Adaptive Frequency-Domain Joint Equalization and Interference Cancellation for Multi–User Space-Time Block-Coded Systems

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***Abstract*— We develop an efficient adaptive receiver for joint equalization and interference cancellation for multi–user space–time block–coded transmissions. The receiver exploits the code structure and allows multiple user transmissions over frequency–selective fading channels with reduced complexity and lower system overhead. The adaptation scheme is based on a recursive least–squares implementation for faster convergence; nevertheless, it exploits the code structure to attain RLS performance at LMS complexity.**

I. Introduction

Single-carrier frequency-domain equalization (SC-FDE) offers several advantages such as low complexity receivers (due to the use of the FFT), and reduced sensitivity to carrier frequency offset and nonlinear distortion in comparison to orthogonal frequency division multiplexing (OFDM) [1]. When combined with space–time block–codes and the Alamouti scheme [2], SC-FDE can help increase system capacity without requiring additional bandwidth [3]–[6]. However, when implemented over frequency-selective channels, the Alamouti scheme should be implemented at a block and not symbol level in order to achieve multipath diversity gains [3]. In this paper, we consider a multi-user multi-antenna scenario with $2M$ transmit antennas and M receive antennas, whereby each user is equipped with 2 antennas. Transmissions from each user are coded in a block space-time manner with cyclic prefixing. Two receiver structures are designed for such multiuser environments in order to perform joint interference cancellation, equalization, and decoding. First, a minimum mean-square-error (MMSE) receiver is developed with an iterative decoding scheme similar to [6]. This structure requires channel information at the receiver, which can be estimated by embedding training sequences into each block. Second, an adaptive receiver is developed that does not require explicit channel information and is therefore useful in reducing the system overhead. The adaptive algorithm operates in two modes: training and tracking and it exploits the structure of the space-time block code to deliver RLS performance at LMS complexity.

II. Transmission over Broadband Channels

A. The Single–User Case

Consider the scheme depicted in Figure 1, where a single user equipped with two antennas is transmitting data over a wireless channel and the receiver has a single antenna. Data are transmitted from the antennas according to the space–time block coding (STBC) scheme depicted in Figure 2 [3]. Denote the n^{th} symbol of the k^{th} transmitted block from antenna i by $\mathbf{x}_i^{(k)}(n)$. At times $k = 0, 2, 4, \cdots$, pairs of length–N blocks $\mathbf{x}_1^{(k)}(n)$ and $\mathbf{x}_2^{(k)}(n)$ (for

This work was partially supported by NSF grant CCR-0208573 and by a gift from AT&T Shannon Laboratory.

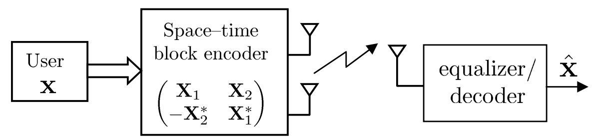

Fig. 1. A single–user system.

$0 \leq n \leq N-1$) are generated by an information source according to the rule:

$$\begin{aligned} \mathbf{x}_1^{(k+1)}(n) &= -\mathbf{x}_2^{*(k)}((-n)_N) \\ \mathbf{x}_2^{(k+1)}(n) &= \mathbf{x}_1^{*(k)}((-n)_N) f \end{aligned} \quad (1)$$

where $\mathbf{x}$ has a covariance matrix equal to $\sigma_x^2 \mathbf{I}_N$ and $(\cdot)^*$ and $(\cdot)_N$ denote complex conjugation and modulo–N operations, respectively. In addition, a cyclic prefix of length ν is added to each transmitted block to eliminate inter–user interference (IBI) and make all channel matrices *circulant*. With two transmit and one receive antenna, the received blocks k and $k+1$ are described by

$$\mathbf{y}^{(j)} = \mathbf{H}_1^{(j)} \mathbf{x}_1^{(j)} + \mathbf{H}_2^{(j)} \mathbf{x}_2^{(j)} + \mathbf{n}^{(j)} \quad \text{for } j = k, k+1 \quad (2)$$

where $\mathbf{n}^{(j)}$ is the noise vector with a covariance matrix equal to $\sigma_n^2 \mathbf{I}_N$, and $\mathbf{H}_1^{(j)}$ and $\mathbf{H}_2^{(j)}$ are the *circulant* channel matrices from the first and second transmit antennas, respectively, over block j, to the receive antenna. Applying the DFT matrix $\mathbf{Q}$ to $\mathbf{y}^{(j)}$, we get (for $j = k, k+1$):

$$\mathbf{Y}^{(j)} \triangleq \mathbf{Q}\mathbf{y}^{(j)} = \mathbf{\Lambda}_1^{(j)} \mathbf{X}_1^{(j)} + \mathbf{\Lambda}_2^{(j)} \mathbf{X}_2^{(j)} + \mathbf{N}^{(j)} \quad (3)$$

where $\mathbf{X}_i^{(j)} = \mathbf{Q}\mathbf{x}_i^{(j)}$, and $\mathbf{N}^{(j)} = \mathbf{Q}\mathbf{n}^{(j)}$, and $\mathbf{\Lambda}_1^{(j)}$ and $\mathbf{\Lambda}_2^{(j)}$ are diagonal matrices given by $\mathbf{\Lambda}_i^{(j)} = \mathbf{Q}\mathbf{H}_i^{(j)}\mathbf{Q}^*$. Using the encoding rule (1) and properties of the DFT [7], and assuming the two channels

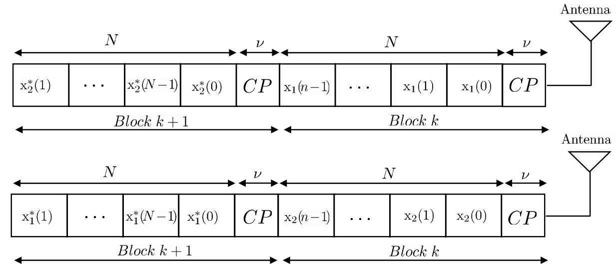

Fig. 2. Block format for SC FDE–STBC transmission scheme.

