DIGITAL COMPENSATION OF RF NONLINEARITIES IN SOFTWARE-DEFINED RADIOS

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ABSTRACT

The wideband RF receiver in a software-defined radio (SDR) system suffers from the nonlinear effects caused by the front-end analog processing. In the presence of strong blocker (interference) signals, nonlinearities introduce severe cross modulation over the desired signals. This paper investigates how the nonlinear distortions can be compensated for by using digital signal processing techniques. In the proposed solution, the SDR scans the wide spectrum and locates the desired signal and strong blocker signals. After down-converting these signals separately into the baseband, the baseband processor processes them jointly to mitigate the cross-modulation interferences. As a result, the sensitivity of the wideband RF receiver to the nonlinearity impairment can be significantly lowered, simplifying the RF and analog circuitry design in terms of implementation cost and power consumption.

Index Terms— Cross modulation, nonlinearity, software-defined radio, wireless communications.

1. INTRODUCTION

A software-defined radio (SDR) system is a radio communication system which can tune to any frequency band and receive any modulation across a large frequency spectrum by means of programmable hardware [1, 2]. It brings the feasibility of providing different kinds of wireless services by using just a single reconfigurable chipset. The power of SDR imposes numerous challenges on the state-of-the-art communication technologies. Traditionally, in order to simultaneously communicate in different frequency bands, the receiver consists of several RF front-end modules so that signals in different bands can be received and processed separately. Fig. 1 shows an RF receiver dedicated to a communication channel with carrier frequency ω_c. Because of the high out-of-band rejection characteristic of the band-selection surface acoustic wave (SAW) filter, interferences at other frequencies are suppressed and hence cause little distortion to the desired signal [3]. Unlike conventional RF receivers, an SDR uses a wideband RF front-end module with several GHz bandwidth. A *tunable* synthesizer and mixer are exploited to lock in the desired frequency band and downconvert the signal to the baseband [4, 5]. Without the SAW filter,[1]

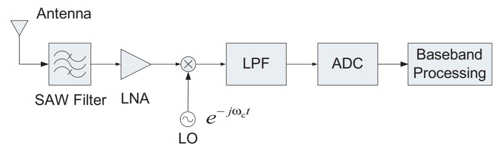

Fig. 1: A traditional receiver dedicated to the frequency band with carrier frequency ω_c.

all the signals and interferences existing in the wide band are amplified and downconverted. Due to the unavoidable nonlinearity in the low-noise amplifier (LNA) and mixer, the presence of strong blocker (interference) signals causes cross modulation over the desired signal. This threat becomes significantly harmful, especially when the desired signal is weak.

While analog/RF designers are striving to improve the linearity of wideband receivers, there have been works in the literature to mitigate this impairment by using digital domain techniques [6, 7]. These digital solutions provide a flexible alternative approach to combat nonlinearities, which is particularly appropriate for SDRs that have an extremely wide bandwidth and a reconfigurable hardware/software structure. However, all the currently available solutions assume a conventional *narrow-band*[2] radio receiver that allows sampling and processing the received signals at the Nyquist rate. This is prohibited in SDRs because their ultra-wide bandwidth requires unbearable sampling rate. A more feasible solution is thus highly desirable for SDRs. In this work, we propose a nonlinearity compensation scheme for the SDR structure recently introduced in [4, 5]. As shown in Fig. 2, the novel SDR system has a wideband RF front-end (0.8-6.0 GHz), and is able to selectively downconvert and sample the signals in desired frequency bands. Our scheme requires two RF signal paths - one is used for capturing the signal in the desired band, while the other is used to locate and acquire the blocker signal.[3] The baseband processor then jointly processes the two discretized signals to alleviate the effects of cross modulation. The next section describes the system model and formulates the effects of RF nonlinearities. For ease of illustration, we shall focus on the scenario with only one blocker signal in the paper. The proposed method can be extended in a straightforward manner to handle multiple blocker signals.

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[1] A SAW filter cannot be used here because it is application-specific with a fixed center frequency and bandwidth. Until now, there is no tunable SAW filter with sufficiently good performance. Furthermore, the SAW filter cannot be integrated on-chip with the receiver circuitry, which means that a multi-standard receiver with many SAW filters will be bulky and expensive.

