Collaborative Algorithms for Bandwidth and Rate Allocation in Wireless Networks

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***Abstract*— In this paper, we propose resource allocation strategies for a class of wireless networks with a clustering protocol. The nodes are assumed stationary and establish connections with the master node according to a priority scheme that relates to their distances from the master node. The paper considers bandwidth and rate allocation.**

I. Introduction

In this paper, we propose an adaptive bandwidth allocation strategy that minimizes an upper bound on the blocking probability across cells for a class of wireless networks. We also propose a rate allocation scheme under buffer and energy constraints assuming the data are correlated. Both resource allocation issues are addressed for a wireless network that adopts a clustering protocol. The protocol is as explained in [1].

II. Results

A. Bandwidth Allocation

Let N denote the number of nodes in a cell. The blocking probability in a cell l is defined as $\text{Prob}(\bar{Z} > Q_l)$, where $\bar{Z}$ is the number of nodes that express a desire to connect with the master node and Q_l is the number of frequency slots available in the cell. It can then be shown, using Azuma's inequality, that for the protocol described in [1], the blocking probability is bounded as

$$P\left\{\bar{Z} \geq Q_l\right\} \leq \exp\left\{-\frac{(Q_l - \frac{1}{1-\alpha})^2}{2N}\right\} \tag{1}$$

for some given positive scalar $\alpha < 1$. The result suggests that we introduce the utility function

$$U(Q_l) = \mu_l \sum_{l=1}^{M+1}\left\{1 - \exp\left(-\frac{\left\{Q_l - \frac{1}{1-\alpha}\right\}^2}{2N}\right)\right\} - \nu_l\left\{Q - \sum_{l=1}^{M+1} Q_l\right\}^2$$

and maximize it over the Q_l. The second term imposes a soft constraint on the total number of frequency slots to be Q. The coefficients μ_l and ν_l are chosen by the designer to proportionally scale the two terms contributing to U. A gradient ascent algorithm can seek the equilibrium [2].

B. Rate Allocation Through Encoding of Correlated Nodes

We also propose a strategy for allocating rates to different nodes in a cell by jointly encoding their data based on the correlation present among them. Consider a cell with N nodes and Q_l frequency slots. Let $\{\mathcal{X}_t^i\}_{t=1}^{\infty}$ for $i = 1, 2, ...Q_l$ be a set of stationary real Gaussian sources. For each $t = 1, 2, ...,$ the sources $\{\mathcal{X}^1, \mathcal{X}^2, ..., \mathcal{X}^i\}$ generate a random vector $\mathbf{x_i}$ with probability density function

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{i/2}|\Lambda_i|^{1/2}} \exp\left\{-\frac{1}{2}\mathbf{x_i^T}\Lambda_i^{-1}\mathbf{x_i}\right\}$$

where $\mathbf{x_i} = (x_1, x_2, x_3, ..., x_i)$ for all $i \leq Q_l$ and Λ_i is the covariance matrix. Consider the simple case when the source node i encodes its data using just one helper node j instead of $Q_l - 1$ helper nodes. Then it is known that for an admissible rate pair (R_i, R_j) and for some distortion $D_j > 0$, a lower bound on the rate distortion region is given by [3]

$$R_i \geq \frac{1}{2}\log^+\left[\frac{\sigma_i^2}{D_i}\left(1 - \rho_{ij}^2 + \rho_{ij}^2 2^{-2R_j}\right)\right] \tag{2}$$

where $\log^+ x = \max\{\log x, 0\}$. As can be seen from the above expression, the rate distortion region even for the one helper case is complicated enough to not allow elegant optimization methods if one seeks to maximize a given utility function in this region. We circumvent this difficulty by introducing an alternate (convex) rate distortion region obtained from a special encoding scheme. Since the rate distortion region that we employ is an outer region, the price paid is that the desired distortion measures $\{D_i\}$ may not be satisfied. Hence, we also provide a worst case distortion measure for the proposed solution. We maximize $U(R_i) = \sum_i \frac{1}{R_i}$ subject to

$$\begin{aligned} R_i &\geq \frac{1}{2}\log^+(|\Lambda_i|/\prod_{j=1}^{i-1}\sigma_j^2 D_i) \\ R_i p_i / f_i &\leq E_i \\ \sum_{i=1}^{Q_l} R_i &\leq b \end{aligned}$$

where b is the buffer capacity, p_i is the transmission power of node i, f_i is the rate of transmission of node i, and E_i is the energy constraint per transmission of node i. We update the rates as follows:

$$\begin{aligned} &\text{if} \sum_{i=1}^{Q_l} R_i \leq b \ \text{ for } \ i = 1, 2, ...Q_l, \ \text{ then} \\ &\qquad R_i(k+1) = [R_i(k) + \alpha_k U'(R_i(k))]_{\mathcal{R}} \\ &\text{else} \qquad R_i(k+1) = R_i(k) - \beta_k \epsilon \end{aligned}$$

with initial conditions $R_i = 1, \ i = 1, 2, ..Q$, and where $\{\alpha_k\}$ and $\{\beta_k\}$ are positive sequences having certain properties [2], and ϵ is a small positive constant. Moreover $[.]_{\mathcal{R}}$ denotes projection onto the set $\mathcal{R}$ defined by

$$\begin{aligned} \mathcal{R} \;=\; &\{R_i \mid R_i p_i / f_i \leq E_i, \qquad R_i \geq \frac{1}{2}\log^+\left(|\Lambda_i|/\prod_{j=1}^{i-1}\sigma_j^2 D_i\right) \\ &\text{for } i = 1, 2, ...Q_l\} \end{aligned}$$

References

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This work was partially supported by NSF grants CCR-0208573 and ECS-0401188.