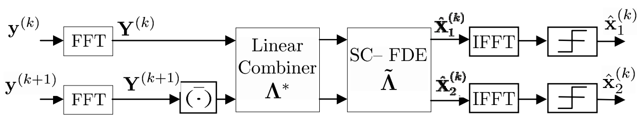

Fig. 3. Receiver structure for a 2-transmit 1-receive antenna system.

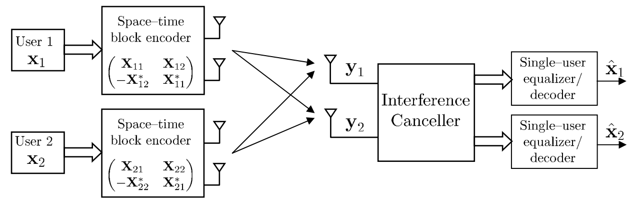

Fig. 4. Two-user system.

are fixed over two consecutive blocks, we get

$$\begin{aligned} \mathbf{X}_1^{(k+1)}(m) &= -\mathbf{X}_2^{*(k)}(m) \\ \mathbf{X}_2^{(k+1)}(m) &= \mathbf{X}_1^{*(k)}(m) \end{aligned} \quad (4)$$

for $m = 0, 1, \cdots, N-1$ and $k = 0, 2, 4, \cdots$. Combining (3) and (4), we arrive at

$$\begin{aligned} \mathbf{Y} = \begin{pmatrix} \mathbf{Y}^{(k)} \\ \bar{\mathbf{Y}}^{(k+1)} \end{pmatrix} &= \begin{pmatrix} \mathbf{\Lambda}_1 & \mathbf{\Lambda}_2 \\ \mathbf{\Lambda}_2^* & -\mathbf{\Lambda}_1^* \end{pmatrix} \begin{pmatrix} \mathbf{X}_1^{(k)} \\ \mathbf{X}_2^{(k)} \end{pmatrix} + \begin{pmatrix} \mathbf{N}^{(k)} \\ \bar{\mathbf{N}}^{(k+1)} \end{pmatrix} \\ &\triangleq \mathbf{\Lambda X} + \mathbf{N} \end{aligned} \quad (5)$$

where $\bar{(\cdot)}$ denotes complex conjugation of the entries of the vector and $\mathbf{\Lambda}$ is the overall frequency–domain channel matrix from the transmit antennas to the receive antenna. The matrix structure of $\mathbf{\Lambda}$ in (5), with diagonal submatrices $\{\mathbf{\Lambda}_1, \mathbf{\Lambda}_2\}$, will appear frequently throughout the paper, and we shall use the terminology *Alamouti–like matrix* to refer to it. The minimum mean–square–error (MMSE) estimator of $\mathbf{X}$ given $\mathbf{Y}$ is then

$$\hat{\mathbf{X}} = \left(\mathbf{\Lambda}^*\mathbf{\Lambda} + \frac{1}{SNR}\mathbf{I}_{2N}\right)^{-1} \mathbf{\Lambda}^*\mathbf{Y} \triangleq \tilde{\mathbf{\Lambda}}\mathbf{\Lambda}^*\mathbf{Y} \quad (6)$$

where $\tilde{\mathbf{\Lambda}}$ is diagonal and SNR is the signal–to noise ratio at the receiver, $SNR = \sigma_x^2/\sigma_n^2$. The SC MMSE–FDE output is transformed back to time–domain where decisions are made. The receiver structure is shown in Figure 3 [8].

B. The Two–User Case

By using a second receive antenna, we can double the number of users. The system block diagram is shown in Figure 4 [9]. With two receive antennas and two users (each equipped with 2 antennas), Eq. (5) generalizes to

$$\begin{pmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{\Lambda}_{11} & \mathbf{\Lambda}_{12} \\ \mathbf{\Lambda}_{21} & \mathbf{\Lambda}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix} + \begin{pmatrix} \mathbf{N}_1 \\ \mathbf{N}_2 \end{pmatrix} \quad (7)$$

where $\mathbf{Y}_i$ is the processed signal from the i–th receive antenna while $\mathbf{N}_i$ is the corresponding noise vector. Moreover, $\mathbf{X}_i$ consists of the two subvectors representing the size-N DFTs of the information blocks transmitted from the i–th user's first and second transmit antennas at time k, i.e.,

$$\mathbf{Y}_i = \begin{pmatrix} \mathbf{Y}_i^{(k)} \\ \bar{\mathbf{Y}}_i^{(k+1)} \end{pmatrix}, \quad \mathbf{X}_i = \begin{pmatrix} \mathbf{X}_{i_1}^{(k)} \\ \mathbf{X}_{i_2}^{(k)} \end{pmatrix}, \quad \mathbf{N}_i = \begin{pmatrix} \mathbf{N}_{i_1}^{(k)} \\ \bar{\mathbf{N}}_{i_2}^{(k)} \end{pmatrix} \quad (8)$$

