Adaptive Differential Pulse-Coded Modulation with Exponential Tracking

Mansour A. Aldajani
Systems Engineering Department
King Fahd University of Petroleum and Minerals
Dhahran 31261, Saudi Arabia
dajani@ccse.kfupm.edu.sa

Ali H. Sayed*
Adaptive Systems Laboratory
Electrical Engineering Department
University of California, Los Angeles, CA 90095
sayed@ee.ucla.edu

Abstract

This paper first investigates a companded differential pulse-coded modulator and derives an expression for its SNR performance. Analysis and simulations show that the coder has superior SNR and dynamic range performance over other coders of similar complexity. The companded modulator is then extended to an adaptive differential pulse-coded modulator with high SNR and dynamic range performance, and it is shown to be BIBO stable.

1. Introduction

Pulse-coded modulation (PCM), delta modulation (DM), and differential pulse-coded modulation (DPCM) have limitations in terms of their signal-to-noise ratio and dynamic range performances. One way to ameliorate this problem is to employ adaptive step-size quantizers, as is the case with adaptive delta modulators (ADM) and adaptive DPCM. Examples of designs involving step-size adaptations can be found in [1]-[3].

In a previous study [4], the authors developed a particular adaptation scheme for sigma-delta modulation (SDM) that was shown to lead to improved SNR and dynamic range performance over conventional sigma-delta modulators, including some earlier adaptive step-size schemes. However, the method of [4] was limited to single-bit quantization. The purpose of this article is to show how to develop a multi-bit adaptive version and how to use it to design an adaptive DPCM system with improved performance.

*The work of A. H. Sayed was supported in part by NSF grant CCR-0208573. The work of M. A. Aldajani was supported by King Fahd University of Petroleum and Minerals.

Fig. 1 shows the adaptation scheme developed in [4], with the specific adaptive step-size scheme illustrated inside the dashed boxed. The scheme itself has the form of a delta modulator with an additional exponential term used to improve tracking performance. It was shown in [4], via a suitable change of variables, that the step-size adaptation block of Fig. 1 can be equivalently represented in the form shown in Fig. 2. In this equivalent form, the adapter behaves like a companded delta modulator. Even more importantly, this alternative representation suggests an extension of the single-bit adaptation scheme to the multi-bit scenario, as we shall now explain. In the process, we shall introduce two new multi-bit converters: a companded DPCM system and an adaptive DPCM system.

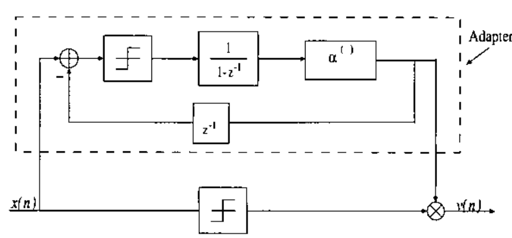

Figure 1. *Step-size adaptation scheme from [4].*

2. A Companded DPCM Structure

One way to extend the adaptation scheme of Fig. 1 is to replace the delta modulator block in Fig. 2 by a more general DPCM block. The resulting structure is shown in Fig. 3. A scaling factor equal to $1/S$ is added before the DPCM block in order to control its tracking performance. In the figure, $x(n)$ is the input signal and $v(n)$ is its representation (estimate). For simplicity of

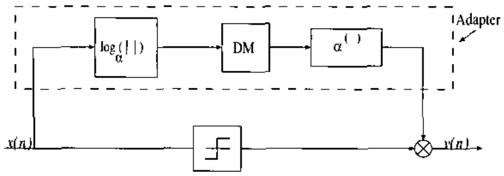

Figure 2. *An equivalent structure for the adaptation scheme of Fig. 1.*

presentation, we shall assume that the DPCM block has a single delay predictor as described by Fig. 4a. The quantizer of the DPCM block is assumed to have $B-1$ bits, so that the total number of bits of the companded system of Fig. 3 is B bits. Part b) of Fig. 3 shows a linearized model for DPCM, with the quantization noise introduced by the quantizer denoted by $e_d(n)$, and modeled as uniformly distributed within the interval $[-\Delta/2, \Delta/2]$ where $\Delta = 1/2^{B-1} = 2/2^B$. This quantization noise will be assumed to be independent of all other variables. In the following, we show that the companded DPCM structure of Fig. 3 is equivalent to a single random gain with known distributions. Moreover, the SNR for this structure will be computed.