[2] In comparison with SDRs that normally have a bandwidth of several GHz.

[3] The secondary RF path, used to acquire the blocker signal, can be implemented with smaller area and less power compared to the main path.

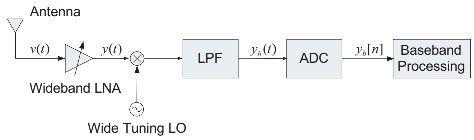

Fig. 2: SDR with a wideband front-end RF receiver.

Throughout this paper, we adopt the following notations. $(\cdot)^T$ denotes the matrix transpose and $(\cdot)^*$ represents the matrix conjugate transpose. $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ return the real and imaginary parts of its argument, respectively. $\mathbf{E}\{\cdot\}$ is the expected value with respect to the underlying probability measure.

2. SYSTEM MODEL

In the presence of nonlinearity, the input-output characteristic of the receiver front-end is modeled as

$$y(t) = \alpha_1 v(t) + \alpha_2 (v(t))^2 + \alpha_3 (v(t))^3 + w(t),$$

where $v(t)$ and $y(t)$ are the real input and output signals (see Fig. 2), $w(t)$ is additive white Gaussian noise, and α_1, α_2, α_3 are real constants with $|\alpha_2| \ll |\alpha_1|$ and $|\alpha_3| \ll |\alpha_1|$ [3]. In this expression, $\alpha_1 v(t)$ represents the linear component in the output, while $\alpha_2(v(t))^2$ and $\alpha_3(v(t))^3$ are the second and third-order nonlinear components in the output. The dynamic range of $v(t)$ depends on the sensitivity of the receive antenna and is limited to within the range $[-1, 1]$.

Assume that the acquired signal $v(t)$ contains a desired signal at frequency ω_1 and a blocker signal at frequency ω_2. Then, $v(t)$ can be represented by

$$v(t) = \text{Re}\left\{ z_1(t)\sqrt{2}e^{j\omega_1 t} + z_2(t)\sqrt{2}e^{j\omega_2 t} \right\},$$

where $z_1(t)$ and $z_2(t)$ are the corresponding complex baseband signals at ω_1 and ω_2. Taking the channel response of the desired channel into account, $z_1(t)$ is given by the convolution of the transmitted baseband signal $x_1(t)$ and the continuous-time baseband channel response $h_1(t)$, i.e.,

$$z_1(t) = \int_{-\infty}^{\infty} h_1(t-\tau)x_1(\tau)d\tau. \quad (1)$$

With proper down-conversion and low-pass filtering, the received complex baseband signal corresponding to the carrier frequency ω_1 is given by

$$y_b(t) = \alpha_1 z_1(t) + \frac{3\alpha_3}{2} z_1(t)|z_1(t)|^2 + 3\alpha_3 z_1(t)|z_2(t)|^2 + w_b(t),$$

where $w_b(t)$ is the additive complex white Gaussian noise in the baseband. This shows that $y_b(t)$ is distorted by the third-order harmonics $\frac{3\alpha_3}{2} z_1(t)|z_1(t)|^2$ and the cross-modulation term $3\alpha_3 z_1(t)|z_2(t)|^2$. If the amplitude of the desired signal is small, i.e., $|z_1(t)| \ll 1$, then the amplitude of $\frac{3\alpha_3}{2} z_1(t)|z_1(t)|^2$ is much smaller than that of $\alpha_1 z_1(t)$ and can be neglected. Its effect becomes significant only when the amplitude of $z_1(t)$ is close to 1, but can be mitigated by properly limiting the dynamic range of the input at little cost of loosing reception sensitivity. In a wideband SDR, the cross modulation is more dangerous because the power of the blocker signal can be as much as 60–70 dB more than that of the desired signal.[4] Since the receiver front-end has to maintain a minimum sensitivity level for the desired signal $z_1(t)$, the simultaneously acquired blocker signal can be quite large, making the term $3\alpha_3 z_1(t)|z_2(t)|^2$ comparable to the desired signal component $\alpha_1 z_1(t)$. Moreover, the distortion caused by cross modulation scales with the desired signal, and therefore, the signal-to-noise ratio does not improve when a stronger desired signal is received.

