case 1: $\mathbf{R}^{-1} = c\mathbf{Q}$		
	Exp. value	Theor. value	Δ%
$\frac{D_{\min}^{\mathrm{RLS}}}{D_{\min}^{\mathrm{LMS}}}$	0.7578	0.8	−5.3%
$\frac{M_{\min}^{\mathrm{RLS}}}{M_{\min}^{\mathrm{LMS}}}$	0.6347	0.6614	−4.0%
	Case 2: $\mathbf{R} = c\mathbf{Q}$		
	Exp. value	Theor. value	Δ%
$\frac{D_{\min}^{\mathrm{RLS}}}{D_{\min}^{\mathrm{LMS}}}$	1.4806	1.5119	−2.1%
$\frac{M_{\min}^{\mathrm{RLS}}}{M_{\min}^{\mathrm{LMS}}}$	1.2064	1.25	−3.5%

Table 3: *Relative experimental performance of RLS and LMS algorithms.*

from normal simulation variance and numerical noise.

5.2. Tracking of Chirped Sinusoid

In this second experiment we consider the tracking of a chirped sinusoid in noise. The deterministic shifts caused by the chirp represent the other extreme of the Markov model described in (2). The chirped input signal is given by

$$s(k) = \sqrt{P_s}e^{j(\omega+\psi k/2)k} , \tag{7}$$

where $\sqrt{P_s}$ denotes the signal amplitude. Noisy measurements of $s(k)$ are available, say

$$y(k) = s(k) + n(k),$$

where $n(k)$ denotes a white-noise sequence with power P_n. The signal-to-noise ratio is denoted by $\rho = P_s/P_n$. A prediction problem is formulated with the objective of estimating $s(k)$ from the noisy data $\{y(k)\}$. More specifically, the "prediction error" $v(k)$ is defined as $v(k) = y(k) - \hat{s}(k)$, where

$$\hat{s}(k) = \mathbf{u}^H(k)\mathbf{w}_o ,$$

and

$$\mathbf{u}^H(k) = [\ y(k-1)\quad y(k-2)\quad \ldots\quad y(k-M)\ ] .$$

The prediction weight-vector $\mathbf{w}_o$ is chosen so as to minimize $E|v(k)|^2$. The state-space model in this case takes the form

$$\begin{aligned} \mathbf{w}_0(n+1) &= \mathbf{F}(\psi)\mathbf{w}_0(n) , \\ y(n) &= \mathbf{u}^H(n)\mathbf{w}_0(n) + v(n) , \end{aligned} \tag{8}$$

which is in agreement with the model studied in Section 4.. The sequence $v(\cdot)$ is taken as a white-noise process with variance $\sigma^2 \approx P_n$. With this model, the LMS and RLS algorithms are used in an ALE (adaptive-line-enhancer) configuration, predicting $y(k)$ using the vector of past inputs $\mathbf{u}(k)$.

The relative performance of the RLS and LMS algorithms for tracking a chirped sinusoid in noise is given by Macchi et al. [9]. The ratio of excess mean-squared errors is

$$\frac{M_{min}^{LMS}}{M_{min}^{RLS}} \approx \left(\frac{\rho}{3M}\right)^{1/3} . \tag{9}$$

When the input chirped signal-to-noise ratio (ρ) is less than $3M$, the performance of the LMS algorithm is superior to that of the RLS algorithm; and for $\rho \gg 3M$ the reverse is true.

To compare the performance of the RLS and LMS algorithms to the ERLS2 algorithm, we simulate two different cases indicated by (9): $\rho \ll 3M$; and $\rho \gg 3M$. The parameters are chosen to complement those in [10] and are given by

1. $(\rho \ll 3M) : \rho = 2,\ M = 2,\ \psi = 10^{-4}$,
2. $(\rho \gg 3M) : \rho = 100,\ M = 2,\ \psi = 10^{-4}$.