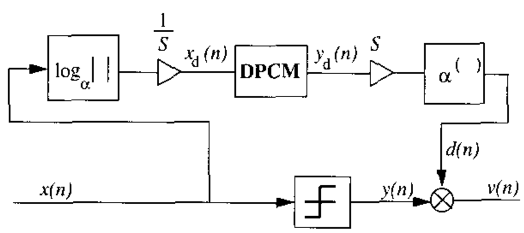

Figure 3. *Structure of the companded DPCM.*

Let $e_c(n) = x(n) - v(n)$ denote the coding error. The SNR performance of the coder will be measured in terms of the ratio of the input variance (σ_x^2) to the variance of $e_c(n)$. In order to derive an expression for the SNR, and thereby evaluate the performance of the system, we shall first show that the overall (nonlinear) companded system of Fig. 3 (which maps $x(n)$ to $v(n)$) can be approximately modeled in terms of a random gain model, i.e., as

$$v(n) = K(n)x(n)$$

for some random scalar $K(n)$ with known distribution. Once this is done, we shall then evaluate the system SNR.

To begin with, from Fig. 3, the input to the DPCM block, $x_d(n)$, is given by

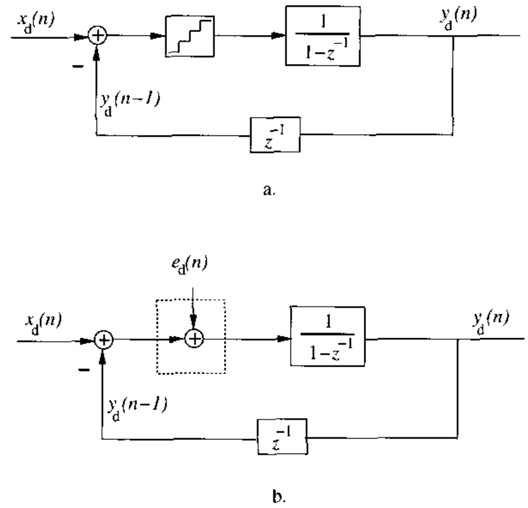

Figure 4. *Simple DPCM structure. a. Original system b. Linearized system.*

$$x_d(n) = \frac{1}{S}\log_\alpha |x(n)| \tag{1}$$

where $x(n)$ is the input to the coder. The output of the DPCM block, denoted by $y_d(n)$, is scaled by S and decompressed by an exponential factor α, so that

$$d(n) = \alpha^{Sy_d(n)} \tag{2}$$

The purpose of the signal $d(n)$ is to track the absolute value of $x(n)$ in an exponential manner. The decoded output $v(n)$ is obtained via

$$v(n) = y(n)d(n).$$

where $y(n)$ denotes the sign of $x(n)$.

Now from Fig. 4b, it is easy to see that $y_d(n)$ is related to $\{x_d(n), e_d(n)\}$ as follows

$$y_d(n) = x_d(n) + e_d(n)$$

so that

$$y_d(n) = \frac{1}{S}\log_\alpha |x(n)| + e_d(n)$$

Substituting into (2) leads to

$$d(n) = |x(n)|\alpha^{Se_d(n)}$$

and hence

$$v(n) = \text{sign}\{x(n)\}|x(n)|\alpha^{Se_d(n)}$$

Therefore, the relation between $v(n)$ and $x(n)$ is effectively given by

$$\boxed{v(n) = K(n)x(n)} \tag{3}$$

where the random variable $K(n)$ is defined by

$$\boxed{K(n) \triangleq \alpha^{Se_d(n)}} \tag{4}$$

We conclude that the companded coder of Fig. 3, which maps $x(n)$ to $v(n)$, can be modelled in terms of the scalar random gain $K(n)$; this gain is a function of the constant S and the quantization noise $e_d(n)$. Since the distribution of $e_d(n)$ is assumed to be uniform, this information can be used to compute the first and second-order moments of $K(n)$ and thereby evaluate the performance of the system. In the sequel we shall assume that all random processes are stationary.

To begin with, it follows from the definition of the coding error $e_c(n)$, and from the fact that $e_d(n)$ is independent of all other variables, that

$$\begin{aligned} E_{e_c} &\triangleq E[e_c(n)] \\ &= E[x(n)] - E[v(n)] \\ &= E_x - E[K(n)x(n)] \\ &= E_x - E_K E_x \end{aligned}$$