and each $\mathbf{\Lambda}_{ij}$ is the Alamouti–like overall frequency domain channel matrix from the i–th user transmit antennas to the j–th receive antenna. The two users can be decoupled by applying the following linear zero–forcing interference canceller:

$$\begin{aligned} \begin{pmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{pmatrix} &\triangleq \begin{pmatrix} \mathbf{I}_{2N} & -\mathbf{\Lambda}_{12}\mathbf{\Lambda}_{22}^{-1} \\ -\mathbf{\Lambda}_{21}\mathbf{\Lambda}_{11}^{-1} & \mathbf{I}_{2N} \end{pmatrix} \begin{pmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{\Sigma} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Delta} \end{pmatrix} \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{pmatrix} + \begin{pmatrix} \tilde{\mathbf{N}}_1 \\ \tilde{\mathbf{N}}_2 \end{pmatrix} \end{aligned} \quad (9)$$

where $\mathbf{\Sigma} = \mathbf{\Lambda}_{11} - \mathbf{\Lambda}_{12}\mathbf{\Lambda}_{22}^{-1}\mathbf{\Lambda}_{21}$ and $\mathbf{\Delta} = \mathbf{\Lambda}_{22} - \mathbf{\Lambda}_{21}\mathbf{\Lambda}_{11}^{-1}\mathbf{\Lambda}_{12}$. It can be verified that both $\mathbf{\Sigma}$ and $\mathbf{\Delta}$ have an Alamouti–like structure. Consequently, the equalization procedure described by (6) can be applied to each user $\{\mathbf{Z}_1, \mathbf{Z}_2\}$ in order to recover the original data $\{\mathbf{X}_1, \mathbf{X}_2\}$ (see [9]).

C. The Multi–User Case

With M users (each equipped with two antennas), we can use M receive antennas to decouple all users and hence, increase the system capacity. Equation (5) generalizes to

$$\underbrace{\begin{pmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \vdots \\ \mathbf{Y}_M \end{pmatrix}}_{2NM\times 1} = \underbrace{\begin{pmatrix} \mathbf{\Lambda}_{11} & \mathbf{\Lambda}_{12} & \dots & \mathbf{\Lambda}_{1M} \\ \mathbf{\Lambda}_{21} & \mathbf{\Lambda}_{22} & \dots & \mathbf{\Lambda}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Lambda}_{M1} & \mathbf{\Lambda}_{M2} & \dots & \mathbf{\Lambda}_{MM} \end{pmatrix}}_{2NM\times 2NM} \underbrace{\begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_M \end{pmatrix}}_{2NM\times 1} + \underbrace{\begin{pmatrix} \mathbf{N}_1 \\ \mathbf{N}_2 \\ \vdots \\ \mathbf{N}_M \end{pmatrix}}_{2NM\times 1} \quad (10)$$

where $\mathbf{Y}_i$, $\mathbf{X}_i$, and $\mathbf{N}_i$ are given by (8) and each $\mathbf{\Lambda}_{ij}$ is the Alamouti–like frequency domain channel matrix from the i–th user transmit antennas to the j–th receive antenna. We can iteratively recover the symbols of each user by successive Schur complementation, starting from the M–th user, proceeding to the $(M-1)$–th user, and down to the first user. To recover the symbols of the M–th user we partition the channel matrix in (10) as

$$\left(\begin{array}{ccc|c} \mathbf{\Lambda}_{11} & \mathbf{\Lambda}_{12} & \dots & \mathbf{\Lambda}_{1M} \\ \mathbf{\Lambda}_{21} & \mathbf{\Lambda}_{22} & \dots & \mathbf{\Lambda}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \hline \mathbf{\Lambda}_{M1} & \mathbf{\Lambda}_{M2} & \dots & \mathbf{\Lambda}_{MM} \end{array}\right) = \left(\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & \mathbf{D} \end{array}\right) \quad (11)$$

with $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ denoting the $2N(M-1)\times 2N(M-1)$ upper–left, $2N(M-1)\times 2N$ upper–right, $2N\times 2N(M-1)$ lower–left, and $2N\times 2N$ lower–right matrices, respectively.

A linear interference canceller similar to the one designed for the two–user case in (9) can be used to suppress the interference from the first $M-1$ users on the M–th user as follows:

$$\begin{aligned} \left(\begin{array}{c} \mathbf{Z}_{1:M-1} \\ \hline \mathbf{Z}_M \end{array}\right) &\triangleq \left(\begin{array}{c|c} \mathbf{I}_{2N\times(M-1)} & -\mathbf{BD}^{-1} \\ \hline -\mathbf{CA}^{-1} & \mathbf{I}_{2N} \end{array}\right) \left(\begin{array}{c} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \vdots \\ \hline \mathbf{Y}_M \end{array}\right) \\ &= \left(\begin{array}{c|c} \mathbf{\Sigma}_{1:M-1} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{\Delta}_M \end{array}\right) \left(\begin{array}{c} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \hline \mathbf{X}_M \end{array}\right) + \left(\begin{array}{c} \tilde{\mathbf{N}}_1 \\ \tilde{\mathbf{N}}_2 \\ \vdots \\ \hline \tilde{\mathbf{N}}_M \end{array}\right) \end{aligned} \quad (12)$$

where $\mathbf{\Sigma}_{1:M-1} = \mathbf{A} - \mathbf{BD}^{-1}\mathbf{C}$ and $\mathbf{\Delta}_M = \mathbf{D} - \mathbf{CA}^{-1}\mathbf{B}$. It could be easily shown that $\mathbf{\Delta}_M$ has an Alamouti–like structure and that $\mathbf{\Sigma}_{1:M-1}$ is an $2N(M-1)\times 2N(M-1)$ block matrix with each $2N \times 2N$ block is an Alamouti–like matrix. This result follows directly from the following properties of Alamouti–like matrices:

