On the Joint Compensation of IQ Imbalances and Phase Noise in MIMO-OFDM Systems

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***Abstract*— OFDM systems are susceptible to receiver impairments such as IQ imbalance and phase noise. These impairments can severely degrade the achievable effective signal-to-noise ratio at the receiver. In this paper, we propose a joint compensation scheme to mitigate the effects of the IQ imbalance and phase noise in multiple-input multiple-output (MIMO) OFDM systems. For ease of notation, a system with one transmit and two receive antennas is used to illustrate how to exploit the structure of MIMO transceivers to efficiently compensate for such impairments. We exploit the fact that the two received signals suffer from the same phase noise distortion, making it possible to achieve better compensation results than the single-input single-output case. The pilot tones in OFDM symbols are exploited to aid data recovery, and the data symbols are jointly estimated with the phase noise components that are parameterized by using the principle component analysis technique. The sensitivity of OFDM receivers to the analog impairments is significantly lowered, which helps simplify the RF and analog circuitry design in terms of implementation cost, power consumption, and silicon fabrication yield.**

I. Introduction

Multiple-input multiple-output (MIMO) OFDM systems are susceptible to the impairments caused by imperfectness in the radio-frequency (RF) signal down-conversion process. The effects of these impairments have been modeled as IQ imbalance and phase noise in the literature [1]. IQ imbalance is the mismatch in amplitude and phase between the I and Q branches in the receiver chains, while phase noise is the random unknown phase difference between the phase of the carrier signal and the phase of the local oscillator. Although much work has been done for the analysis and compensation of IQ imbalance and phase noise in single-input single-output (SISO) OFDM systems, e.g., [2] and [3], only limited work is available in the literature for MIMO-OFDM systems. The effects of IQ imbalance and phase noise on MIMO-OFDM receivers have been investigated in previous works, such as [4], [5]. Some algorithms have also been developed for the compensation of IQ imbalance [6] or the compensation of phase noise [7], separately.

In this paper, we pursue an explicit formulation for the joint effects of IQ imbalance and phase noise, and propose a compensation scheme with improved performance for MIMO-OFDM systems. It is shown that the joint compensation can be decomposed into the IQ imbalance compensation followed by the phase noise compensation. OFDM symbols that contain both data symbols and pilot symbols are transmitted in the payload portion of each packet such that the data symbols and phase noise components can be jointly estimated at the receiver[1]. We also exploit the fact that all the receive branches share a common oscillator and hence experience the same phase noise, which helps improve the phase noise compensation.

Throughout the paper, we use $(\cdot)^T$ to represent the matrix transpose and $(\cdot)^*$ the matrix conjugate transpose. $\mathbf{E}\{\cdot\}$ returns the expected value with respect to the underlying probability measure. $\text{diag}\{\beta_1, \beta_2, \ldots, \beta_N\}$ denotes the diagonal matrix whose diagonal elements are β_1, β_2, $\ldots$, β_N.

II. System Model

For ease of notation, we focus on the MIMO-OFDM system with one transmit and two receive antennas (1-Tx-2-Rx) throughout the paper, as illustrated in Fig. 1. At the OFDM transmitter, the information bits are first mapped into constellation symbols, and then converted into a block of N symbols $x[k]$, $k = 0, 1, \ldots, N-1$, by a serial-to-parallel converter. The N symbols are the frequency components to be transmitted using the N subcarriers of the OFDM modulator, and are converted to OFDM symbols by the unitary inverse Fast Fourier Transform (IFFT). After adding a cyclic prefix of length P, the resulting $N+P$ time-domain symbols are converted into a continuous-time signal $x(t)$ for transmission.

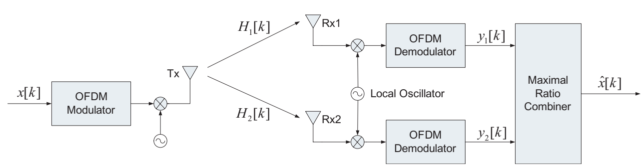

Fig. 1. A MIMO-OFDM system with one transmit and two receive antennas.

