

# OFDM CHANNEL ESTIMATION IN THE PRESENCE OF FREQUENCY OFFSET, IQ IMBALANCE AND PHASE NOISE

Q. Zou, A. Tarighat, K. Y. Kim, and A. H. Sayed

Electrical Engineering Department

University of California

Los Angeles, CA 90095

Email: {eqyzou,tarighat,kykim,sayed}@ee.ucla.edu

## ABSTRACT

OFDM systems are susceptible to receiver impairments like frequency offset, IQ imbalance and phase noise. In this paper, OFDM channel estimation in the presence of these impairments is studied, and an iterative algorithm is proposed to jointly estimate the channel coefficients and the impairment parameters. It is shown by computer simulations that the algorithm performs close to its associated Cramer-Rao lower bound.

**Index Terms**— Channel estimation, frequency offset, IQ imbalance, OFDM, phase noise.

## 1. INTRODUCTION

The surging interest in the Orthogonal Frequency Division Multiplexing (OFDM) modulation technique has resulted in research activities to make the implementation of OFDM receivers more reliable and less costly in practice. However, OFDM modulation is susceptible to the impairments present in low-cost low-power silicon implementation of OFDM receivers. The impairments include frequency offset, IQ imbalance and phase noise [1]. Frequency offset is the oscillator frequency difference between the transmitter and the receiver, IQ imbalance is the mismatch in amplitude and phase between the I and Q branches in the receiver chain, and phase noise is the random unknown phase difference between the carrier signal and the local oscillator. The effects of these impairments on OFDM receivers have been investigated in previous works [2–5], and some compensation algorithms have also been developed [6–9].

Although much work has been done to improve data symbol recovery in the presence of receiver impairments, relatively less work is available for channel estimation. In [10], a channel estimation scheme is proposed for the presence of frequency offset and phase noise. It consists of a pre-FFT frequency offset correction followed by time-domain channel estimation. The channel length in the time domain is iteratively estimated in order to mitigate interferences. In [11], the maximum a posteriori (MAP) channel estimator is derived

also for the case when both frequency offset and phase noise are present. In this paper, we propose an iterative algorithm to jointly estimate the channel response in the presence of the impairments, including frequency offset, IQ imbalance and phase noise. Instead of estimating the channel coefficients and phase noise in the frequency domain, we estimate them in the time domain in order to reduce the number of unknowns. Also, the phase noise is parameterized by using the principle component analysis (PCA) technique. The performance of the proposed algorithm is analyzed in terms of the Cramer-Rao lower bound, and is also compared with the ideal case when there is no receiver impairment.

Throughout the paper, we use  $(\cdot)^T$  to represent the matrix transpose and  $(\cdot)^*$  the matrix conjugate transpose.  $\mathbf{E}\{\cdot\}$  returns the expected value with respect to the underlying probability measure.  $\mathbf{I}_K$  denotes the identity matrix of size  $K \times K$ , and  $\mathbf{I}_\theta$  denotes the Fisher information matrix associated with the parameter vector  $\theta$ . We also denote by  $\text{diag}\{\beta_1, \beta_2, \dots, \beta_N\}$  the diagonal matrix whose diagonal elements are  $\beta_1, \beta_2, \dots, \beta_N$ .

## 2. SYSTEM MODEL

At the OFDM transmitter, the information bits are first mapped into constellation symbols, and then converted into a block of  $N$  symbols  $x[k]$ ,  $k = 0, 1, \dots, N - 1$ , by a serial-to-parallel converter. The  $N$  symbols are the frequency components to be transmitted using the  $N$  subcarriers of the OFDM modulator, and are converted to OFDM symbols by the unitary inverse Fast Fourier Transform (IFFT). After adding a cyclic prefix of length  $P$ , the resulting  $N + P$  time-domain symbols are converted into a continuous-time signal  $x(t)$  for transmission.

Fig. 1 shows the block diagram of an RF (radio frequency) receiver with frequency offset  $\Delta f$ , IQ imbalance  $\alpha$  and  $\theta$ , unknown phase error  $\gamma^1$ , and phase noise  $\phi(t)$ . Let  $T_s$  be the sampling period. The *normalized* frequency offset in the discrete-time domain is defined as  $\epsilon = N\Delta f T_s$ , which is the accumulated phase shift caused by the frequency offset over one OFDM symbol duration. It can be shown that the output symbols  $y[k]$ ,  $k = 0, 1, \dots, N - 1$ , after OFDM demodulation

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<sup>1</sup>The unknown phase error is the accumulated phase shift with respect to the carrier signal that is caused by the frequency offset and phase noise.