- The sum or difference of two Alamouti–like matrices is an Alamouti–like matrix.
- The inverse of an Alamouti–like matrix is Alamouti–like.
- The inverse of a block matrix with Alamouti–like subblocks is a block matrix with Alamouti–like subblocks.

The next step is to apply the single–user SC MMSE–FDE of (6) to $\mathbf{Z}_M$ in order to get an estimate for the DFT of the symbols of the M–th user, $\hat{\mathbf{X}}_M$. Then, we apply IDFT to $\hat{\mathbf{X}}_M$ to get an estimate for the corresponding symbols, $\hat{\mathbf{x}}_M$ The symbols of the $(M-1)$–th user can be recovered by repeating the procedure on the reduced system:

$$\underbrace{\begin{pmatrix} \mathbf{Z}_{1:M-1} \end{pmatrix}}_{2N(M-1)\times 1} = \underbrace{\begin{pmatrix} \mathbf{\Sigma}_{1:M-1} \end{pmatrix}}_{2N(M-1)\times 2N(M-1)} \underbrace{\begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_{M-1} \end{pmatrix}}_{2N(M-1)\times 1} + \underbrace{\begin{pmatrix} \tilde{\mathbf{N}}_1 \\ \tilde{\mathbf{N}}_2 \\ \vdots \\ \tilde{\mathbf{N}}_{M-1} \end{pmatrix}}_{2N(M-1)\times 1} \quad (13)$$

Again, we partition $\mathbf{\Sigma}_{1:M-1}$ in a way similar to (11) with $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ now being the $2N(M-2)\times 2N(M-2)$ upper–left, $2N(M-2)\times 2N$ upper–right, $2N \times 2N(M-2)$ lower–left, and $2N \times 2N$ lower–right matrices, respectively. We then apply an interference canceller scheme similar to (12), followed by a single–user SC MMSE–FDE to get an estimate for the symbols of the $(M-1)$–th user, $\hat{\mathbf{x}}_{M-1}$. We then proceed until we recover the symbols of all M users. The block diagram of the multi–user receiver is shown in Figure 5.

III. Adaptive Scheme

The joint interference–cancellation and equalization technique described above requires the channels to be known at the receiver. Channel estimation is done by adding a training sequence to each block, which increases the system overhead. In this section, we develop an adaptive receiver for joint interference cancellation and equalization of the FDE–STBC. The adaptive receiver uses a few training blocks during initialization, then it tracks the channel variations in a decision–directed mode.

By inspecting the structure of the interference canceller described in Section II, we find that it successively multiplies the received symbols of each user with alamouti–like matrices in order to decouple them. We can verify that the overall response of the interference canceller and the MMSE equalizers, i.e., the mapping from the $\{\mathbf{Y}_i\}$ to the $\{\hat{\mathbf{X}}_i\}$ has the following form

$$\begin{pmatrix} \hat{\mathbf{X}}_1 \\ \hat{\mathbf{X}}_2 \\ \vdots \\ \hat{\mathbf{X}}_M \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \dots & \mathbf{A}_{1M} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \dots & \mathbf{A}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{M1} & \mathbf{A}_{M2} & \dots & \mathbf{A}_{MM} \end{pmatrix} \begin{pmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \vdots \\ \mathbf{Y}_M \end{pmatrix} \quad (14)$$

where each $\mathbf{A}_{ij}, i, j = 1, \cdots, M$, has the following Alamouti–like structure:

$$\mathbf{A}_{ij} = \begin{pmatrix} \mathbf{A}_{ij_1} & \mathbf{A}_{ij_2} \\ \mathbf{A}^*_{ij_2} & -\mathbf{A}^*_{ij_1} \end{pmatrix} \quad (15)$$

For the two–user case, the entries of $\mathbf{A}_{ij}$, $i, j = 1, 2$, are given explicitly in [9]. However, the explicit knowledge of the entries of these matrices is not needed for the development of the adaptive solution. The adaptive solution proposed here works for any $\mathbf{A}'_{ij}s$ regardless of their entries as long as they have an Alamouti-like structure. Equation (14) can be rewritten as

