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Adaptive Channel Estimation for Space-Time Block Coded MIMO–OFDM Communications

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***Abstract*— We develop adaptive channel estimation techniques for space–time block–coded (STBC) multiple–input multiple–output (MIMO) orthogonal frequency division multiplexing (OFDM) communications. The structure of the space–time code is exploited to lower the complexity of the adaptive receiver.**

I. Introduction

Wireless communications systems with multiple transmit and receive antennas provide large capacity gains, especially in rich scattering environment [1]. In order to achieve such high transmission rates with high performance and reliability, the receiver needs accurate information about the channel between the transmitter and the receiver. In multi–user multi–antenna scenarios, the channel estimation problem is challenging due the large number of subchannels.

Space-time block codes (STBC) have been originally developed to provide high performance and transmission rate over flat fading channels (e.g., Alamouti–STBC [2]). However, their performance is severely degraded by inter–symbol–interference (ISI) when transmitted over frequency selective channels. Orthogonal frequency division multiplexing (OFDM) is known to provide high transmission rate over frequency–selective channels due to its immunity to ISI. Recently, STBC–OFDM [3] has been proposed as a promising transmission scheme that enjoys better performance and diversity gain over such frequency–selective channels. However, the receivers require explicit knowledge of the impulse response of all sub-channels for efficient decoding. Several channel estimation techniques have been reported in literature (e.g.,[4], [5]). These techniques do not exploit the structure of the code to reduce complexity. In this paper, we develop adaptive channel estimation techniques for STBC MIMO–OFDM over frequency selective fading channels. We also show how to exploit the STBC structure to reduce the computational cost of the algorithm.

II. Problem Formulation

Consider a system consisting of M users, each equipped with two antennas. Each user transmits STBC–OFDM data from its two antennas. The receiver is equipped with M receive antennas. The block diagram of the system is shown in Figure 1. For each user, data are transmitted from its two antennas according to the space–time block coding scheme shown in Figure 2. At times $k = 0, 2, 4, \cdots$, N-symbol data blocks $\mathbf{x}_{k,1}^{(i)}$ and $\mathbf{x}_{k,2}^{(i)}$ are generated by an information source. The data blocks are then transmitted from the antennas of the i–th user according to the following rule:

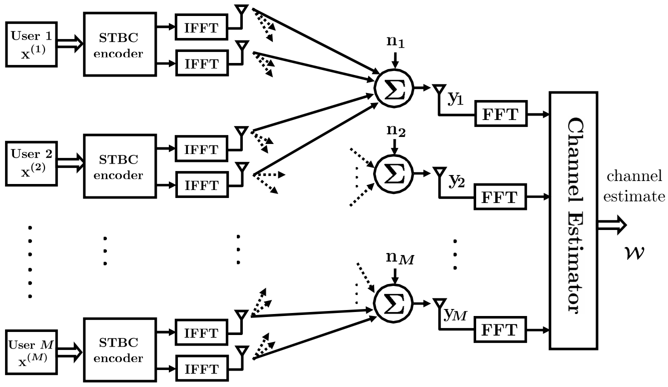

Fig. 1. Block diagram of an M-user STBC–OFDM communications system.

$$
\begin{array}{ll|cc}
 & & \multicolumn{2}{c}{\textit{Block}} \\
 & & k & k+1 \\
\hline
\textit{Antenna} & 1 & \mathbf{Q}^*\mathbf{x}_{k,1}^{(i)} & -\mathbf{Q}^*\mathbf{x}_{k,2}^{*(i)} \\
 & 2 & \mathbf{Q}^*\mathbf{x}_{k,2}^{(i)} & \mathbf{Q}^*\mathbf{x}_{k,1}^{*(i)}
\end{array} \quad (1)
$$

where $\mathbf{Q}^*$ is the IFFT matrix of size $N \times N$. A cyclic prefix (CP) of length ν, where ν is the memory of the longest subchannel, is added to each transmitted block after the IFFT operation to eliminate inter–block interference (IBI) and to make all channel matrices *circulant* as indicated in Figure 2. The received blocks k and $k+1$ at the l–th antenna, in the presence of additive white noise, are described by

$$
\begin{aligned}
\mathbf{y}_{k,l} &= \sum_{i=1}^{M} \left(\mathbf{H}_{1,l}^{(i)}\mathbf{Q}^*\mathbf{x}_{k,1}^{(i)} + \mathbf{H}_{2,l}^{(i)}\mathbf{Q}^*\mathbf{x}_{k,2}^{(i)} \right) + \mathbf{n}_{k,l} \\
\mathbf{y}_{k+1,l} &= \sum_{i=1}^{M} \left(-\mathbf{H}_{1,l}^{(i)}\mathbf{Q}^*\mathbf{x}_{k,2}^{*(i)} + \mathbf{H}_{2,l}^{(i)}\mathbf{Q}^*\mathbf{x}_{k,1}^{*(i)} \right) + \mathbf{n}_{k+1,l} \quad (2)
\end{aligned}
$$