where the symbols $\{E_x, E_K\}$ refer to the means

$$E_x \triangleq E[x(n)], \quad E_K \triangleq E[K(n)].$$

If we assume a zero-mean input signal then $E_x = 0$ and

$$\boxed{E_{e_c} = 0} \tag{5}$$

Moreover, by using

$$e_c^2(n) = x^2(n) - 2x(n)v(n) + v^2(n)$$

we have that

$$E_{e_c^2} = E_{x^2} - 2E_{xv} + E_{v^2}$$

Now since

$$E_{xv} = E\{K(n)x^2(n)\} = E_K E_{x^2}$$

and

$$E_{v^2} = E\{K^2(n)x^2(n)\} = E_{K^2} E_{x^2}$$

we get

$$E_{e_c^2} = (1 - 2E_K + E_{K^2})E_{x^2}$$

where E_{K^2} refers to the second moment of $K(n)$, i.e.,

$$E_{K^2} \triangleq E[K^2(n)]$$

We conclude that

$$\boxed{\mathrm{SNR} = \frac{1}{1-2E_K+E_{K^2}}} \tag{6}$$

which provides an expression for the SNR performance of the coder in terms of E_K and E_{K^2}. These moments are evaluated as follows:

$$\begin{aligned} E_K \triangleq E\{K\} &= \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \frac{1}{\Delta}\alpha^{S\eta} d\eta \\ &= \frac{1}{\Delta S \ln(\alpha)}\left(\alpha^{S\Delta/2} - \alpha^{-S\Delta/2}\right) \end{aligned}$$

and

$$\begin{aligned} E_{K^2} \triangleq E\{K^2\} &= \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \frac{1}{\Delta}\alpha^{2S\eta} d\eta \\ &= \frac{1}{2\Delta S \ln(\alpha)}\left(\alpha^{S\Delta} - \alpha^{-S\Delta}\right) \end{aligned}$$

We thus find that the SNR expression of the companded DPCM system is independent of the input signal strength, which translates into a theoretically infinite dynamic range.

3. An Adaptive DPCM Structure

Besides its values as a coder in its own right, the companded DPCM of Fig. 3 could be used as a sub-block within a single-bit delta modulator. Specifically, it could be used to adapt the step-size of a delta modulator, as shown in Fig. 5. In this case, the input to the companded DPCM system (dashed line) is $e_a(n)$, the difference between the input signal and its estimate.

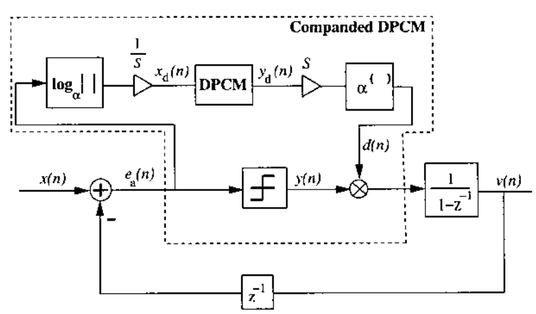

Figure 5. *Structure of the proposed ADPCM coder.*

The ADPCM structure of Fig. 5 differs from standard ADPCM coders in three respects. First, it employs a new step-size adaptation scheme. Second, the prediction part appears here as a single delay[1]. Third, the input signal in standard DPCM and ADPCM coders is usually buffered and normalized before coding. In contrast, since the proposed structure exhibits a high dynamic range, the input signal can be directly coded.

In [5], we studied the stability of the single-bit version of the structure shown in Fig. 5 (i.e., the case in which the DPCM block is simply a single-bit DM). The same analysis can be extended to the present context to establish the following result.

Lemma 1 (Stability of the ADPCM) *If α is chosen inside the open interval*

$$2^{-\frac{2}{S\Delta}} < \alpha < 2^{\frac{2}{S\Delta}} \tag{7}$$

and if the input signal $x(n)$ has a bound Λ, then there exists a finite number L such that

$$|v(n)| \leq \frac{\alpha^{S\Delta/2}\Lambda L}{1-L}. \tag{8}$$

In other words, the modulator is BIBO stable under condition (7).

Proof: The output signal $v(n)$ can be rewritten as

$$v(n) = \big(1 - K(n)\big)v(n-1) + K(n)x(n) \tag{9}$$

or

$$v(n) = \sum_{i=1}^{n}\prod_{j=i}^{n}\big(1 - K(j)\big)K(i)x(i) \tag{10}$$

If the input signal $x(n)$ has a bound Λ, then

$$|K(n)x(n)| \leq \Lambda|K(n)| \tag{11}$$

Now, if we choose

$$2^{-\frac{2}{S\Delta}} < \alpha < 2^{\frac{2}{S\Delta}} \tag{12}$$

then a bound L can be found such that

$$|1 - K(n)| \leq L < 1 \tag{13}$$

Using this result, we can now write

$$|v(n)| \leq \alpha^{S\Delta/2}\Lambda\sum_{i=1}^{n} L^{j} \tag{14}$$

which leads to (8).