Define the effective signal-to-noise ratio at the receiver as

$$\text{SNR}_{\text{effective}} = \frac{\mathbf{E}\left\{\alpha_1^2 |z_1(t)|^2\right\}}{\mathbf{E}\left\{ \left| \frac{3\alpha_3}{2} z_1(t)|z_1(t)|^2 + 3\alpha_3 z_1(t)|z_2(t)|^2 + w_b(t) \right|^2 \right\}}.$$

Then the ideal signal-to-noise ratio in the absence of nonlinear distortion, i.e., $\alpha_2 = \alpha_3 = 0$, is given by

$$\text{SNR}_0 = \mathbf{E}\left\{\alpha_1^2 |z_1(t)|^2\right\} / \mathbf{E}\left\{ |w_b(t)|^2 \right\}.$$

Fig. 3 plots $\text{SNR}_{\text{effective}}$ vs. SNR_0 for $\alpha_1 = 10$, $\alpha_2 = 0$, $\alpha_3 = -1$, $\mathbf{E}\{|z_1(t)|^2\} = 5 \times 10^{-7}$ and $\mathbf{E}\{|z_2(t)|^2\} = 0.5$ [4, 5]. When SNR_0 is high, $\text{SNR}_{\text{effective}}$ is dominated by the cross modulation and saturates at 13.5 dB. This shows that a strong blocker signal can cause significant distortion to the desired signal. In the next section, we propose a compensation scheme to mitigate this effect by using digital signal processing techniques.

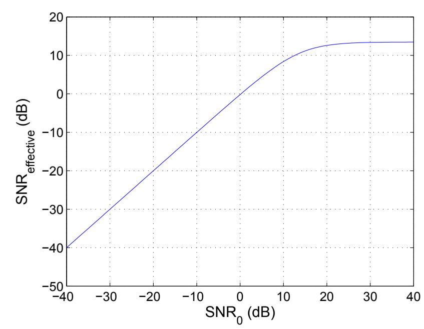

Fig. 3: Plot of $\text{SNR}_{\text{effective}}$ vs. SNR_0 for $\alpha_1 = 10$, $\alpha_2 = 0$, $\alpha_3 = -1$, $\mathbf{E}\{|z_1(t)|^2\} = 5 \times 10^{-7}$ and $\mathbf{E}\{|z_2(t)|^2\} = 0.5$.

3. PROPOSED COMPENSATION SCHEME

In the proposed scheme, the SDR provides two separate RF signal paths. One path is used to capture the signal in the desired band, while the other path is used to acquire the blocker signal, as illustrated in Fig. 4.[5] The two-channel signals are jointly processed in the baseband to alleviate the nonlinear effect. There are two stages in this scheme. In the first stage, the SDR exploits the pilot sequence in the desired signal to estimate the channel response and the nonlinearity parameters. The estimates are then used in the second stage to recover the

[4] In a traditional narrow-band RF receiver, the blocker signal $z_2(t)$ is greatly suppressed by the SAW filter, and then the cross-modulation term is negligible.

[5] The blocker signal can be detected and located by an SDR through scanning the spectrum of its received signals.

transmitted data symbols. Since today's wireless communication standards provide pilot symbols at the beginning of every packet for synchronization and channel estimation, the proposed scheme does not require any modification to the packet structure and can be applied to existing standards. As explained in Section 2, we will only focus on the most problematic case of $\mathbf{E}\{|z_1(t)|^2\} \ll \mathbf{E}\{|z_2(t)|^2\}$ in the following discussion. It is also assumed that $\mathbf{E}\{|z_1(t)|^2\} \leq 1$ and $\mathbf{E}\{|z_2(t)|^2\} \leq 1$.

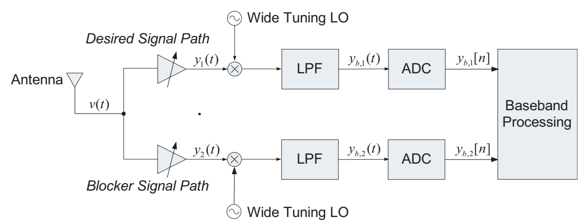

Fig. 4: An SDR with two signal paths.