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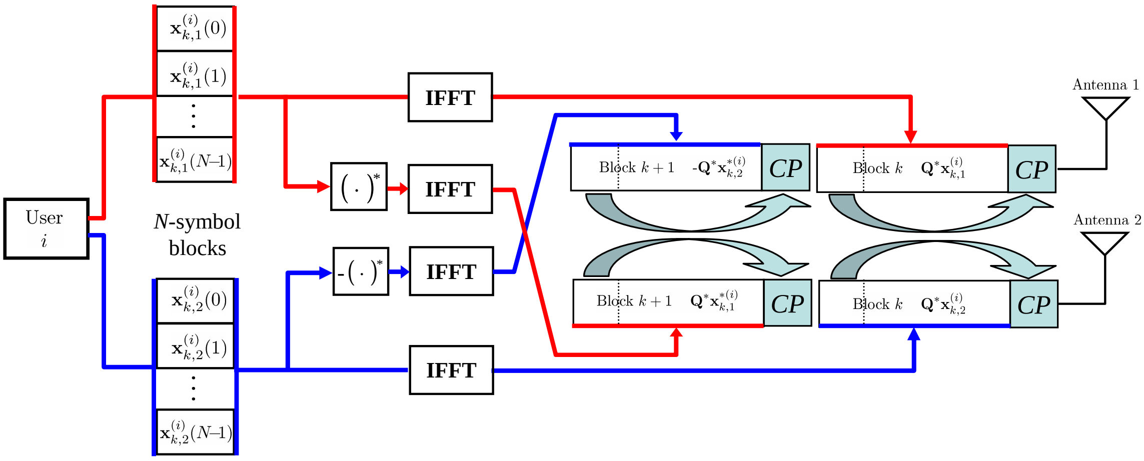

Fig. 2. STBC–OFDM transmission over frequency–selective channels.

where $\mathbf{H}_{1,l}^{(i)}$ and $\mathbf{H}_{2,l}^{(i)}$ have circulant structures of the form

$$\mathbf{H} = \begin{pmatrix} h(0) & 0 & \cdots & 0 & h(\nu) & \cdots h(1) \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \; \vdots \\ h(\nu-1) & \cdots & h(0) & 0 & \cdots & 0 \; h(\nu) \\ h(\nu) & h(\nu-1) & \cdots & h(0) & 0 & \cdots \; 0 \\ 0 & h(\nu) & h(\nu-1) \cdots & h(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \; 0 \\ 0 & \cdots & 0 & h(\nu)h(\nu-1) \cdots h(0) \end{pmatrix} \quad (3)$$

in terms of an impulse response sequence $\mathbf{h} \triangleq [h(0), h(1), \cdots, h(\nu)]$. Note that all indices associated with $\mathbf{H}$ in (3) have been dropped for compactness. Applying the DFT matrix $\mathbf{Q}$ to $\mathbf{y}_{k,l}$ and $\mathbf{y}_{k+1,l}$ in (2), we get a relation in terms of frequency–transformed variables:

$$\begin{aligned} \mathbf{Y}_{k,l} &= \sum_{i=1}^{M} \left(\mathbf{\Lambda}_{1,l}^{(i)} \mathbf{x}_{k,1}^{(i)} + \mathbf{\Lambda}_{2,l}^{(i)} \mathbf{x}_{k,2}^{(i)} \right) + \mathbf{N}_{k,l} \\ \mathbf{Y}_{k+1,l} &= \sum_{i=1}^{M} \left(-\mathbf{\Lambda}_{1,l}^{(i)} \mathbf{x}_{k,2}^{*(i)} + \mathbf{\Lambda}_{2,l}^{(i)} \mathbf{x}_{k,1}^{*(i)} \right) + \mathbf{N}_{k+1,l} \end{aligned} \quad (4)$$

where $\mathbf{Y} = \mathbf{Q}\mathbf{y}$, $\mathbf{N} = \mathbf{Q}\mathbf{n}$, and $\mathbf{\Lambda}_{1,l}^{(i)}$ and $\mathbf{\Lambda}_{2,l}^{(i)}$ are diagonal matrices given by $\mathbf{\Lambda}_{1,l}^{(i)} = \mathbf{Q}\mathbf{H}_{1,l}^{(i)}\mathbf{Q}^*$ and $\mathbf{\Lambda}_{2,l}^{(i)} \triangleq \mathbf{Q}\mathbf{H}_{1,l}^{(i)}\mathbf{Q}^*$, respectively. From (4), we get the linear relation (5). Let

$$\mathcal{Y}_{k,l}(m) = \begin{pmatrix} \mathbf{Y}_{k,l}(m) \\ \mathbf{Y}_{k+1,l}(m) \end{pmatrix}$$