$\diamond$

[1]The proposed ADPCM can be easily extended to higher order prediction. In this paper, however, the single-delay predictor is used for simplicity of analysis.

4. Simulations

The performance of the proposed ADPCM coder is compared against the companded DPCM coder of Fig. 3, as well as $\mu-$law PCM and A-law PCM coders. All these schemes have similar complexities. In particular, they do not involve elaborate prediction operations as in standard DPCM and ADPCM coders. Moreover, they are all on-line (real-time) schemes meaning that they operate on the input signal on a sample-by-sample basis.

A speech waveform is coded at a bit rate of 32KHz using the companded DPCM and the ADPCM coders. The parameters α, B, and S are chosen as 1.8, 3, and 5, respectively. The SNR is used as a qualitative measure of the quality of the decoded speech. In order to test the dynamic range of the coder, we apply different attenuation factors κ to the input speech and the SNR is measured for each value of κ. The result is shown in Fig. 6 together with that obtained using PCM, μ-law PCM, and A-law PCM. The figure shows that the SNR values obtained by the companded DPCM and proposed ADPCM coders are independent of the input strength. Moreover, the proposed ADPCM coder shows improvement of about 9dB over μ-law PCM in this case.

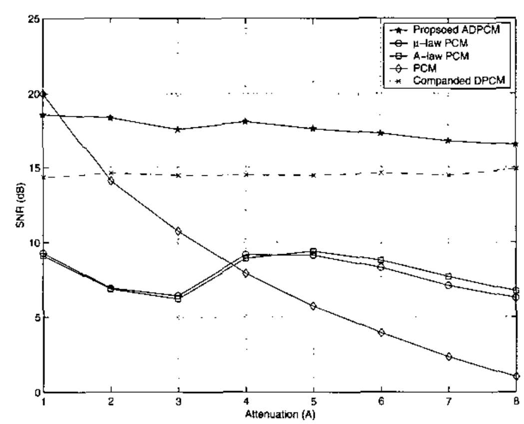

Figure 6. *SNR performance of five coding schemes with different attenuation factors for $B = 3$.*

In a different experiment, we investigate the effect of the input sampling rate on the performance of the ADPCM coder. Fig. 7 shows the SNR performance versus sampling rate of the input speech at different number of bits. The SNR changes approximately in a linear fashion with respect to the sampling rate.

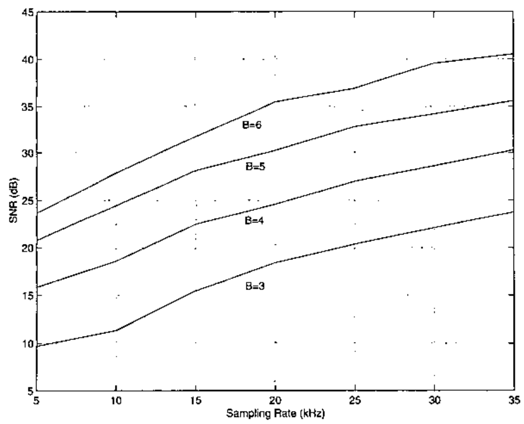

Figure 7. *SNR of the ADPCM coder versus sampling rate for different values of B.*

5. Conclusion

We investigated a companded DPCM coder and proposed an adaptive DPCM coder. Analysis and simulations show that the coders exhibit both high dynamic ranges and SNR performances.

References

[1] C. Chakravarthy, "An amplitude controlled adaptive delta sigma modulators," *Radio and Electronic Engineering,* 49(1):49–54, January 1979.

[2] M. Jaggi and C. Chakravarthy, "Instantaneous adaptive delta sigma modulator," *Canadian Electrical Engineering Journal,* 11(1):3–6, January 1986.

[3] J. Yu, M. Sandler, and R. Hwaken, "Adaptive quantization for one bit delta sigma modulation," *IEEE Proceedings G (Circuits, Devices and Systems),* 139(1):39–44, February 1992.

[4] M. A. Aldajani and A. H. Sayed, "Stability and performance analysis of an adaptive sigma-delta modulator." *IEEE Trans. on Circuits and Systems II: Analog and Digital Signal Processing,* 48(3):233–44, March 2001.

[5] M. A. Aldajani and A. H. Sayed, "A stable adaptive structure for delta modulation with improved performance," *Proc. ICASSP,* Salt Lake City, Utah, 4:2621–24, May 2001.