After successful synchronization and sampling,[6] we obtain the discrete-time version of the received baseband signals at the carrier frequencies ω_1 and ω_2 (see Fig. 4):

$$\begin{aligned} y_{b,1}[n] &= \alpha_1 z_1[n] + \frac{3\alpha_3}{2} z_1[n]|z_1[n]|^2 + 3\alpha_3 z_1[n]|z_2[n]|^2 \\ &\quad + w_{b,1}[n], \\ y_{b,2}[n] &= \alpha_1' z_2[n] + \frac{3\alpha_3'}{2} z_2[n]|z_2[n]|^2 + 3\alpha_3'|z_1[n]|^2 z_2[n] \\ &\quad + w_{b,2}[n], \end{aligned}$$

where α_1, α_3 are the model parameters associated with the signal path of $y_{b,1}(t)$, and α_1', α_3' are the model parameters associated with the signal path of $y_{b,2}(t)$. Also, it follows from (1) that

$$z_1[n] = \sum_{l=0}^{L-1} x_1[n-l]h_1[l], \tag{2}$$

where L is the length of the discrete-time channel response $h_1[n]$, i.e., $h_1[n] = 0$ if $n \notin \{0, 1, \ldots, L-1\}$. Since $\mathbf{E}\{|z_1(t)|^2\} \ll \mathbf{E}\{|z_2(t)|^2\} \leq 1$ and $|\alpha_3| \ll |\alpha_1|$, the secondary-path signal $y_{b,2}(t)$ is dominated by

$$y_{b,2}[n] \approx \alpha_1' z_2[n] + w_{b,2}[n]. \tag{3}$$

In $y_{b,1}[n]$, the third-order harmonics $\frac{3\alpha_3}{2} z_1[n]|z_1[n]|^2$ is negligible and hence

$$y_{b,1}[n] \approx \alpha_1 z_1[n] + 3\alpha_3 z_1[n]|z_2[n]|^2 + w_{b,1}[n]. \tag{4}$$

In the channel estimation stage, the receiver utilizes the pilot symbols transmitted along with the desired signal to estimate the channel response and the nonlinearity parameters. Then, $x_1[n]$, $n = 0, 1, \ldots, N-1$, are known to the receiver, where N is the length of the pilot sequence and $N > L$. By (4), α_1, α_3 and $h_1[n]$, $n = 0, 1, \ldots, L-1$, can be estimated by solving the following optimization problem:

$$\min_{\widehat{\alpha}_1, \widehat{\alpha}_3, \widehat{h}_1[n]} \sum_{n=0}^{N-1} \left| y_{b,1}[n] - \widehat{\alpha}_1 z_1[n] - 3\widehat{\alpha}_3 z_1[n]|\widehat{z}_2[n]|^2 \right|^2$$

[6]Even when the signal-to-noise ratio is low, this can be achieved by sending a long training sequence.

where $z_1[n]$ is related to $x_1[n]$ and $\widehat{h}_1[n]$ through (2) and $\widehat{z}_2[n]$ is given by

$$\widehat{z}_2[n] = \frac{1}{\widehat{\alpha}_1'} y_{b,2}[n]$$

according to (3). In this formulation, we have the product of $\widehat{\alpha}_1$ and $z_1[n]$ and the product of $\widehat{\alpha}_3$ and $z_1[n]|\widehat{z}_2[n]|^2$, which causes an ambiguity of a scaling factor in the estimate of α_1, α_3 and $h_1[n]$. To resolve this ambiguity, we add the following constraint to the original problem:

$$\widehat{\alpha}_1 = \widehat{\alpha}_1' = 1.$$

With this constraint, the estimated α_3 and $h_1[n]$ should be a scaled version of their actual values. When compensating for the distortion, these scaled estimates are used without the need to resolve the ambiguity. The original problem now becomes

$$\min_{\widehat{\alpha}_3, \widehat{h}_1[n]} \sum_{n=0}^{N-1} \left| y_{b,1}[n] - z_1[n] - 3\widehat{\alpha}_3 z_1[n]|y_{b,2}[n]|^2 \right|^2,$$