Then the m–th entry of received vectors from the l–th receive antenna can be written as

$$\begin{aligned} \mathcal{Y}_{k,l}(m) &= \sum_{i=1}^{M} \begin{pmatrix} \mathbf{x}_{k,1}^{(i)}(m) & \mathbf{x}_{k,2}^{(i)}(m) \\ -\mathbf{x}_{k,2}^{*(i)}(m) & \mathbf{x}_{k,1}^{*(i)}(m) \end{pmatrix} \begin{pmatrix} \mathbf{\Lambda}_{1,l}^{(i)}(m) \\ \mathbf{\Lambda}_{2,l}^{(i)}(m) \end{pmatrix} + \begin{pmatrix} \mathbf{N}_{k,l}(m) \\ \mathbf{N}_{k+1,l}(m) \end{pmatrix} \\ &\triangleq \sum_{i=1}^{M} \mathcal{X}_k^{(i)}(m) \mathbf{\Lambda}_l^{(i)}(m) + \mathcal{N}_{k,l}(m) \end{aligned} \quad (6)$$

where each $\mathcal{X}_k^{(i)}(m)$ has an Alamouti structure. We further collect the m–th entry of the received vectors from all receive antennas into the following matrix form:

$$\mathcal{Y}_k(m) = \mathcal{X}_k(m)\mathbf{\Lambda}(m) + \mathcal{N}_k(m), \quad m = 0, \cdots, N-1 \quad (7)$$

where

$$\begin{aligned} \mathcal{Y}_k(m) &= \underbrace{\begin{pmatrix} \mathcal{Y}_{k,1}(m) & \ldots & \mathcal{Y}_{k,M}(m) \end{pmatrix}}_{2\times M}, \\ \mathcal{X}_k(m) &= \underbrace{\begin{pmatrix} \mathcal{X}_k^{(1)}(m) & \ldots & \mathcal{X}_k^{(M)}(m) \end{pmatrix}}_{2\times 2M}, \\ \mathbf{\Lambda}(m) &= \underbrace{\begin{pmatrix} \mathbf{\Lambda}_1^{(1)}(m) & \ldots & \mathbf{\Lambda}_M^{(1)}(m) \\ \vdots & \ddots & \vdots \\ \mathbf{\Lambda}_1^{(M)}(m) & \ldots & \mathbf{\Lambda}_M^{(M)}(m) \end{pmatrix}}_{2M\times M}, \\ \mathcal{N}_k(m) &= \underbrace{\begin{pmatrix} \mathcal{N}_{k,1}(m) \ldots \mathcal{N}_{k,M}(m) \end{pmatrix}}_{2\times M} \end{aligned} \quad (8)$$

We want to estimate $\mathbf{\Lambda}(m)$ for $m = 0, \cdots, N-1$. Equation (7) shows that this MIMO STBC–OFDM channel estimation problem decouples into N estimation problems of size $2M \times M$ each. Each one of these problems is similar to the channel estimation problem for M–user Alamouti–STBC transmissions over flat–fading channels.

III. Adaptive Channel Estimation

Our goal is to estimate the entries of $\mathbf{\Lambda}(m)$ in (7). This can be done by using a few blocks of known data to train an adaptive filter. Let the adaptive estimate of $\mathbf{\Lambda}(m)$ at time k be denoted by

$$\mathcal{W}_k(m) = \begin{pmatrix} \mathcal{W}_{k,1}^{(1)}(m) & \ldots & \mathcal{W}_{k,M}^{(1)}(m) \\ \vdots & \ddots & \vdots \\ \mathcal{W}_{k,1}^{(M)}(m) & \ldots & \mathcal{W}_{k,M}^{(M)}(m) \end{pmatrix} \quad (9)$$