Fig. 2 shows the block diagram of an RF receiver with IQ imbalances α_i, θ_i, $i = 1, 2$, and phase noise $\phi(t)$. The RF chains of the two receive antennas share a common oscillator, resulting in the same phase noise injected into the two received signals. Compared to SISO-OFDM systems, this structure can benefit from phase noise compensation, as will be seen later. IQ imbalances might have more sources than phase noise, and can originate from imperfectness of not

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[1]Most OFDM standards, such as the IEEE 802.11a/g standard and the IEEE 802.15.3a Multiband-OFDM proposal, support such a comb-type pilot structure. Hence, the proposed scheme does not require any significant modification to the packet structure and can be applied to existing standards.

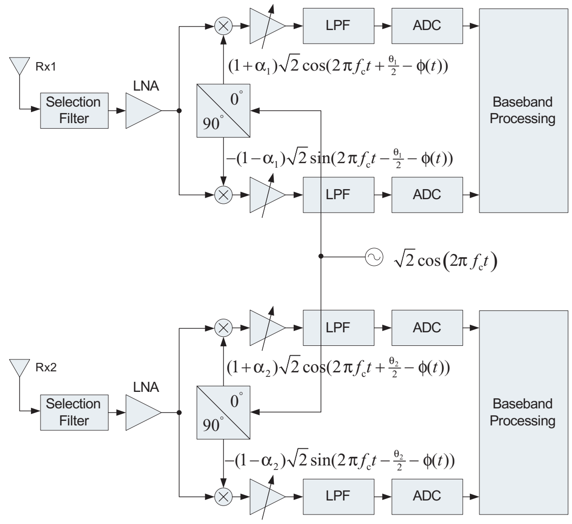

Fig. 2. An RF receiver with IQ imbalances α_i, θ_i and phase noise $\phi(t)$, $i = 1, 2$.

only the phase shifters but also other analog components. We use α_i and θ_i, $i = 1, 2$, to represent the IQ imbalances in the two RF chains, respectively. Let T_s be the sampling period. It can be shown that the output symbols $y_i[k]$, $i = 1, 2$, $k = 0, 1, \ldots, N-1$, after OFDM demodulation are related to the data symbols $x[k]$, $k = 0, 1, \ldots, N-1$, by

$$y_i[k] = \mu_i \sum_{r=0}^{N-1} a[r] H_i[(k-r)_N] x[(k-r)_N] \tag{1}$$
$$+ \nu_i \sum_{r=0}^{N-1} a^*[r] H_i^*[(N-k-r)_N] x^*[(N-k-r)_N] + w_i[k],$$

where the notations are defined as follows:

1) $(k)_N$ stands for $(k \bmod N)$.
2) μ_i and ν_i account for the IQ imbalances and are related to α_i and θ_i by [2]

$$\mu_i = \cos(\theta_i/2) - j\alpha_i \sin(\theta_i/2),$$
$$\nu_i = \alpha_i \cos(\theta_i/2) + j \sin(\theta_i/2).$$

3) $a[r]$, $r = 0, 1, \ldots, N-1$, are determined by the phase noise through

$$a[r] = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\phi(nT_s)} e^{-j\frac{2\pi rn}{N}}. \tag{2}$$

4) $H_i[k]$, $i = 1, 2$, $k = 0, 1, \ldots, N-1$, are the channel response of the k^{th} subcarrier from the transmitter to the i^{th} receive antenna. Let $h_i[n]$, $n = 0, 1, \ldots, L-1$, be the discrete-time impulse response of the baseband channel from the transmitter to the i^{th} receive antenna, $i = 1, 2$. Then, $H_i[k]$ is given by the discrete-time Fourier transform of $h_i[n]$, i.e.,

$$H_i[k] = \sum_{n=0}^{L-1} h_i[n] e^{-j\frac{2\pi kn}{N}}.$$

5) $w_i[k]$ is the additive noise in the k^{th} subcarrier.