which is nonlinear and nonconvex. For every *fixed* $\widehat{\alpha}_3$, the associated optimal $\widehat{h}_1[n]$ can be obtained by solving

$$\min_{\widehat{h}_1[n]} \sum_{n=0}^{N-1} \left| y_{b,1}[n] - \left(1 + 3\widehat{\alpha}_3|y_{b,2}[n]|^2\right) \left(\sum_{l=0}^{L-1} x_1[n-l]\widehat{h}_1[l] \right) \right|^2$$

which can be formulated as a linear least-squares problem [8]:

$$\min_{\mathbf{h}} \| \mathbf{y} - \mathbf{AXh} \|^2, \tag{5}$$

where $\mathbf{y}$, $\mathbf{h}$, $\mathbf{A}$ and $\mathbf{X}$ are defined as

$$\begin{aligned} \mathbf{y} &= \left[y_{b,1}[0] \quad y_{b,1}[1] \quad \ldots \quad y_{b,1}[N-1] \right]^T, \\ \mathbf{h} &= \left[\widehat{h}_1[0] \quad \widehat{h}_1[1] \quad \ldots \quad \widehat{h}_1[L-1] \right]^T, \\ \mathbf{A} &= \mathrm{diag}\left\{ 1 + 3\widehat{\alpha}_3|y_{b,2}[0]|^2, 1 + 3\widehat{\alpha}_3|y_{b,2}[1]|^2, \ldots, \right. \\ &\qquad \left. 1 + 3\widehat{\alpha}_3|y_{b,2}[N-1]|^2 \right\}, \\ \mathbf{X} &= \begin{bmatrix} x_1[0] & 0 & \ldots & 0 \\ x_1[1] & x_1[0] & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ x_1[N-2] & x_1[N-3] & \ldots & x_1[N-L-1] \\ x_1[N-1] & x_1[N-2] & \ldots & x_1[N-L] \end{bmatrix}. \end{aligned}$$

The closed-form solution of (5) is

$$\mathbf{h}_o = (\mathbf{X}^*\mathbf{A}^*\mathbf{AX})^{-1}\mathbf{X}^*\mathbf{A}^*\mathbf{y},$$

and its associated residual error is

$$\| \mathbf{y} - \mathbf{AXh}_o \|^2 = \mathbf{y}^*\mathbf{y} - \mathbf{y}^*\mathbf{AX}(\mathbf{X}^*\mathbf{A}^*\mathbf{AX})^{-1}\mathbf{X}^*\mathbf{A}^*\mathbf{y}.$$

Note that the above residual error is a function of $\widehat{\alpha}_3$ because $\mathbf{A}$ depends on $\widehat{\alpha}_3$. Let

$$f(\widehat{\alpha}_3) = \mathbf{y}^*\mathbf{y} - \mathbf{y}^*\mathbf{AX}(\mathbf{X}^*\mathbf{A}^*\mathbf{AX})^{-1}\mathbf{X}^*\mathbf{A}^*\mathbf{y}.$$

Then a one-dimensional search is conducted to find the optimal $\widehat{\alpha}_3$ that minimizes $f(\widehat{\alpha}_3)$. That is, the estimate of α_3 is given by

$$\widetilde{\alpha}_3 = \arg\min_{\widehat{\alpha}_3} \left\{ \mathbf{y}^*\mathbf{y} - \mathbf{y}^*\mathbf{AX}(\mathbf{X}^*\mathbf{A}^*\mathbf{AX})^{-1}\mathbf{X}^*\mathbf{A}^*\mathbf{y} \right\}.$$

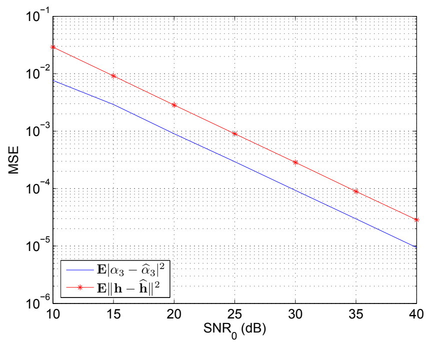

Fig. 5: Mean-square error (MSE) of the channel estimation.