$$\mathcal{Y}_{k,l} \triangleq \begin{pmatrix} \mathbf{Y}_{k,l}(0) \\ \mathbf{Y}_{k+1,l}(0) \\ \hline \vdots \\ \hline \mathbf{Y}_{k,l}(N-1) \\ \mathbf{Y}_{k+1,l}(N-1) \end{pmatrix} = \sum_{i=1}^{M} \begin{pmatrix} \mathbf{x}_{k,1}^{(i)}(0) & \mathbf{x}_{k,2}^{(i)}(0) & & & \\ -\mathbf{x}_{k,2}^{*(i)}(0) & \mathbf{x}_{k,1}^{*(i)}(0) & & & \\ & & \ddots & & \\ & & & \mathbf{x}_{k,1}^{(i)}(N-1) & \mathbf{x}_{k,2}^{(i)}(N-1) \\ & & & -\mathbf{x}_{k,2}^{*(i)}(N-1) & \mathbf{x}_{k,1}^{*(i)}(N-1) \end{pmatrix} \begin{pmatrix} \mathbf{\Lambda}_{1,l}^{(i)}(0) \\ \mathbf{\Lambda}_{2,l}^{(i)}(0) \\ \hline \vdots \\ \hline \mathbf{\Lambda}_{1,l}^{(i)}(N-1) \\ \mathbf{\Lambda}_{2,l}^{(i)}(N-1) \end{pmatrix} + \begin{pmatrix} \mathbf{N}_{k,l}(0) \\ \mathbf{N}_{k+1,l}(0) \\ \hline \vdots \\ \hline \mathbf{N}_{k,l}(N-1) \\ \mathbf{N}_{k+1,l}(N-1) \end{pmatrix} \quad (5)$$

The matrix $\mathcal{W}_k(m)$ can be updated, for instance, by using the block RLS algorithm [6] according to the following recursion for $k = 0, 2, 4, \cdots$:

$$\mathcal{W}_{k+2}(m) = \mathcal{W}_k(m) + \lambda^{-1}\mathcal{P}_k(m)\mathcal{X}_{k+2}^*(m)\mathbf{\Pi}_{k+2}(m) \times [\mathcal{Y}_{k+2}(m) - \mathcal{X}_{k+2}(m)\mathcal{W}_k(m)] \quad (10)$$

where $\mathcal{P}_k(m)$ is updated as follows:

$$\mathcal{P}_{k+2}(m) = \lambda^{-1}\left[\mathcal{P}_k(m) - \lambda^{-1}\mathcal{P}_k(m)\mathcal{X}_{k+2}^*(m) \times \mathbf{\Pi}_{k+2}^*(m)\mathcal{X}_{k+2}(m)\mathcal{P}_k(m)^*\right] \quad (11)$$

and

$$\mathbf{\Pi}_{k+2}(m) = \left(\mathbf{I}_2 + \lambda^{-1}\mathcal{X}_{k+2}(m)\mathcal{P}_k(m)\mathcal{X}_{k+2}^*(m)\right)^{-1} \quad (12)$$

The quantities $\{\mathcal{W}_k(m), \mathcal{P}_k(m)\}$ are updated over k for each $m = 0, \cdots, N-1$. The initial conditions are $\mathcal{W}_0 = \mathbf{0}$ and $\mathcal{P}_0 = \delta\mathbf{I}_{2M}$, where δ is a large number. The following result is a consequence of the structure of the code.

Lemma 1: $\mathcal{P}_k(m)$ has a Hermitian block structure with 2×2 subblocks, where the off diagonal subblocks $\mathbf{P}_{k,l}^{(i)}, i \neq l$ are 2×2 Alamouti matrices while the diagonal blocks are scaled multiples of $\mathbf{I}_2$. Moreover, $\mathbf{\Pi}_k(m)$ is a scaled multiple of $\mathbf{I}_2$ as well.

Proof: Let $\mathbf{A}$ and $\mathbf{B}$ be two Alamouti matrices with entries

$$\mathbf{A} = \begin{pmatrix} a_1 & a_2 \\ -a_2^* & a_1^* \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_1 & b_2 \\ -b_2^* & b_1^* \end{pmatrix}$$

Then it holds that

1) $\mathbf{A}+\mathbf{B}$, $\mathbf{A}-\mathbf{B}$, $\mathbf{AB}$, and $\mathbf{A}^{-1}$ are also Alamouti matrices.
2) $\mathbf{A}+\mathbf{A}^* = 2\text{Re}\{a_1\}\mathbf{I}_2$.
3) $\mathbf{AA}^* = \mathbf{A}^*\mathbf{A} = (|a_1|^2+|a_2|^2)\mathbf{I}_2$.

Using these properties, we now verify that $\mathcal{P}_k(m)$ and $\mathbf{\Pi}_k(m)$ have the mentioned structures by induction. Without loss of generality, assume $M=3$ and let $\mathcal{X}_k(m) = \left(\mathcal{X}_k^{(1)}\mathcal{X}_k^{(2)}\mathcal{X}_k^{(3)}\right)$ with Alamouti subblocks $\mathcal{X}_k^{(i)}$,

$$\mathcal{X}_k^{(i)} = \begin{pmatrix} \mathbf{x}_{k,1}^{(i)}(m) & \mathbf{x}_{k,2}^{(i)}(m) \\ -\mathbf{x}_{k,2}^{*(i)}(m) & \mathbf{x}_{k,1}^{*(i)}(m) \end{pmatrix}$$

