DISTRIBUTED BEAMFORMING AND MODE SELECTION BASED ON INSTANTANEOUS SYSTEM THROUGHPUT

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ABSTRACT

In this paper, we design cooperative beamforming weights for source, relay and destination nodes based on a minimum meansquare-error (MMSE) formulation under network power constraints. We also propose a mode selection procedure based on the instantaneous system throughput. Simulation results indicate that the MMSE cooperative beamforming method with mode selection achieves better performance compared to other beamforming methods: direct beamforming, relay beamforming, and cooperative beamforming without mode selection.

Index Terms— Beamforming, minimum mean-square-error (MMSE), communication mode selection.

1. INTRODUCTION

In slow fading channels, non-orthogonal amplify-and-forward (NAF) relay schemes have been studied to increase diversity gain and spectral efficiency [1,2]. In NAF protocols, the source (denoted by S) transmits new data at each phase to circumvent spectral efficiency loss from a half-duplex relay (denoted by R), which cannot transmit and receive data simultaneously. The relay R forwards previously received data to the destination (denoted by D) as shown in Fig. 1(a). For this reason, some data cannot enjoy the cooperative gain achieved through the relay R. On the other hand, under static channel conditions, when the channel does not vary over a few frames, the diversity gain of the communication system might decrease. In this case, beamforming schemes can be used to obtain high processing gain [3].

In this paper, we design distributed (cooperative) beamforming weights based on a minimum mean-square-error (MMSE) formulation with network power constraints in a manner similar to [4]. Here, we assume perfect channel state information (CSI) at each node. The communication scenario for obtaining the CSI and numerical results with uncertain CSI will be presented later. The difference from [4] is consideration of the direct path between S and D. We also employ network power constraints for the S and R nodes. The cooperative beamforming method involves simultaneous beamforming of the *same data* with *increased modulation size* from the S and R nodes to the D node as shown in Fig. 1(b), so that every symbol can achieve cooperative gain. However, from the perspective of network power and transmission data rates, it happens that cooperative Ali H. Sayed

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Fig. 1. Illustration of relay protocols over two phases, t_1 and t_2 . (a) NAF scheme. (b) Cooperative beamforming scheme.

beamforming performs worse than direct beamforming from S to D when the direct channel is good and the relay channel is bad (in terms of signal-to-noise ratio (SNR) conditions). Hence, we propose to use a mode selection procedure based on the instantaneous system throughput [1]. Numerical results illustrate that the designed MMSE cooperative beamforming method with mode selection performs better than other beamforming methods without mode selection.

Notation. Throughout this paper, for any vector or matrix, the superscripts 'T' and '*' denote transposition and complex conjugate transposition, respectively. 'E' stands for expectation of a random variable; for any scalar q, vector q, and matrix Q, the notation |q|, ||q||, and $||Q||_F$ denote the absolute value of q, 2-norm of q, and Frobenius-norm of Q, respectively; tr(Q) represents the trace of matrix Q; I_q is a q-dimensional identity matrix; 0 represents zero vector or matrix; and $\text{Re}(\cdot)$ takes the real value of its argument.

2. COOPERATIVE SYSTEM AND SIGNAL MODEL

The source, relay, and destination nodes have N_S , N_R , and N_D antennas, respectively. The channel matrix of the direct channel between the S and the D is represented by $\boldsymbol{H} \in \mathbb{C}^{N_D \times N_S}$ and the relay channel matrices of the first and second-hops are represented by $\boldsymbol{F} \in \mathbb{C}^{N_R \times N_S}$ and $\boldsymbol{G} \in \mathbb{C}^{N_D \times N_R}$, respectively. The elements of \boldsymbol{H} , \boldsymbol{F} and \boldsymbol{G} are i.i.d and zero-mean complex Gaussian random variables with variances σ_H^2 , σ_F^2 , and σ_G^2 , respectively, as determined by the path loss effects of shadowing and large scale fading. It is assumed that every channel remains static during several transmission data blocks (or frames). The additive white Gaussian noise (AWGN) at the R and D are denoted by $\boldsymbol{n}_r(t) \in \mathbb{C}^{N_R \times 1}$ and $\boldsymbol{n}_d(t) \in \mathbb{C}^{N_D \times 1}$, respectively, where $\mathbb{E} \boldsymbol{n}_r \boldsymbol{n}_r^* = \sigma_{n_r}^2 \boldsymbol{I}_{N_R}$ and $\mathbb{E} \boldsymbol{n}_d \boldsymbol{n}_d^* = \sigma_{n_d}^2 \boldsymbol{I}_{N_D}$.