$$\begin{aligned} \begin{pmatrix} \hat{\mathbf{X}}^{(k)}_{i1} \\ \bar{\hat{\mathbf{X}}}^{(k)}_{i2} \end{pmatrix} &= \sum_{j=1}^{M} \begin{pmatrix} \text{diag}(\mathbf{Y}^{(k)}_j) & \text{diag}(\bar{\mathbf{Y}}^{(k+1)}_j) \\ \text{diag}(\mathbf{Y}^{(k+1)}_j) & -\text{diag}(\bar{\mathbf{Y}}^{(k)}_j) \end{pmatrix} \begin{pmatrix} \mathbf{W}_{ij_1} \\ \mathbf{W}_{ij_2} \end{pmatrix} \\ &\triangleq \sum_{j=1}^{M} \mathbf{U}^j_k \mathcal{W}_{ij} \\ &= \begin{pmatrix} \mathbf{U}^1_k & \dots & \mathbf{U}^M_k \end{pmatrix} \begin{pmatrix} \mathcal{W}_{i1} \\ \vdots \\ \mathcal{W}_{iM} \end{pmatrix} = \mathcal{U}_k \mathcal{W}^i \end{aligned} \quad (16)$$

where $i = 1 \cdots M$, and $\mathbf{W}_{ij_1}$ and $\mathbf{W}_{ij_2}$ are the vectors containing the diagonal elements of $\mathbf{A}_{ij_1}$ and $\mathbf{A}_{ij_2}$, respectively. Moreover, $\mathcal{W}_{ij}$ is a $2N \times 1$ vector containing the elements of $\{\mathbf{W}_{ij_1}$ and $\mathbf{W}_{ij_2}\}$, and $\mathbf{U}^i_k$ is an alamouti–like matrix of size $2N \times 2N$ containing the received symbols at antenna i from blocks k and $k+1$. Equation (16) reveals the special structure of the interference canceller for the STBC problem. In the non-adaptive scenario, the coefficients of $\mathcal{W}_{ij}$ are calculated from a channel estimate at every block. Equation (16) suggests that $\mathcal{W}^i$ can be computed adaptively, e.g., by using an RLS algorithm (for faster convergence) or some other adaptive filter. In the case of RLS, the receiver coefficients are updated every two blocks according to the recursions:

$$\mathcal{W}^i_{k+2} = \mathcal{W}^i_k + \mathcal{P}_{k+2}\mathcal{U}^*_{k+2}\left[\mathbf{D}^i_{k+2} - \mathcal{U}_k\mathcal{W}^i_k\right] \quad (17)$$

where

$$\mathcal{P}_{k+2} = \lambda^{-1}[\mathcal{P}_k - \lambda^{-1}\mathcal{P}_k\mathcal{U}^*_{k+2}\mathbf{\Pi}(k)\mathcal{U}_{k+2}\mathcal{P}_k] \quad (18)$$

and

$$\mathbf{\Pi}(k) = \left(\mathbf{I}_{2N} + \lambda^{-1}\mathcal{U}_{k+2}\mathcal{P}_k\mathcal{U}^*_{k+2}\right)^{-1} \quad (19)$$

and λ is a forgetting factor that is close to 1. The initial conditions are $\mathcal{W}^i_0 = \mathbf{0}$ and $\mathcal{P}_0 = \delta\mathbf{I}_{2MN}$, δ is a large number, and $\mathbf{I}_{2MN}$ is the $2MN \times 2MN$ identity matrix. $\mathbf{D}^i_{k+2}$ is the desired response vector given by

$$\mathbf{D}^1_{k+2} = \begin{cases} \begin{pmatrix} \mathbf{X}^{(k+2)}_{i_1} \\ \bar{\mathbf{X}}^{(k+2)}_{i_2} \end{pmatrix} & \text{for training} \\ \begin{pmatrix} \check{\mathbf{X}}^{(k+2)}_{i_1} \\ \bar{\check{\mathbf{X}}}^{(k+2)}_{i_2} \end{pmatrix} & \text{for decision–directed tracking} \end{cases}$$

The block diagram of the adaptive receiver is shown in Figure 6. The received signals from both antennas are transformed to the frequency domain using FFT, then the matrices $\mathbf{U}^1_k, \cdots, \mathbf{U}^M_k$ in (16) are formed. The data matrix $\mathcal{U}_k$ is passed through the filters to form the frequency domain estimates for the M users' transmitted data $\hat{\mathbf{X}}_i$. The filter outputs are transformed back to the time domain using IFFT and a decision device is used to generate the receiver outputs. The receiver operates in a training mode where known training data are used to generate the error vectors and update the receiver coefficients until they converge, then it switches to a decision–directed mode where previous decisions are used to update the receiver coefficients for tracking. For decision–directed operation, the reconstructed data are transformed back to frequency domain and compared to the corresponding receiver outputs to generate error vectors. The error vectors are used to update the coefficients according to the RLS algorithm. When tracking channels with fast variations, retraining blocks might be needed to prevent divergence of the algorithm.