- At $k=0$, we have $\mathcal{P}_0(m) = \delta\mathbf{I}_{2M}$ which has the desired structure (with off diagonal subblocks = 0). Then

$$\mathcal{X}_2(m)\mathcal{P}_0(m)\mathcal{X}_2^*(m) = \delta\left(\sum_{i=1}^{3}\mathcal{X}_2^{(i)}\mathcal{X}_2^{*(i)}\right) = \zeta_2\mathbf{I}_2$$

where $\zeta_2 = \delta\left(\sum_{i=1}^{3}(|\mathbf{x}_{k,1}^{(i)}|^2 + |\mathbf{x}_{k,2}^{(i)}|^2)\right)$ and $\mathbf{\Pi}_2 = (1+\zeta_2)^{-1}\mathbf{I}_2 = \alpha_2^{-1}\mathbf{I}_2$. Using this result, we evaluate $\mathcal{P}_2(m)$ as

$$\begin{aligned}\mathcal{P}_2(m) &= \lambda^{-1}\mathcal{P}_0(m) - \lambda^{-2}\delta^2\alpha_2^{-1}\mathcal{X}_2^*(m)\mathcal{X}_2(m) \\ &= \lambda^{-1}\delta\mathbf{I}_{2M} - \lambda^{-2}\delta^2\alpha_2^{-1} \\ &\quad\times\begin{pmatrix} \mathcal{X}_2^{*(1)}\mathcal{X}_2^{(1)} & \mathcal{X}_2^{*(1)}\mathcal{X}_2^{(2)} & \mathcal{X}_2^{*(1)}\mathcal{X}_2^{(3)} \\ \mathcal{X}_2^{*(2)}\mathcal{X}_2^{(1)} & \mathcal{X}_2^{*(2)}\mathcal{X}_2^{(2)} & \mathcal{X}_2^{*(2)}\mathcal{X}_2^{(3)} \\ \mathcal{X}_2^{*(3)}\mathcal{X}_2^{(1)} & \mathcal{X}_2^{*(3)}\mathcal{X}_2^{(2)} & \mathcal{X}_2^{*(3)}\mathcal{X}_2^{(3)} \end{pmatrix}\end{aligned}$$

It is obvious that the diagonal blocks are multiples of $\mathbf{I}_2$ and the off diagonal blocks are Alamouti.

- At time k, assume that $\mathbf{\Pi}_k$ and $\mathcal{P}_k$ have the desired structures, i.e.,

$$\mathbf{\Pi}_k = \alpha_k^{-1}\mathbf{I}_2, \quad \mathcal{P}_k = \begin{pmatrix} \gamma_{k,1}\mathbf{I}_2 & \mathbf{P}_{k,2}^{(1)} & \mathbf{P}_{k,3}^{(1)} \\ \mathbf{P}_{k,2}^{*(1)} & \gamma_{k,2}\mathbf{I}_2 & \mathbf{P}_{k,3}^{(2)} \\ \mathbf{P}_{k,3}^{*(1)} & \mathbf{P}_{k,3}^{*(2)} & \gamma_{k,3}\mathbf{I}_2 \end{pmatrix}$$

where $\mathbf{P}_{k,l}^{(i)}$ are Alamouti. We need to show that these structures are preserved at $k+2$.

$$\mathcal{X}_{k+2}(m)\mathcal{P}_k(m)\mathcal{X}_{k+2}^*(m) = \sum_{i=1}^{3}\gamma_{k,i}\mathcal{X}_{k+2}^{(i)}\mathcal{X}_{k+2}^{*(i)} + \sum_{i=2}^{3}\sum_{j=1}^{i-1}\left(\mathcal{X}_{k+2}^{(i)}\mathbf{P}_{k,i}^{(j)}\mathcal{X}_{k+2}^{*(j)} + \mathcal{X}_{k+2}^{(j)}\mathbf{P}_{k,i}^{*(j)}\mathcal{X}_{k+2}^{*(i)}\right) = \zeta_{k+2}\mathbf{I}_2$$

Note that the first term is a multiple of $\mathbf{I}_2$. Moreover, the second term is the sum of an Alamouti matrix and its complex conjugate, which is also a multiple of $\mathbf{I}_2$. Therefore, $\mathbf{\Pi}_{k+2} = (1+\zeta_{k+2})^{-1}\mathbf{I}_2 = \alpha_{k+2}^{-1}\mathbf{I}_2$. Now let