In the first phase at t_1 , S broadcasts a symbol vector $a_1d \in \mathbb{C}^{N_S \times 1}$, where d is a data symbol; $E|d|^2 = 1$; and $a_1 \in \mathbb{C}^{N_S \times 1}$ is a transmit beamforming vector at the first phase. The received

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signals at the D and R nodes are then given by

$$\boldsymbol{y}(t_1) = \boldsymbol{H}\boldsymbol{a}_1\boldsymbol{d} + \boldsymbol{n}_d(t_1) \tag{1}$$

and

$$\boldsymbol{r}(t_1) = \boldsymbol{F}\boldsymbol{a}_1 \boldsymbol{d} + \boldsymbol{n}_r(t_1) \tag{2}$$

respectively. In the next phase at t_2 , the R and S nodes perform cooperative beamforming. While the R multiplies $r(t_1)$ by the relay transceiver beamforming matrix $\boldsymbol{W} \in \mathbb{C}^{N_R \times N_R}$, and forwards $\boldsymbol{x} = \boldsymbol{W}\boldsymbol{r}(t_1) \in \mathbb{C}^{N_R \times 1}$ to D, the S node simultaneously retransmits a signal $\boldsymbol{a}_2 d$ synchronized to arrive at D at the same time as the relayed signal, where $\boldsymbol{a}_2 \in \mathbb{C}^{N_S \times 1}$ is a transmit beamforming vector. Therefore, at D, the received signal vector $\boldsymbol{y}(t_2) \in \mathbb{C}^{N_D \times 1}$ is represented by

$$y(t_2) = GWr(t_1) + Ha_2d + n_d(t_2).$$
 (3)

The D combines two consecutively received signals $y(t_1)$ and $y(t_2)$ as follows:

$$\hat{d} = \boldsymbol{b}_1^* \boldsymbol{y}(t_1) + \boldsymbol{b}_2^* \boldsymbol{y}(t_2) \tag{4}$$

by using receive beamforming vectors $\boldsymbol{b}_1 \in \mathbb{C}^{N_D imes 1}$ and $\boldsymbol{b}_2 \in \mathbb{C}^{N_D imes 1}$.

3. MMSE PROBLEM FORMULATION

We now jointly design the set of beamforming weights $\{a_1, a_2, b_1, b_2, W\}$ in order to minimize the MSE under transmit power constraints at the source and relay nodes. Using (1)–(4), the overall signal model is

$$\hat{d} = (b_1^* H a_1 + b_2^* GWF a_1 + b_2^* H a_2)d
+ (b_1^* n_d(t_1) + b_2^* n_d(t_2) + b_2^* GW n_r(t_1)).$$
(5)

When the network power is limited by P_T , the desired MMSE problem is as follows:

$$\arg \min_{\substack{\{a_1, a_2, b_1, b_2, W\}}} \mathbb{E} |d - \hat{d}|^2$$

s.t. $\mathbb{E} ||a_1 d||^2 + \mathbb{E} ||a_2 d||^2 + \mathbb{E} ||x||^2 \le P_T$ (6)