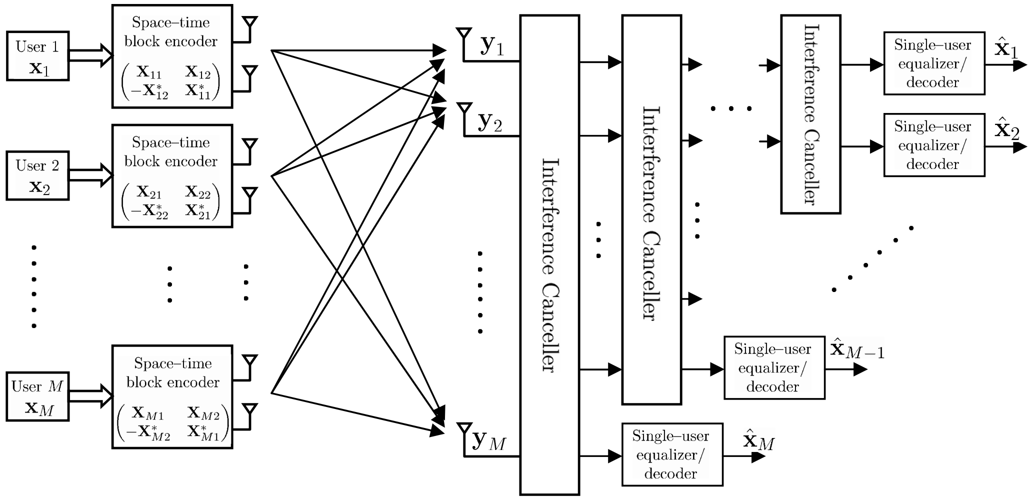

Fig. 5. A receiver structure for multi–user STBC transmissions.

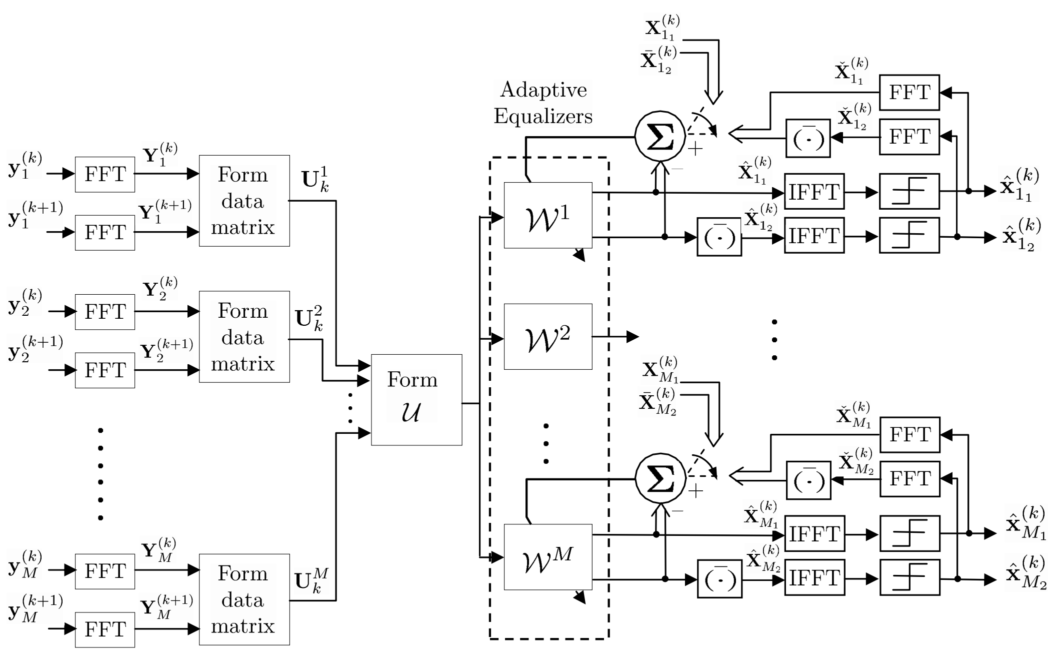

Fig. 6. An adaptive receiver structure for an M–user system with 2M–transmit and M–receive antennas.

A. Exploiting STBC structure

Although matrix inversion is needed in (18) for operation of the RLS algorithm, the computational complexity can be significantly reduced and matrix inversion can be avoided by exploiting the special STBC structure, in a manner similar to what we did in the two–user case in [9]. Actually, the complexity of the algorithm can be shown to be similar to that of an LMS implementation. We then end up with RLS performance at LMS cost. The rationale behind complexity reduction is as follows. Starting with Eq.(18) at $k = 0$, we get

$$\mathcal{P}_2 = \lambda^{-1}[\delta \mathbf{I}_{2MN} - \lambda^{-1}\delta^2 \mathcal{U}_2^* \mathbf{\Pi}(0)\mathcal{U}_2] \tag{20}$$

However, $\mathbf{\Pi}(0)$ evaluates to

$$\mathbf{\Pi}(0) = \left(\mathbf{I}_{2N} + \lambda^{-1}\delta \mathcal{U}_2 \mathcal{U}_2^*\right)^{-1} = \begin{pmatrix} \mathbf{\Pi}_N & \mathbf{0}_N \\ \mathbf{0}_N & \mathbf{\Pi}_N \end{pmatrix} \tag{21}$$

where $\mathbf{\Pi}(0)$ is $2N \times 2N$ and $\mathbf{\Pi}_N$ is $N \times N$ diagonal and given by

$$\mathbf{\Pi}_N = \left(\mathbf{I}_N + \lambda^{-1}\delta \sum_{i=1}^{M} \mathrm{diag}\left(|\mathbf{Y}_i^0|^2 + |\mathbf{Y}_i^1|^2\right)\right)^{-1}$$

Substituting (21) into (20), we find that $\mathcal{P}_2$ is given by

$$\begin{aligned} \mathcal{P}_2 &= \lambda^{-1}\delta \mathbf{I}_{2MN} - (\lambda^{-1}\delta)^2 \mathcal{U}_2^* \mathbf{\Pi}(0) \mathcal{U}_2 \\ &= \begin{pmatrix} \mathbf{P}_{11}(2) & \dots & \mathbf{P}_{1\ 2M}(2) \\ \vdots & \ddots & \vdots \\ \mathbf{P}_{2M\ 1}(2) & \dots & \mathbf{P}_{2M\ 2M}(2) \end{pmatrix} \end{aligned} \tag{22}$$