$$\begin{aligned}\mathbf{\Phi}_{k+2} &= \mathcal{P}_k(m)\mathcal{X}_{k+2}^*(m) \\ &= \begin{pmatrix} \gamma_{k,1}\mathcal{X}_{k+2}^{*(1)} + \mathbf{P}_{k,2}^{(1)}\mathcal{X}_{k+2}^{*(2)} + \mathbf{P}_{k,3}^{(1)}\mathcal{X}_{k+2}^{*(3)} \\ \mathbf{P}_{k,2}^{*(1)}\mathcal{X}_{k+2}^{*(3)} + \gamma_{k,2}\mathcal{X}_{k+2}^{*(2)} + \mathbf{P}_{k,3}^{(2)}\mathcal{X}_{k+2}^{*(3)} \\ \mathbf{P}_{k,3}^{*(1)}\mathcal{X}_{k+2}^{*(1)} + \mathbf{P}_{k,3}^{*(2)}\mathcal{X}_{k+2}^{*(2)} + \gamma_{k,3}\mathcal{X}_{k+2}^{*(3)} \end{pmatrix} = \begin{pmatrix} \mathbf{\Phi}_{k+2}^{(1)} \\ \mathbf{\Phi}_{k+2}^{(2)} \\ \mathbf{\Phi}_{k+2}^{(3)} \end{pmatrix}\end{aligned}$$

where each $\mathbf{\Phi}_{k+2}^{(i)}$ is Alamouti. The update equation for $\mathcal{P}_{k+2}$ is

$$\mathcal{P}_{k+2} = \lambda^{-1}\mathcal{P}_k - \lambda^{-2}\alpha_{k+2}^{-1}\mathbf{\Phi}_{k+2}\mathbf{\Phi}_{k+2}^* = \lambda^{-1}\mathcal{P}_k - \lambda^{-2} \times \alpha_{k+2}^{-1}\begin{pmatrix} \mathbf{\Phi}_{k+2}^{(1)}\mathbf{\Phi}_{k+2}^{*(1)} & \mathbf{\Phi}_{k+2}^{(1)}\mathbf{\Phi}_{k+2}^{*(2)} & \mathbf{\Phi}_{k+2}^{(1)}\mathbf{\Phi}_{k+2}^{*(3)} \\ \mathbf{\Phi}_{k+2}^{(2)}\mathbf{\Phi}_{k+2}^{*(1)} & \mathbf{\Phi}_{k+2}^{(2)}\mathbf{\Phi}_{k+2}^{*(2)} & \mathbf{\Phi}_{k+2}^{(2)}\mathbf{\Phi}_{k+2}^{*(3)} \\ \mathbf{\Phi}_{k+2}^{(3)}\mathbf{\Phi}_{k+2}^{*(1)} & \mathbf{\Phi}_{k+2}^{(3)}\mathbf{\Phi}_{k+2}^{*(2)} & \mathbf{\Phi}_{k+2}^{(3)}\mathbf{\Phi}_{k+2}^{*(3)} \end{pmatrix}$$

The product $\mathbf{\Phi}_{k+2}\mathbf{\Phi}_{k+2}^*$ has the same structure as $\mathcal{P}_k$. Therefore, $\mathcal{P}_{k+2}$ has the same structure as $\mathcal{P}_k$.

∎

It follows from Lemma 1 that the structures of $\mathcal{P}_k(m)$ and $\mathbf{\Pi}_k(m)$ are as follows:

$$\mathcal{P}_k = \underbrace{\begin{pmatrix} \mathbf{P}_{k,1}^{(1)} & \mathbf{P}_{k,2}^{(1)} & \cdots & \mathbf{P}_{k,M}^{(1)} \\ \mathbf{P}_{k,1}^{(2)} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{P}_{k,M}^{(M-1)} \\ \mathbf{P}_{k,1}^{(M)} & \cdots & \mathbf{P}_{k,M-1}^{(M)} & \mathbf{P}_{k,M}^{(M)} \end{pmatrix}}_{2M\times 2M}$$

$$\triangleq \begin{pmatrix} \gamma_{k,1}\mathbf{I}_2 & \mathbf{P}_{k,2}^{(1)} & \cdots & \mathbf{P}_{k,M}^{(1)} \\ \mathbf{P}_{k,2}^{*(1)} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{P}_{k,M}^{(M-1)} \\ \mathbf{P}_{k,M}^{*(1)} & \cdots & \mathbf{P}_{k,M}^{*(M-1)} & \gamma_{k,M}\mathbf{I}_2 \end{pmatrix} \quad (13)$$

$$\mathbf{\Pi}_{k+2} \triangleq \alpha_{k+2}^{-1}\mathbf{I}_2 \quad (14)$$

where the index (m) has been dropped in (13) and (14) for compactness. Table I shows how we exploit the STBC structure to update the entries of $\mathbf{\Pi}_k$ and $\mathcal{P}_k$. Note that the index (m) has been dropped in Table I. The reduction in computational complexity due to the STBC structure as a function of the number of users is illustrated in Figure 3.