The minimization problem (6) with inequality constraints can be transformed into

$$\arg\min_{\{a_1,a_2,b_1,b_2,W,\lambda\}} J \tag{7}$$

where the Lagrange cost J is

$$J = E |d - \hat{d}|^{2} + \lambda \left(E ||\boldsymbol{a}_{1}d||^{2} + E ||\boldsymbol{a}_{2}d||^{2} + E ||\boldsymbol{x}||^{2} - P_{T} \right)$$
(8)

and λ is a non-negative Lagrange multiplier. Although J in (8) is not guaranteed to be jointly convex over all the variables $\{a_1, a_2, b_1, b_2, W\}$, it is obviously convex over each of the variables. Therefore, alternating minimization procedures, where variables are optimized one at a time while keeping all others fixed [5], are applicable to get a feasible local optimal solution. Differentiating J with respect to its variables and equating the derivatives to zero (Karush-Kuhn-Tucker (KKT) conditions [6]), we can get the beamforming weights. Using the techniques of complex matrix derivatives and linear algebra [7,8], the derivative of J with respect to W is obtained and equated to zero. As a result, we get

$$G^{*}b_{2}(1 - b_{1}^{*}Ha_{1} - b_{2}^{*}Ha_{2})a_{1}^{*}F^{*}$$

= $(G^{*}b_{2}b_{2}^{*}G + \lambda I_{N_{R}})W(Fa_{1}a_{1}^{*}F^{*} + \sigma_{n_{r}}^{2}I_{N_{R}}).$ ⁽⁹⁾

Using the matrix inversion lemma [7], when $\lambda > 0$, the solution is

$$W = \frac{(1 - b_1^* H a_1 - b_2^* H a_2) G^* b_2 a_1^* F^*}{(\|G^* b_2\|^2 + \lambda) (\|Fa_1\|^2 + \sigma_{n_r}^2)}.$$
 (10)

We can also show that (10) is a minimum Frobenius-norm solution to (9) when $\lambda = 0$. For the transmit beamforming vector, the derivative of J with respect to a_1 is obtained and equated to zero. As a result, we get

$$ig(\|m{H}^*m{b}_1 + m{F}^*m{W}^*m{G}^*m{b}_2 \|^2 m{I}_{N_S} + \lambda m{I}_{N_S} + \lambda m{F}^*m{W}^*m{W}m{F} ig) a_1 \ = (m{H}^*m{b}_1 + m{F}^*m{W}^*m{G}^*m{b}_2)(1 - m{b}_2^*m{H} a_2). \ (11)$$

When $\lambda \neq 0$, we get

$$a_{1} = \left(\| \boldsymbol{H}^{*} \boldsymbol{b}_{1} + \boldsymbol{F}^{*} \boldsymbol{W}^{*} \boldsymbol{G}^{*} \boldsymbol{b}_{2} \|^{2} \boldsymbol{I}_{N_{S}} + \lambda \boldsymbol{I}_{N_{S}} + \lambda \boldsymbol{F}^{*} \boldsymbol{W}^{*} \boldsymbol{W} \boldsymbol{F} \right)^{-1} \\ \times \left(\boldsymbol{H}^{*} \boldsymbol{b}_{1} + \boldsymbol{F}^{*} \boldsymbol{W}^{*} \boldsymbol{G}^{*} \boldsymbol{b}_{2} \right) (1 - \boldsymbol{b}_{2}^{*} \boldsymbol{H}^{*} \boldsymbol{a}_{2})$$
(12)

which can be reformulated as

$$a_{1} = \frac{(1 - b_{2}^{*}Ha_{2})(H^{*}b_{1} + F^{*}W^{*}G^{*}b_{2})}{\|H^{*}b_{1} + F^{*}W^{*}G^{*}b_{2}\|^{2} + \lambda + \lambda\|WF\|_{F}^{2}}$$
(13)