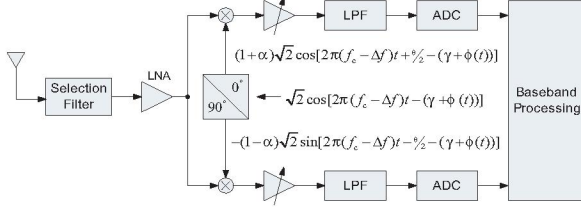


Fig. 1. An RF receiver with analog impairments.

are related to the data symbols  $x[k]$ ,  $k = 0, 1, \dots, N-1$ , by

$$y[k] = \mu \sum_{r=0}^{N-1} a[r] H[(k-r)_N] x[(k-r)_N] + \nu \sum_{r=0}^{N-1} a^*[r] H^*[(N-k-r)_N] x^*[(N-k-r)_N] + w[k] \quad (1)$$

where  $(k)_N$  stands for  $(k \bmod N)$ ,  $\mu$  and  $\nu$  account for the IQ imbalance and are related to  $\alpha$  and  $\theta$  by [7]

$$\mu = \cos(\theta/2) - j\alpha \sin(\theta/2), \quad \nu = \alpha \cos(\theta/2) + j \sin(\theta/2),$$

$a[r]$ ,  $r = 0, 1, \dots, N-1$ , are determined by the frequency offset and phase noise through [9]

$$a[r] = \frac{1}{N} \sum_{n=0}^{N-1} e^{j[\frac{2\pi\epsilon n}{N} + \gamma + \phi(nT_s)]} e^{-j\frac{2\pi r n}{N}}, \quad (2)$$

$H[k]$ ,  $k = 0, 1, \dots, N-1$ , are the discrete-time Fourier transform of the baseband channel impulse response  $h[n]$ ,  $n = 0, 1, \dots, L-1$ , i.e.,

$$H[k] = \sum_{n=0}^{L-1} h[n] e^{-j\frac{2\pi k n}{N}}, \quad (3)$$

and  $w[k]$  is the additive noise in the  $k^{th}$  subcarrier. Using matrix notation, (1) can be represented by

$$\mathbf{y} = \mu \mathbf{A} \mathbf{H} \mathbf{x} + \nu \tilde{\mathbf{A}} \tilde{\mathbf{H}} \tilde{\mathbf{x}} + \mathbf{w}, \quad (4)$$

where

$$\begin{aligned} \mathbf{y} &= [y[0] \ y[1] \ \dots \ y[N-1]]^T, \\ \mathbf{x} &= [x[0] \ x[1] \ \dots \ x[N-1]]^T, \\ \tilde{\mathbf{x}} &= [x^*[0] \ x^*[1] \ \dots \ x^*[N-1]]^T, \\ \mathbf{A} &= \begin{bmatrix} a[0] & a[N-1] & \dots & a[1] \\ a[1] & a[0] & \dots & a[2] \\ \vdots & \vdots & \ddots & \vdots \\ a[N-1] & a[N-2] & \dots & a[0] \end{bmatrix}, \\ \tilde{\mathbf{A}} &= \begin{bmatrix} a^*[0] & a^*[N-1] & \dots & a^*[1] \\ a^*[N-1] & a^*[N-2] & \dots & a^*[0] \\ \vdots & \vdots & \ddots & \vdots \\ a^*[1] & a^*[0] & \dots & a^*[2] \end{bmatrix}, \\ \mathbf{H} &= \text{diag}\{H[0], H[1], \dots, H[N-1]\}, \\ \tilde{\mathbf{H}} &= \text{diag}\{H^*[0], H^*[1], \dots, H^*[N-1]\}, \\ \mathbf{w} &= [w[0] \ w[1] \ \dots \ w[N-1]]^T. \end{aligned}$$