IV. Simulation Results

We simulate an M–user system with each user equipped with two transmit antennas. The data are transmitted from the each user's antennas according to STBC–OFDM. The number of receive antennas is equal to the number of users. The channels from each transmit antenna to each receive antenna are assumed to be independent. The data bits of each user are mapped into an 8-PSK signal constellation. The symbols are grouped into blocks of 32 symbols and a cyclic prefix is added to each block by copying the first ν symbols after the last symbol of the same block. and the processed symbols are transmitted at a rate equal to 271 KSymbols/sec. The signal to noise ratios of all users at the receiver are assumed to be equal. A Typical Urban (TU) channel model with overall channel impulse response memory ν equals to 3 is considered for all channels. The MSE results are obtained by running the adaptive algorithm until it converges and the steady state MSE is calculated. Moreover, the results are averaged over 1000 different channel realizations. Figure 4 shows the overall system steady state MSE associated with the channel estimation technique presented in this paper for two and three users.

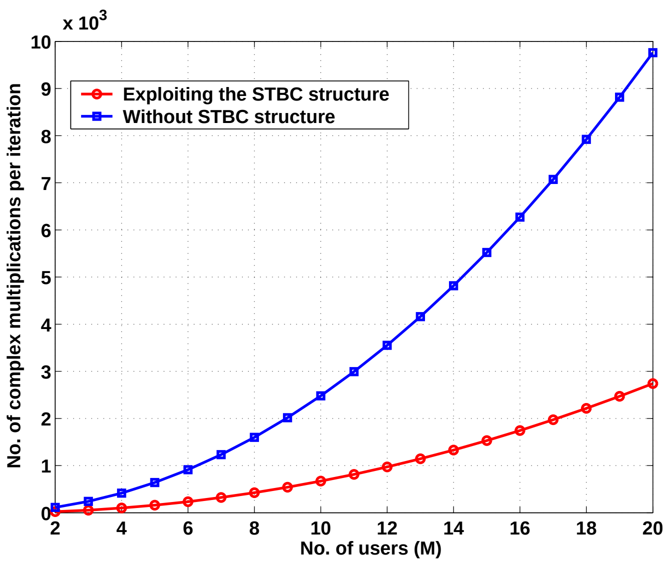

Fig. 3. Number of complex multiplications per iteration needed for updating $\mathcal{P}_k(m)$.

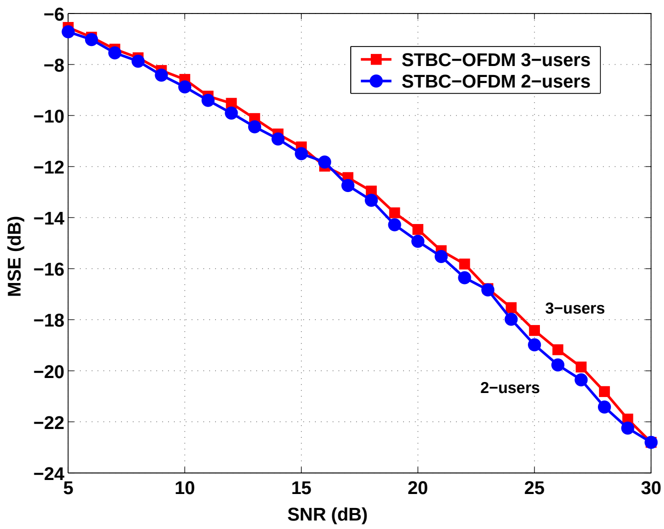

Fig. 4. MSE performance of the adaptive channel estimator.

V. Conclusions

In this paper, we developed an adaptive channel estimation technique for MIMO STBC–OFDM transmissions over frequency selective fading channels. In particular, we showed that the channel estimation for OFDM-STBC collapses to the problem of estimating N parallel subchannels where N is the block size. Moreover, we showed that certain quantities propagated by the RLS estimator alsoexhibit Alamouti structures as a result of the space–time block code.

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TABLE I

ADAPTATION ALGORITHM FOR M USERS.

Let the entries of the 2×2 matrices $\mathbf{P}_{k,l}^{(i)}$ and $\mathcal{X}_k^{(i)}$ be

$$\mathbf{P}_{k,l}^{(i)} = \begin{pmatrix} p_{k,l}^{(i)}(1) & p_{k,l}^{(i)}(2) \\ -p_{k,l}^{*(i)}(2) & p_{k,l}^{*(i)}(1) \end{pmatrix} \quad \mathcal{X}_k^{(i)} = \begin{pmatrix} \mathbf{x}_{k,1}^{(i)} & \mathbf{x}_{k,2}^{(i)} \\ -\mathbf{x}_{k,2}^{*(i)} & \mathbf{x}_{k,1}^{*(i)} \end{pmatrix}$$