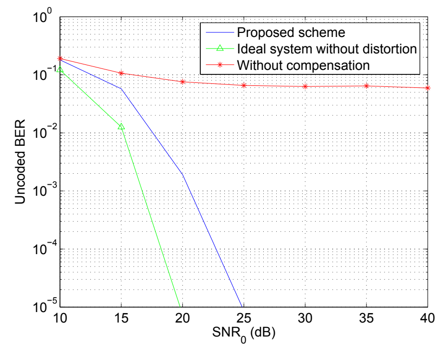

Fig. 6: Uncoded bit error rate (BER) of the proposed scheme.

The optimal $\mathbf{h}_o$ associated with $\widetilde{\alpha}_3$ gives an estimate of $h_1[n]$, $n = 0, 1, \ldots, L-1$, that is denoted by $\widetilde{h}_1[n]$, $n = 0, 1, \ldots, L-1$. The obtained $\widetilde{\alpha}_3$ and $\widetilde{h}_1[n]$, $n = 0, 1, \ldots, L-1$, are used in the data transmission stage to recover data symbols.

To estimate the transmitted data symbols from the distorted signal, we solve the following optimization problem:

$$\min_{x_1[n]} \sum_{n=0}^{M-1} \left| y_{b,1}[n] - z_1[n] - 3\widetilde{\alpha}_3 z_1[n] |y_{b,2}[n]|^2 \right|^2 \qquad (6)$$

where M is the length of the data symbol block. Its exact solution can be obtained as follows. We first get the estimate of $z_1[n]$ as

$$\widehat{z}_1[n] = \frac{y_{b,1}[n]}{1 + 3\widetilde{\alpha}_3 |y_{b,2}[n]|^2}, \quad n = 0, 1, \ldots, M-1.$$

It then follows from (2) that the estimates of $x_1[n]$ are given by

$$\widehat{x}_1[n] = \frac{1}{\widetilde{h}_1[0]} \left[\widehat{z}_1[n] - \sum_{l=1}^{L-1} \widehat{x}_1[n-l] \widetilde{h}_1[l] \right], \qquad (7)$$

where $\widehat{x}_1[n-l]$, $l = 1, 2, \ldots, L-1$, are the previously estimated data symbols.[7]

4. COMPUTER SIMULATIONS

In the simulations, the constellation used for the desired signal is 16-QAM. The simulated channel response has length 6. Its first tap has unity gain, and other taps are independently Rayleigh distributed with the power profile specified by 3 dB decay per tap. We simulate the case when there is only one blocker signal. The average received signal power is set to be $\mathbf{E}\{|z_1(t)|^2\} = 5 \times 10^{-7}$ and $\mathbf{E}\{|z_2(t)|^2\} = 0.5$. The model parameters of the desired channel and the blocker signal channel are specified as[8] $\alpha_1 = 10$, $\alpha_3 = -1$, $\alpha_1' = 0.1$, and $\alpha_3' = -10^{-6}$. The proposed channel estimation method uses pilot sequences of length $N = 64$, and assumes that the channel length in the time domain is $L = 10$. Its mean-square errors (MSE) are plotted vs. the ideal signal-to-noise ratio SNR$_0$ at the receiver in Fig. 5. Fig. 6 compares the bit error rate (BER) performance of the proposed scheme with that of an ideal receiver without nonlinear distortion and a distorted receiver without any compensation. It shows that the proposed scheme can significantly improve the system performance.

5. CONCLUSIONS

In this paper, the effects of RF nonlinearity in a software-defined radio (SDR) receiver are studied from a communication theoretic point of view. A digital compensation scheme is proposed and the simulation demonstrates its effectiveness. Since receivers with less analog impairments usually have the disadvantage of high implementation cost and power consumption, our technique enables the use of low-cost receivers for the next-generation wireless communications that are built on the platform of SDRs.

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[7] This is similar to a decision-directed method.

[8] Compared to the desired channel, this is equivalent to a $20\log 10(\alpha_1/\alpha_1') = 40$ dB attenuation in the blocker signal channel.