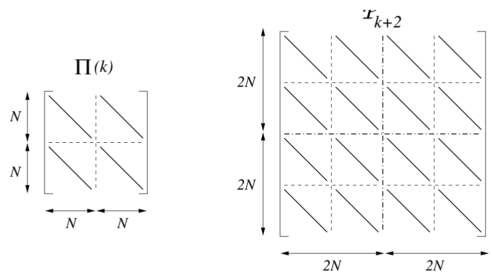

Fig. 7. Matrix structures of $\mathbf{\Pi}(k)$ and $\mathcal{P}_{k+2}$ for $M = 2$.

where $\mathbf{P}_{ij}(2)$, $i, j = 1 \cdots 2M$ are $N \times N$ diagonal. Proceeding for $k = 2$,

$$\mathbf{\Pi}(2) = \left(\mathbf{I}_{2N} + \lambda^{-1}\mathcal{U}_4\mathcal{P}_2\mathcal{U}_4^*\right)^{-1} \tag{23}$$

After simple algebra, it can be verified that $\mathbf{\Pi}(2)$ has the form:

$$\mathbf{\Pi}(2) \triangleq \begin{pmatrix} \mathbf{\Psi}_{11}(2) & \mathbf{\Psi}_{12}(2) \\ \mathbf{\Psi}_{21}(2) & \mathbf{\Psi}_{22}(2) \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{\Pi}_{11}(2) & \mathbf{\Pi}_{12}(2) \\ \mathbf{\Pi}_{21}(2) & \mathbf{\Pi}_{22}(2) \end{pmatrix} \tag{24}$$

where $\mathbf{\Psi}_{ij}(2), i, j = 1, 2$, are $N \times N$ diagonal matrices. Block matrix inversion can be used to evaluate $\mathbf{\Pi}(2)$. It can be easily shown that no matrix inversion is needed since $\mathbf{\Psi}_{ij}(2), i, j = 1, 2$, are all diagonal matrices. Moreover, the $\mathbf{\Pi}_{ij}(2), i, j = 1, 2$, are $N \times N$ diagonal matrices. $\mathcal{P}_4$ is then found to be

$$\mathcal{P}_4 = \lambda^{-1}\left[\mathcal{P}_2 - \lambda^{-1}\mathcal{P}_2\mathcal{U}_4^*\mathbf{\Pi}(2)\mathcal{U}_4\mathcal{P}_2\right] \tag{25}$$

It follows that $\mathcal{P}_4$ is a $2MN \times 2MN$ block matrix that consists of $4M^2$ $N \times N$ diagonal matrices. This means that the number of entries to be calculated is much lower than for a full matrix. If we proceed further beyond $k = 2$, we will find that the structures for $\mathbf{\Pi}(k)$ and $\mathcal{P}_{k+2}$ stay the same. The matrix structures for the two–user case, $M = 2$, are shown in Figure 7. Table I shows how we can use the structure of $\mathcal{P}_{k+2}$ to update its entries. It is worth mentioning that all $N \times N$ matrices in Table I are diagonal. This means that any matrix multiplication is simply evaluated by N scalar multiplications.

IV. Simulation Results

In this section, we provide simulation results for the performance of the adaptive interference canceller and equalizer for STBC. We simulated two different scenarios, one for the two–user case and the other for the three–user case. A Typical Urban (TU) channel is considered with a linearized GMSK transmit pulse shape. Furthermore, all channels are assumed to be independent. The overall channel impulse response memory of the channel is $\nu = 3$. In the two user scenario, each user is equipped with two transmit antennas. The receiver is equipped with 2 receive antennas. In the three–user scenario, the receiver is equipped with three antennas. 8-PSK signal constellation is used. The Signal–to–interference–noise ratio (SINR) is set to 0dB, i.e., all users are transmitting at the same power. Data blocks of 32 symbols plus 3 symbols for the cyclic prefix are used. Figure 8 shows the bit–error–rate performance of the system compared to the single user case at two different Doppler frequencies. From this figure, it is clear that the adaptive interference cancellation technique can separate co–channel users without sacrificing performance. However, at higher Doppler frequencies, the RLS algorithm might not be able to track the channel variations. In this case, training more often can improve the system performance at the expense of increasing system overhead. It was shown in [8] that smaller training blocks can be used to maintain low system overhead.

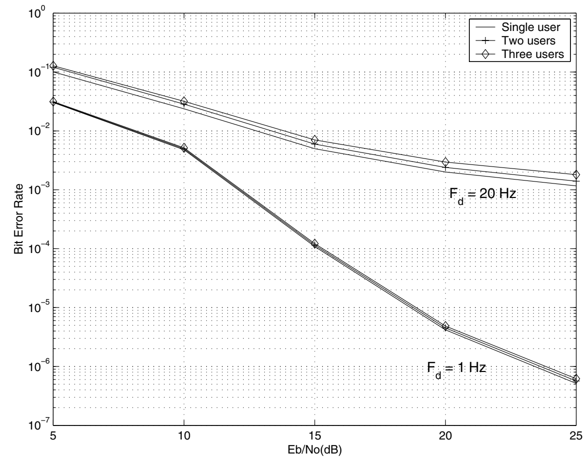

Fig. 8. BER performance of the adaptive receiver.