In the absence of IQ imbalance and phase noise, we have the traditional relation

$$y_i[k] = H_i[k]x[k] + w[k], \quad i = 1, 2, \tag{3}$$

which follows by setting $\mu_i = 1$, $\nu_i = 0$, $a[0] = 1$, and $a[r] = 0$ for $r \neq 0$. Using matrix notation, (1) can be represented by

$$\mathbf{y}_i = \mu_i \mathbf{A}\mathbf{H}_i\mathbf{x} + \nu_i \widetilde{\mathbf{A}}\widetilde{\mathbf{H}}_i\widetilde{\mathbf{x}} + \mathbf{w}_i, \quad i = 1, 2, \tag{4}$$

where

$$\begin{aligned}
\mathbf{y}_i &= \begin{bmatrix} y_i[0] & y_i[1] & \ldots & y_i[N-1] \end{bmatrix}^T, \\
\mathbf{x} &= \begin{bmatrix} x[0] & x[1] & \ldots & x[N-1] \end{bmatrix}^T, \\
\widetilde{\mathbf{x}} &= \begin{bmatrix} x^*[0] & x^*[1] & \ldots & x^*[N-1] \end{bmatrix}^T, \\
\mathbf{A} &= \begin{bmatrix} a[0] & a[N-1] & \ldots & a[1] \\ a[1] & a[0] & \ldots & a[2] \\ \vdots & \vdots & \ddots & \vdots \\ a[N-1] & a[N-2] & \ldots & a[0] \end{bmatrix}, \\
\widetilde{\mathbf{A}} &= \begin{bmatrix} a^*[0] & a^*[N-1] & \ldots & a^*[1] \\ a^*[N-1] & a^*[N-2] & \ldots & a^*[0] \\ \vdots & \vdots & \ddots & \vdots \\ a^*[1] & a^*[0] & \ldots & a^*[2] \end{bmatrix}, \\
\mathbf{H}_i &= \mathrm{diag}\{H_i[0],\ H_i[1],\ \ldots,\ H_i[N-1]\}, \\
\widetilde{\mathbf{H}}_i &= \mathrm{diag}\{H_i^*[0],\ H_i^*[1],\ \ldots,\ H_i^*[N-1]\}, \\
\mathbf{w}_i &= \begin{bmatrix} w_i[0] & w_i[1] & \ldots & w_i[N-1] \end{bmatrix}^T.
\end{aligned} \tag{5}$$

In the absence of IQ imbalance and phase noise, (3) can be similarly represented by

$$\mathbf{y}_i = \mathbf{H}_i\mathbf{x} + \mathbf{w}_i, \quad i = 1, 2.$$

III. Proposed Compensation Algorithm

Given the system model (4), we are interested in estimating the transmitted vector $\mathbf{x}$ from $\mathbf{y}$, assuming[2] that the receiver has acquired the channel response $\mathbf{H}_i$ and the IQ imbalance parameters μ_i, ν_i. It is noticed that expression (4) can be represented as

$$\mathbf{y}_i = \mu_i \mathbf{z}_i + \nu_i \widetilde{\mathbf{z}}_i + \mathbf{w}_i, \quad i = 1, 2,$$

where $\mathbf{z}_i = \mathbf{A}\mathbf{H}_i\mathbf{x}$ is denoted by

$$\mathbf{z}_i = \begin{bmatrix} z_i[0] & z_i[1] & \ldots & z_i[N-1] \end{bmatrix}^T,$$

and $\widetilde{\mathbf{z}}_i$ is defined accordingly as

$$\widetilde{\mathbf{z}}_i = \begin{bmatrix} z_i^*[0] & z_i^*[N-1] & \ldots & z_i^*[1] \end{bmatrix}^T.$$

Clearly, the problem can be decomposed into two separate compensation problems: IQ imbalance compensation and phase noise compensation, as illustrated in Fig. 3. First, $\mathbf{z}_i$, $i = 1, 2$, is estimated from $\mathbf{y}_i$, $i = 1, 2$, by using an IQ imbalance compensation method; then, $\mathbf{x}$ is estimated from $\widehat{\mathbf{z}}_i$, $i = 1, 2$, by using a phase noise compensation method.