The equality between (12) and (13) can be shown by using (10), the matrix inversion lemma, and $\|Q\|_F^2 = \operatorname{tr}(QQ^*)$. We can also show that (13) is a minimum 2-norm solution of (11) when $\lambda = 0$. Similarly, equating the derivatives of J with respect to a_2 , b_1 and b_2 to zero, we obtain

$$a_{2} = \frac{(1 - b_{2}^{*}GWFa_{1} - b_{1}^{*}Ha_{1})H^{*}b_{2}}{\|H^{*}b_{2}\|^{2} + \lambda},$$
 (14)

$$b_1 = \frac{(1 - a_1^* F^* W^* G^* b_2 - a_2^* H^* b_2) H a_1}{\|Ha_1\|^2 + \sigma_{n_d}^2}$$
(15)

and

$$\boldsymbol{b}_{2} = \frac{(1 - \boldsymbol{a}_{1}^{*}\boldsymbol{H}^{*}\boldsymbol{b}_{1}) \big(\boldsymbol{GWF}\boldsymbol{a}_{1} + \boldsymbol{H}\boldsymbol{a}_{2}\big)}{\|\boldsymbol{GWF}\boldsymbol{a}_{1} + \boldsymbol{H}\boldsymbol{a}_{2}\|^{2} + \sigma_{n_{r}}^{2} \|\boldsymbol{GW}\|_{F}^{2} + \sigma_{n_{d}}^{2}}.$$
 (16)

Next, by equating the derivative of J with respect to λ to zero, the equality

$$P_T = \|\boldsymbol{a}_1\|^2 + \|\boldsymbol{a}_2\|^2 + \|\boldsymbol{W}\boldsymbol{F}\boldsymbol{a}_1\|^2 + \sigma_{n_r}^2 \|\boldsymbol{W}\|_F^2$$
(17)

is obtained. Substituting (10), (13) and (14) into (17), we arrive at a degree six polynomial equation, f(x), and we can get the optimal Lagrange multiplier from the largest real root x_o of $f(x_o) = 0$, i.e.,

$$\lambda_o = (x_o)^+,\tag{18}$$

where $(v)^+ = \max(0, v)$. Since the optimum values are functions of one another, direct computation of $\{a_1, a_2, b_1, b_2, W, \lambda\}$ is a formidable task. An iterative procedure where variables are optimized one at a time while keeping all others fixed [5] is applicable to circumvent this difficulty. At the *k*th iteration, denoting the MSE and the beamforming weights by J_k and $\{a_{1,k}, a_{2,k}b_{1,k}, b_{2,k}, W_k\}$, respectively, the proposed iterative algorithm is described in Table 1. The difference between J_{k-1} and J_k can be used as a stopping criterion with a positive design factor ϵ in Step 3. It is easily seen that if $a_{1,0} = a_{2,0} = b_{1,0} = b_{2,0} = 0$, there is no feasible solution of (18). Such undesirable initial points should be avoided while implementing the algorithms.



Fig. 2. Illustration for signaling procedure and communication mode. The information available at each node is depicted in $\{\cdot\}$. Especially, feedback information is boxed.

Table 1. Iterative Algorithm

Step 1:	Initialization, $k = 0$
	$a_{1,0} = a_{2,0} = b_{1,0} = b_{2,0} = [1 \cdots 1],$
	$\lambda = 1, J_0 = 0.$
Step 2:	Iteration: $k \leftarrow k + 1$
	$W_k = f_W \left(a_{1,k-1}, a_{2,k-1}, b_{1,k-1}, b_{2,k-1} \lambda \right)$ in (10)
	$\boldsymbol{a}_{1,k} = f_{\boldsymbol{a}_1} \left(\boldsymbol{W}_k, \boldsymbol{a}_{2,k-1}, \boldsymbol{b}_{1,k-1}, \boldsymbol{b}_{2,k-1}, \lambda \right) $ in (13)
	$\boldsymbol{a}_{2,k} = f_{\boldsymbol{a}_2} \left(\boldsymbol{W}_k, \boldsymbol{a}_{1,k}, \boldsymbol{b}_{1,k-1}, \boldsymbol{b}_{1,k-1}, \lambda \right)$ in (14)
	$\boldsymbol{b}_{1,k} = f_{\boldsymbol{b}_2} \left(\boldsymbol{W}_k, \boldsymbol{a}_{1,k}, \boldsymbol{a}_{2,k}, \boldsymbol{b}_{2,k-1} \right)$ in (15)
	$\boldsymbol{b}_{2,k} = f_{\boldsymbol{b}_1} \left(\boldsymbol{W}_k, \boldsymbol{a}_{1,k}, \boldsymbol{a}_{2,k}, \boldsymbol{b}_{1,k} \right)$ in (16)
	$\lambda = f_{\lambda} \left(\tilde{\boldsymbol{W}}_{k}, \boldsymbol{a}_{1,k}, \boldsymbol{a}_{2,k}, \boldsymbol{b}_{1,k}, \boldsymbol{b}_{2,k}, \lambda \right) \text{ in (18)}$
	$J_k = f_J \left(\boldsymbol{W}_k, \boldsymbol{a}_{1,k}, \boldsymbol{a}_{2,k}, \boldsymbol{b}_{1,k}, \boldsymbol{b}_{2,k}, \lambda \right)$ in (8)
Step 3:	If $0 \le J_{k-1} - J_k \le \epsilon$ stop, otherwise go back Step 2.