V. Conclusions

An adaptive scheme for joint interference cancellation and equalization of multi–user space–time block–coded transmission is developed. The scheme is based on a modified low–complexity RLS algorithm that exploits the rich structure of STBC to separate M equal–powered co–channel users. Both training and tracking performance results of the scheme are presented. It is shown that the system capacity can be increased by using this scheme while maintaining low system complexity and overhead and without sacrificing performance or bandwidth.

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Table I. Adaptation Algorithm

Define

$$\mathcal{P}_k \triangleq \begin{pmatrix} \mathbf{P}_{11}(k) & \mathbf{P}_{12}(k) & \dots & \mathbf{P}_{1\ 2M}(k) \\ \mathbf{P}_{21}(k) & \mathbf{P}_{22}(k) & \dots & \mathbf{P}_{2\ 2M}(k) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{2M\ 1}(k) & \mathbf{P}_{2M\ 2}(k) & \dots & \mathbf{P}_{2M\ 2M}(k) \end{pmatrix}$$

where each $\mathbf{P}_{ij}(k)$ is $N \times N$ diagonal. Let

$$\begin{aligned} \mathcal{U}_k &\triangleq \begin{pmatrix} \mathbf{U}_k^1 & \mathbf{U}_k^2 & \dots & \mathbf{U}_k^M \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{U}_{11}(k) & \mathbf{U}_{21}(k) & \dots & \mathbf{U}_{2M\ 1}(k) \\ \mathbf{U}_{12}(k) & \mathbf{U}_{22}(k) & \dots & \mathbf{U}_{2M\ 2}(k) \end{pmatrix} \end{aligned}$$

where the entries of $\mathcal{U}_k$ are given by Equation (16). Also, each $\mathbf{U}_{ij}(k)$ is $N \times N$ diagonal. $\mathcal{P}_k$ is now updated as follows:

1) Let

$$\mathbf{\Psi}_k = \mathbf{I}_{2N} + \lambda^{-1}\mathcal{U}_{k+2}\mathcal{P}_k\mathcal{U}_{k+2}^* = \begin{pmatrix} \mathbf{\Psi}_{11}(k) & \mathbf{\Psi}_{12}(k) \\ \mathbf{\Psi}_{21}(k) & \mathbf{\Psi}_{22}(k) \end{pmatrix}$$

2) Compute its block entries $\mathbf{\Psi}_{ij}(k)$, $i, j = 1, 2$, as follows

$$\mathbf{\Psi}_{ij}(k) = \xi_{ij} + \lambda^{-1} \cdot \sum_{l=1}^{2M} \sum_{m=1}^{2M} \mathbf{U}_{mi}(k+2)\mathbf{P}_{ml}(k)\mathbf{U}_{lj}^*(k+2)$$

where $\xi_{ij} = \mathbf{I}_N$ when $i = j$, and zero otherwise. Again, each $\mathbf{\Psi}_{ij}(k)$ is $N \times N$ diagonal.

3) Compute

$$\mathbf{\Pi}(k) = \begin{pmatrix} \mathbf{\Sigma}_{\Psi}^{-1}(k) & \mathbf{\Sigma}_{\Psi}^{-1}(k)\mathbf{\Psi}_{12}\mathbf{\Psi}_{22}^{-1} \\ \mathbf{\Psi}_{22}^{-1}\mathbf{\Psi}_{21}\mathbf{\Sigma}_{\Psi}^{-1}(k) & \mathbf{\Psi}_{22}^{-1}\mathbf{\Psi}_{21}\mathbf{\Sigma}_{\Psi}^{-1}(k)\mathbf{\Psi}_{12}\mathbf{\Psi}_{22}^{-1} \end{pmatrix}$$

where $\mathbf{\Sigma}_{\Psi}(k) = \mathbf{\Psi}_{11}(k) - \mathbf{\Psi}_{12}(k)\mathbf{\Psi}_{22}^{-1}(k)\mathbf{\Psi}_{21}(k)$ is the Schur complement of $\mathbf{\Psi}_{22}(k)$ and the time index (k) has been dropped for compactness. The $\mathbf{\Pi}_{ij}(k)$ blocks of $\mathbf{\Pi}(k)$ are also $N \times N$ diagonal.

4) Define $\mathbf{\Phi}_k = \mathcal{U}_{k+2}^*\mathbf{\Pi}(k)\mathcal{U}_{k+2}$. It has a structure similar to $\mathcal{P}_k$, then the $N \times N$ diagonal matrices $\mathbf{\Phi}_{ij}(k)$, $i, j = 1 \cdots 2M$, are given by

$$\mathbf{\Phi}_{ij}(k) = \sum_{l=1}^{2} \sum_{m=1}^{2} \mathbf{U}_{il}^*(k+2)\mathbf{\Pi}_{lm}(k)\mathbf{U}_{jm}(k+2)$$

5) Update $\mathbf{P}_{ij}(k+2)$ as

$$\mathbf{P}_{ij}(k+2) = \lambda^{-1}\mathbf{P}_{ij}(k) - \lambda^{-2} \sum_{l=1}^{2M} \sum_{m=1}^{2M} \mathbf{P}_{im}(k)\mathbf{\Phi}_{ml}(k)\mathbf{P}_{lj}(k)$$

6) Repeat the previous steps for each iteration.