[2]This can be done by performing channel estimation at the beginning of each packet, because channel response and IQ imbalances are usually slowly time-varying.

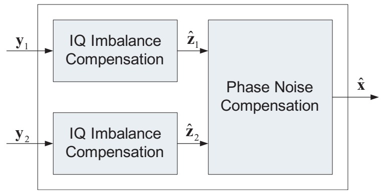

Fig. 3. Block diagram of the joint compensation algorithm. It can be decomposed into the IQ imbalance compensation blocks and the phase noise compensation block.

A. IQ Imbalance Compensation

We estimate $\mathbf{z}_i$ from $\mathbf{y}_i$ by using [2]:

$$\widehat{z}_i[k] = \frac{\mu_i^* y_i[k] - \nu_i y_i^*[(N-k)_N]}{|\mu_i|^2 - |\nu_i|^2},$$

for $k = 0, 1, \ldots, N-1$.

B. Phase Noise Compensation

The relation $\mathbf{z}_i = \mathbf{A}\mathbf{H}_i\mathbf{x}$, $i = 1, 2$, can be compactly represented as

$$\overline{\mathbf{z}} = \overline{\mathbf{A}}\mathbf{H}\mathbf{x},$$

where

$$\overline{\mathbf{z}} = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}, \quad \overline{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix}. \tag{6}$$

Then, $\mathbf{x}$ can be estimated by solving the following optimization problem:

$$\min_{\mathbf{x},\mathbf{A}} \| \overline{\mathbf{z}} - \overline{\mathbf{A}}\mathbf{H}\mathbf{x} \|^2. \tag{7}$$

Note that there are N unknowns in $\mathbf{A}$ and N unknowns in $\mathbf{x}$. The system is normally solvable[3], but the MIMO diversity gain can be greatly reduced. To improve system performance, we can reduce the number of unknowns in $\mathbf{A}$ by exploiting its statistical information. Phase noise can be modeled as a wide-sense stationary process, and its statistics can be obtained by online or offline measurements in the analog or digital domain. Using the principle component analysis (PCA) technique, phase noise can be approximated within a tolerable error range by fewer parameters. Let $\mathbf{c}$ denote the vector of phase noise, i.e.,

$$\mathbf{c} = \begin{bmatrix} e^{j\phi(0)} & e^{j\phi(T_s)} & \ldots & e^{j\phi((N-1)T_s)} \end{bmatrix}^T.$$

Assume its autocorrelation matrix is $\mathbf{R_c}$, whose singular value decomposition (SVD) is given by

$$\mathbf{R_c} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^*.$$

We choose the columns of $\mathbf{U}$ corresponding to the largest M ($M \leq N$) singular values of $\mathbf{R_c}$ as a basis for representing the phase noise vector $\mathbf{c}$. Denote the matrix of the basis by $\mathbf{P}$. Then, $\mathbf{c}$ can be approximated by

$$\mathbf{c} \approx \mathbf{P}\mathbf{c}', \tag{8}$$

[3]In contrast, there are relatively too many unknowns in SISO-OFDM systems compared to the system size [3]. In this case, the two RF chains share a common oscillator and hence have the same phase noise.

where $\mathbf{c}'$ is the parameter vector of length M that characterizes each realization of the phase noise in the subspace spanned by the principle components in $\mathbf{P}$. Combining (2) and (8) gives

$$\mathbf{a} \approx \frac{1}{N}\mathbf{F_a}\mathbf{P}\mathbf{c}', \tag{9}$$

where

$$\mathbf{a} = \begin{bmatrix} a[0] & a[1] & \ldots & a[N-1] \end{bmatrix}^T,$$

$$\mathbf{F_a} = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & e^{-j\frac{2\pi}{N}} & \ldots & e^{-j\frac{2\pi(N-1)}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\frac{2\pi(N-1)}{N}} & \ldots & e^{-j\frac{2\pi(N-1)^2}{N}} \end{bmatrix}.$$

Instead of estimating $\mathbf{a}$, we can estimate $\mathbf{c}'$, which reduces the number of unknown phase noise components from N to M.