4. MODE SELECTION AND COMMUNICATION SCENARIO

When H = 0 in the MMSE solution in the previous section, the direct link is not used for communications, i.e., a_2 and b_1 become zero vectors and the solutions $\{a_1, b_2, W\}$ become identical to the MMSE solution in [4] considering only the relay path. This method will be named relay beamforming in our simulations. On the other hand, when F = 0 or G = 0, the relay link is not used for communication, i.e., W = 0, and the first and second beamforming vectors become identical, i.e., $a_1 = a_2$ and $b_1 = b_2$. This means that S repeatedly transmits the same date twice with the same transmit power $\frac{\overline{P_T}}{2}$. Note that the two-phase (repeated transmission) direct communication is different from one-phase direct communication with respect to the modulation: since the former uses two-time resources while the latter uses one-time resource, the modulation size of the former should be twice as large as that of the latter to fulfill a comparable transmission rate, resulting in the performance difference between two- and one-phase direct communications. Also noting that the MSE formulation in section III comes from the two-phase communication, unfortunately, the proposed solutions in (10) and (13)-(16) do not yield one-phase direct communication scheme. Therefore, a criterion to dynamically decide the communication mode is required to further improve the network performance and it can be based on system throughput as in [1].

Since it is natural in practice to decide communication modes before data communication, we propose a signaling period before data communication. Throughout the signaling period, as shown in Fig. 2, the data communication mode is determined and each node can get CSI based on the received signal and feedback information according to the communication mode. The first signaling comes from S. In the first signaling step, R and D receive a training signal from S and estimate $\{F, \sigma_{n_r}^2\}$ and $\{H, \sigma_{n_d}^2\}$, respectively. In the second signaling step, R transmits a training signal so that the S and D can estimate F and G, respectively. The R also transmits feedback information illustrated in the boxes $\sigma_{n_r}^2$ and $\{\sigma_{n_r}^2, F\}$ to S and D, respectively. Using CSI, D can then determine the communication mode as follows:

$$\operatorname{comm.} \begin{cases} \operatorname{w/relay,} & \text{if } R_r > R_d, \text{ feed back } + 1 \\ \operatorname{w/o relay,} & \text{if } R_r \le R_d, \text{ feed back } - 1 \end{cases}$$
(19)

where the achievable rate the of direct beamforming system is

$$R_d = \log\left(1 + \frac{|\boldsymbol{b}^* \boldsymbol{H} \boldsymbol{a}|^2}{\sigma_{n_d^2}^2 \|\boldsymbol{b}\|^2}\right)$$
(20)

and the achievable rate of the relay beamforming system is

$$R_{r} = \frac{1}{2} \log \left(1 + \frac{|\boldsymbol{b}_{1}^{*} \boldsymbol{H} \boldsymbol{a}_{1} + \boldsymbol{b}_{2}^{*} \boldsymbol{G} \boldsymbol{W} \boldsymbol{F} \boldsymbol{a}_{1} + \boldsymbol{b}_{2}^{*} \boldsymbol{H} \boldsymbol{a}_{2}|^{2}}{\sigma_{n_{d}^{2}}^{2} (\|\boldsymbol{b}_{1}\|^{2} + \|\boldsymbol{b}_{2}\|^{2}) + \sigma_{n_{r}}^{2} \|\boldsymbol{b}_{2}^{*} \boldsymbol{G} \boldsymbol{W}\|^{2}} \right).$$
(21)