Starting from $k=0$, update the entries of $\mathbf{\Pi}_k$, $\mathcal{P}_k$, and $\mathcal{W}_k$ as follows:

1) Compute the entries of $\mathbf{\Pi}_{k+2}$, i.e., α_{k+2} as

$$\begin{aligned}\alpha_{k+2} &= 1+\sum_{j=1}^{M}\gamma_{k,i}\left(|\mathbf{x}_{k+2,1}^{(i)}|^2+|\mathbf{x}_{k+2,2}^{(i)}|^2\right)\\ &\quad +2\mathrm{Re}\left\{\sum_{i=2}^{M}\sum_{j=1}^{i-1}\begin{pmatrix}\mathbf{x}_{k+2,1}^{(i)} & \mathbf{x}_{k+2,1}^{(i)}\end{pmatrix}\mathbf{P}_{k,i}^{(j)}\begin{pmatrix}\mathbf{x}_{k+2,1}^{*(j)}\\ \mathbf{x}_{k+2,2}^{*(j)}\end{pmatrix}\right\}\end{aligned}$$

2) Let $\mathbf{\Phi}_{k+2} = \mathcal{P}_k\mathcal{X}_{k+2}^{*} \triangleq \begin{pmatrix}\mathbf{\Phi}_{k+2}^{(1)} & \cdots & \mathbf{\Phi}_{k+2}^{(M)}\end{pmatrix}^T$, where $(\cdot)^T$ indicates the matrix transposition. Let the entries of the 2×2 subblocks $\mathbf{\Phi}_{k+2}^{(i)}$ be

$$\mathbf{\Phi}_{k+2}^{(i)} = \begin{pmatrix}\mathbf{\Phi}_{k+2,1}^{(i)} & \mathbf{\Phi}_{k+2,2}^{(i)}\\ -\mathbf{\Phi}_{k+2,2}^{*(i)} & \mathbf{\Phi}_{k+2,1}^{*(i)}\end{pmatrix}$$

Then $\mathbf{\Phi}_{k+2,1}^{(i)}$ and $\mathbf{\Phi}_{k+2,2}^{(i)}$, $i=1,\cdots,M$, are given by

$$\begin{aligned}\mathbf{\Phi}_{k+2,1}^{(i)} &= \gamma_{k,i}\mathbf{x}_{k+2,1}^{(i)} + \sum_{j=1,\ j\neq i}^{M} p_{k,j}^{(i)}(1)\mathbf{x}_{k+2,1}^{*(j)} + p_{k,j}^{(i)}(2)\mathbf{x}_{k+2,2}^{*(j)}\\ \mathbf{\Phi}_{k+2,2}^{(i)} &= \gamma_{k,i}\mathbf{x}_{k+2,2}^{(i)} - \sum_{j=1,\ j\neq i}^{M} p_{k,j}^{(i)}(1)\mathbf{x}_{k+2,1}^{(j)} - p_{k,j}^{(i)}(2)\mathbf{x}_{k+2,2}^{(j)}\end{aligned}$$

3) Compute the entries of $\mathbf{P}_{k+2,l}^{(i)}$, $l,i=1,\cdots,M$, as follows

$$\begin{aligned}p_{k+2,l}^{(i)}(1) &= \begin{cases}\lambda^{-1}\gamma_{k,i}-\lambda^{-2}\alpha_{k+2}^{-1}\left(|\mathbf{\Phi}_{k+2,1}^{(i)}|^2+|\mathbf{\Phi}_{k+2,2}^{(i)}|^2\right) & i=l\\ \lambda^{-1}p_{k,l}^{(i)}(1)-\lambda^{-2}\alpha_{k+2}^{-1}\left(\mathbf{\Phi}_{k+2,1}^{(i)}\mathbf{\Phi}_{k+2,1}^{*(l)}+\mathbf{\Phi}_{k+2,2}^{(i)}\mathbf{\Phi}_{k+2,2}^{*(l)}\right) & i\neq l\end{cases}\\ p_{k+2,l}^{(i)}(2) &= \begin{cases}0 & i=l\\ \lambda^{-1}p_{k,l}^{(i)}(2)-\lambda^{-2}\alpha_{k+2}^{-1}\left(\mathbf{\Phi}_{k+2,2}^{(i)}\mathbf{\Phi}_{k+2,2}^{(l)}-\mathbf{\Phi}_{k+2,1}^{(i)}\mathbf{\Phi}_{k+2,1}^{(l)}\right) & i\neq l\end{cases}\end{aligned}$$

4) Repeat the previous steps for each iteration over k.

5) Repeat the previous procedure for $m=0,\cdots,N-1$.

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[6] A. H. Sayed, *Fundamentals of Adaptive Filtering*. NY: Wiley, 2003.