Moreover, note that there exists a scalar ambiguity between the estimates of $\mathbf{A}$ and $\mathbf{x}$ in (7). To resolve this ambiguity, we use some data tones of $\mathbf{x}$ to transmit pilot symbols that are known to the receiver. Consequently, the original problem (7) is transformed to [3]:

$$\min_{\mathbf{c}',\mathbf{x}_{data}} \| \overline{\mathbf{z}} - \overline{\mathbf{A}}_{pilot}\mathbf{H}_{pilot}\mathbf{x}_{pilot} - \overline{\mathbf{A}}_{data}\mathbf{H}_{data}\mathbf{x}_{data} \|^2,$$

where $\mathbf{x}_{pilot}$ is the sub-vector of $\mathbf{x}$ that consists of all the pilot symbols and $\overline{\mathbf{A}}_{pilot}$, $\mathbf{H}_{pilot}$ are its associated sub-matrices from $\overline{\mathbf{A}}$ and $\mathbf{H}$, and $\mathbf{x}_{data}$ is the sub-vector of $\mathbf{x}$ that consists of all the data symbols and $\overline{\mathbf{A}}_{data}$, $\mathbf{H}_{data}$ are its associated sub-matrices from $\overline{\mathbf{A}}$ and $\mathbf{H}$. The optimization problem is nonlinear and nonconvex, and a sub-optimal solution is given in Algorithm 1. In Step 5), given a previously estimated $\mathbf{c}'$, we find an optimal $\mathbf{x}_{data}$. In Step 6), given a previously estimated $\mathbf{x}_{data}$, we find an optimal $\mathbf{c}'$. The objective function is decreasing with $i = 1, 2, \ldots$, and eventually converges to a local minimum.

IV. Computer Simulations

In the simulations, the system bandwidth is 20 MHz, i.e., $T_s = 0.05\ \mu s$, and the constellation used for symbol mapping is 64-QAM. The OFDM symbol size is $N = 64$ and the prefix length is $P = 20$. The channel length is 6 and each tap is independently Rayleigh distributed. We simulate an OFDM receiver with the IQ imbalances specified by $\alpha_1 = \alpha_2 = 0.1$ and $\theta_1 = \theta_2 = 10°$. The spectrum of the simulated phase noise is shown in Fig. 4.

The proposed joint compensation algorithm is simulated in comparison with the ideal OFDM receiver with no impairment and the IQ+CPE (common phase error) correction scheme proposed in [9]. During the payload portion of OFDM packets, 16 out of the 64 subcarriers are used for pilot tones, i.e., $Q = 16$. Fig. 5 shows the uncoded bit error rate (BER) performance of the system when the receiver has the perfect channel information. Compared to the IQ+CPE scheme, the proposed method achieves lower BERs, because it not only corrects the common phase rotation of the

Algorithm 1 IQ Imbalance and Phase Noise Compensation

1: Let $k_{pilot,j}$, $j = 1, 2, \ldots, Q$, be the subcarrier indices of the Q pilot tones. Then, $\widehat{a}[0]$ is given by

$$\widehat{a}[0] = \frac{\sum_{i=1}^{2} \sum_{j=1}^{Q} (H_i[k_{pilot,j}])^* \, (x[k_{pilot,j}])^* \, \widehat{z}_i[k_{pilot,j}]}{\sum_{i=1}^{2} \sum_{j=1}^{Q} |H_i[k_{pilot,j}]|^2 \, |x[k_{pilot,j}]|^2}.$$

2: Let

$$\widehat{\mathbf{c}}'_0 = \left[\, \widehat{a}[0] \quad \widehat{a}[0] \quad \ldots \quad \widehat{a}[0] \,\right]^T.$$