Beamforming vectors a and b in (20) can be obtained from the iterative algorithm without relay path, i.e., setting W = 0 and $a_2 = b_2 = 0$ in Table 1, by using $\frac{P_T}{2}$ since the average power consumption per phase of both systems is $\frac{P_T}{2}$. In (21), the pre-log factor $\frac{1}{2}$ is from the fact that the relay communication consumes two time phases. In the third signaling step, D transmits a training signal and feedback information $\{\pm 1\}$ according to the communication mode. If R is involved in communication, D feeds back $\{+1, G, \sigma_{n_d}^2\}$ and $\{+1, H, \sigma_{n_d}^2\}$ to S and R, respectively, and also broadcasts a training signal so that the S and R can estimate H and G, respectively. Otherwise, D feeds back $\{-1, \sigma_{n_d}^2\}$ and $\{-1\}$ to S and R, respectively, and transmits a training signal so that S can estimate H. Consequently, data communication is performed in the subsequent period according to the feedback information including the communication mode.

5. SIMULATION AND DISCUSSION

The BER performance of the proposed system is evaluated and discussed when $N_S = N_R = N_D = 2$. The transmitted signals



Fig. 3. BER performance comparison when $N_S = N_R = N_D = 2$, $\sigma_F^2/\sigma_{n_r}^2 = 12 \text{ dB}$, $\sigma_{n_d}^2 = \sigma_{n_r}^2 = 10^{-4}$ and $P_T = 2$ (solid line for perfect CSI and dashed line for uncertain CSI). (a) Direct beamforming. (b) Relay beamforming. (c) Cooperative beamforming. (d) Mode selection between (a) and (c).



Fig. 4. MSE over iteration numbers when $P_T = 2$.

from the sources are modulated by QPSK and 16-QAM for direct and cooperative beamforming, respectively. One frame consists of 100 symbols. Channels are fixed during signaling and two data frame transmissions, but they vary independently over the next signaling period.

In Fig. 3, the BER performance is evaluated over various σ_H^2 and σ_G^2 when $\sigma_F^2/\sigma_{n_r}^2 = 12 \,\mathrm{dB}$, $P_T = 2$, $\sigma_{n_d}^2 = \sigma_{n_r}^2 = 10^{-4}$ and $\epsilon = 10^{-4}$. For comparison purposes, the BER performance is evaluated for four different scenarios: (a) direct beamforming without relay link, (b) relay beamforming without direct link, (c) cooperative beamforming with direct and relay links, and (d) mode selection between (a) and (c). The solid line illustrates the BER performance when the CSIs are perfectly estimated and fed back to every node according to the signaling scenario in Fig. 2. The dashed line shows the BER performance with uncertain CSIs $\{\hat{H}, \hat{G}, \hat{F}\}$, where $\hat{A} = A + \Delta_A$ and the variance of the elements of Δ_A is $\sigma_A^2 \times 0.05$, i.e., 5% uncertain CSIs are used to generate beamforming vectors at each node. As we expected, the performance of direct beamforming (a) and relay beamforming (b) are independent of σ_G^2 and σ_H^2 , respectively, while that of the designed cooperative beamforming (c) enhances as σ_G^2 or σ_H^2 increases. Furthermore, it is seen that the mode selection get more performance gain as shown in (d).

Figure 4 shows the MSE J_k in (8) over the iterative steps in Table 1. It is observed that the MSE is decreasing through the proposed iterative algorithm.

6. CONCLUSION

In this paper, we designed cooperative beamforming weights based on an MMSE formulation under network power constraints and proposed a beamforming mode selection procedure. From the simulation results, it is seen that the designed cooperative beamforming method with mode selection performs better than direct and relay beamforming methods.

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