The same chirped signal was used for all three algorithms: LMS, standard RLS, and ERLS2. The estimates of the misadjustment were measured as the mean of $|y(k) - \mathbf{u}^H(k)\hat{\mathbf{w}}(k|k-1)|^2$, over 1500 iterations in steady state. This was repeated 10 times to find a mean value for the misadjustment. In order to illustrate that the ERLS2 algorithm can estimate an unknown chirp rate, the initial value of Π_0 was set to the identity matrix, and the initial guess for the chirp ψ was set to 0. The fact that Π_0 was incorrect was compensated by using a larger value for σ^2 in the algorithm. We found that the error decreased as σ^2 is increased for this case. The simulations results shown below are for σ^2 that is $200X$ larger than the actual. The results are summarized in Table 4. It can be

	Case I ($\rho = 2$)			
	μ_{min}		M_{min}/P_n	
	Theor.	Exp.	Theor.	Exp.
LMS	0.0037	$2\mu_{opt}$	-20.85dB	-24.37dB
RLS	0.0039	μ_{opt}	-20.55dB	-22.19dB
ERLS2	-	-	-	-27.64dB
	Case II ($\rho = 100$)			
	μ_{min}		M_{min/P_n}	
	Theor.	Exp.	Theor.	Exp.
LMS	0.0261	$2\mu_{opt}$	-12.31dB	-15.11dB
RLS	0.0144	μ_{opt}	-14.89dB	-16.23dB
ERLS2	-	-	-	-31.33dB

Table 4: *Misadjustment of LMS, RLS, and ERLS1 for the chirped sinusoid problem.*

seen that neither the LMS nor the RLS is superior in all cases but, in this chirped-tone example, the LMS might be favored since interest usually lies in the low SNR regions. Note also that the misadjustments for the LMS and RLS actually increase as the SNR (ρ) is increased, since neither algorithm is estimating the chirp rate. The ERLS2 algorithm uses the additional SNR to improve its estimate of the chrip rate thereby, decreasing the misadjustment. It was observed that, on a run by run basis, the ERLS2 always performed better.

6. Conclusions

The Kalman filter is known to be the linear optimum tracker on the basis of second-order statistics. Building on this fact and exploiting the one-to-one correspondences between the RLS and Kalman variables, we may derive extended forms of the RLS algorithm that inherit the good tracking behaviour of the Kalman filter. In this paper we have considered two particular forms of this extension:

- ERLS-1, pertaining to a system identification problem.
- ERLS-2, pertaining to the tracking of a chirped sinusoid in noise.

In each case, prior knowledge about the original dynamical system model is built into the formulation of the extended form of the RLS algorithm, making it the optimum linear tracking device for the particular applciation of interest.

References

1. B. Widrow and S. D. Stearns, *Adaptive Signal Processing*, Prentice Hall, 1985.
2. S. Haykin, *Adaptive Filter Theory*, third edition, Prentice Hall, 1996.
3. E. Eleftheriou and D. D. Falconer, "Tracking properties and steady-state performance of RLS adaptive filter algorithsm," *IEEE Transactions on Acoustics, Speech, and Signal Processing,* vol. ASSP-34, pp. 1097–1110, 1986.
4. A. Benveniste, "Design of adaptive algorithms for the tracking of time-varying systems," *International Journal of Adaptive Control and Signal Processing*, vol. 1, pp. 3–29, 1987.
5. E. Ewade, "Comparison of RLS, LMS, and sign algorithms for tracking randomly time-varying channels," *IEEE Transactions on Signal Processing,* vol. 42, pp. 2937–2944, 1994.
6. O. Macchi, "Optimization of adaptive identification for time-varying filters," *IEEE Transactions on Automatic Control*, vol. AC-31, pp. 283–287, 1986.
7. A. H. Sayed and T. Kailath, "A state-space approach to adaptive RLS filtering," *IEEE Signal Processing Magazine*, vol. 11, no. 3, pp. 18–60, July 1994.
8. D. N. Godard, "Channel equalization using a Kalman filter for fast data transmission," *IBM J. Res. Dev.*, vol. 18, pp. 267–273, 1974.
9. S. Haykin, A. H. Sayed, J. Zeidler, P. Yee, and P. Wei, "Tracking of linear time-variant systems," submitted to *IEEE Transactions on Signal Processing*, 1995.
10. O. Macchi, N. Bershad, and Mboup, "Steady-state superiority of LMS over LS for time-varying line enhancer in noisy environment," *IEE Proceedings F (Radar and Signal Processing)*, vol. 138, pp. 354-360, Aug. 1991.