3: $i = 1$

4: **repeat**

5: Let $\widehat{\mathbf{a}}_{i-1} = \frac{1}{N}\mathbf{F_a P}\widehat{\mathbf{c}}'_{i-1}$ and construct $\overline{\mathbf{A}}_{i-1}$ from $\widehat{\mathbf{a}}_{i-1}$ according to (5) and (6). Find the associated optimal $\widehat{\mathbf{x}}_{data,i-1}$ by solving the following least-squares problem [8]:

$$\widehat{\mathbf{x}}_{data,i-1} = \arg \min_{\mathbf{x}_{data}} \| \overline{\mathbf{z}} - \overline{\mathbf{A}}_{pilot,i-1}\mathbf{H}_{pilot}\mathbf{x}_{pilot} - \overline{\mathbf{A}}_{data,i-1}\mathbf{H}_{data}\mathbf{x}_{data} \|^2$$

6: Find the optimal $\widehat{\mathbf{c}}'_i$ by solving the following least-squares problem:

$$\widehat{\mathbf{c}}'_i = \arg \min_{\mathbf{c}'} \| \overline{\mathbf{z}} - \overline{\mathbf{A}}_{pilot}\mathbf{H}_{pilot}\mathbf{x}_{pilot} - \overline{\mathbf{A}}_{data}\mathbf{H}_{data}\widehat{\mathbf{x}}_{data,i-1} \|^2$$

where $\overline{\mathbf{A}}_{pilot}$ and $\overline{\mathbf{A}}_{data}$ are determined by $\mathbf{c}'$ according to (9), (5) and (6).

7: $i = i + 1$

8: **until** there is no significant improvement in the objective function.

received constellation but also suppresses part of the inter-carrier interference caused by phase noise. It shows that the performance of the existing IQ+CPE algorithm saturates at about BER $= 5 \times 10^{-3}$ for the simulated impairments; on the other hand, the proposed algorithm can achieve an uncoded BER close to 10^{-4}.

V. Conclusions

In this paper, the joint effects of IQ imbalance and phase noise on MIMO-OFDM systems are studied. A joint compensation scheme is proposed to aid data recovery. The simulations show that the compensation scheme can effectively improve the system performance and reduce the sensitivity of MIMO-OFDM receivers to the analog impairments. Since receivers with less analog impairments usually have the disadvantage of high implementation cost, our technique enables the use of low-cost receivers for OFDM communications.

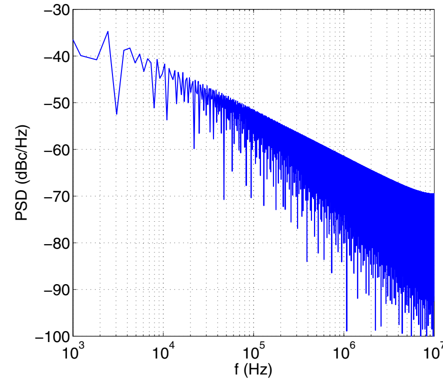

Fig. 4. Power spectral density (PSD) of the simulated phase noise. The PSD is measured in dB with respect to the carrier power, namely, dBc.

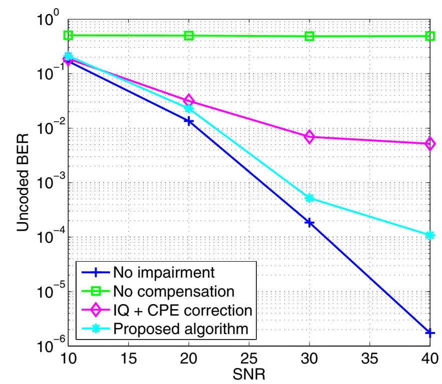

Fig. 5. Plots of uncoded BER vs. received signal-to-noise ratio when the receiver has the perfect channel information. Four scenarios are simulated: i) There is no impairment. ii) Both IQ imbalance and phase noise are present, but no compensation is applied. iii) Both IQ imbalance and phase noise are present, and the IQ+CPE (common phase error) correction scheme is applied. iv) Both IQ imbalance and phase noise are present, and the proposed compensation algorithm is applied.

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