### 3. CHANNEL ESTIMATION

The proposed algorithm uses block-type pilot symbols, in which all subcarriers are used for pilot tones<sup>2</sup>. For convenience of exposition, we assume that each time only *one* OFDM symbol is used as the block-type pilot symbol for channel estimation. Since the OFDM demodulation output  $\mathbf{y}$  is related to the training symbol  $\mathbf{x}$  through expression (4), the proposed algorithm is based on the following optimization problem:

$$\min_{\mu, \nu, \mathbf{A}, \mathbf{H}} \|\mathbf{y} - \mu \mathbf{A} \mathbf{H} \mathbf{x} - \nu \tilde{\mathbf{A}} \tilde{\mathbf{H}} \tilde{\mathbf{x}}\|^2 \quad (5)$$

We notice that there are  $N$  unknowns in  $\mathbf{H}$  (i.e.,  $H[k]$ ),  $N$  unknowns in  $\mathbf{A}$  (i.e.,  $a[k]$ ), plus two additional unknowns  $\mu$  and  $\nu$ . Hence, the solution to (5) is not unique. To overcome this difficulty, we can reduce the number of unknowns by properly modeling the channel and the phase noise process with fewer parameters. Since the length  $L$  of the discrete-time baseband channel impulse response is normally less than the OFDM symbol size  $N$ , we can relate  $H[k]$ ,  $k = 0, 1, \dots, N-1$ , to  $h[n]$ ,  $n = 0, 1, \dots, L-1$ , through

$$\mathbf{h} = \mathbf{F}_h \mathbf{h}'$$

where

$$\begin{aligned} \mathbf{h} &= [H[0] \ H[1] \ \dots \ H[N-1]]^T, \\ \mathbf{h}' &= [h[0] \ h[1] \ \dots \ h[L-1]]^T, \end{aligned}$$

and  $\mathbf{F}_h$  is the discrete Fourier transform matrix of appropriate size according to (3). Instead of estimating  $\mathbf{h}$ , we can estimate  $\mathbf{h}'$ . This reduces the number of unknown channel coefficients from  $N$  to  $L$ .

For the phase noise, by assuming that the receiver has information about its statistics<sup>3</sup>, we can approximate it using fewer parameters based on the principle component analysis (PCA) technique. Let  $\mathbf{c}$  denote the vector of phase noise, i.e.,

$$\mathbf{c} = [e^{j\phi(0)} \ e^{j\phi(T_s)} \ \dots \ e^{j\phi((N-1)T_s)}]^T.$$

Assume its autocorrelation matrix is  $\mathbf{R}_c$ , whose singular value decomposition (SVD) is given by

$$\mathbf{R}_c = \mathbf{U} \mathbf{\Sigma} \mathbf{U}^*.$$

We choose the columns of  $\mathbf{U}$  corresponding to the largest  $M$  ( $M \leq N$ ) singular values of  $\mathbf{R}_c$  as a basis for representing the phase noise vector  $\mathbf{c}$ . Denote the matrix of the basis by  $\mathbf{P}$ . Then,  $\mathbf{c}$  can be approximated by

$$\mathbf{c} \approx \mathbf{P} \mathbf{c}', \quad (6)$$

where  $\mathbf{c}'$  is the parameter vector of length  $M$  that characterizes each realization of the phase noise in the subspace spanned by the principle components in  $\mathbf{P}$ . Combining (2) and (6) gives

$$\mathbf{a} = \frac{1}{N} e^{j\gamma} \mathbf{F}_a [\mathbf{f} \odot \mathbf{c}] \approx \frac{1}{N} e^{j\gamma} \mathbf{F}_a [\mathbf{f} \odot (\mathbf{P} \mathbf{c}')], \quad (7)$$

<sup>2</sup>All standardized OFDM systems today provide such full pilot symbols at the beginning of every packet.

<sup>3</sup>The phase noise can be modeled as a wide-sense stationary process. Its statistics can be obtained by online or offline measurements in the analog or digital domain.

where  $\odot$  denotes the elementwise product,

$$\mathbf{a} = [a[0] \ a[1] \ \dots \ a[N-1]]^T,$$

$$\mathbf{f} = \begin{bmatrix} 1 & e^{j\frac{2\pi\epsilon}{N}} & \dots & e^{j\frac{2\pi\epsilon(N-1)}{N}} \end{bmatrix}^T,$$

and  $\mathbf{F}_a$  is the discrete Fourier transform matrix. Instead of estimating  $\mathbf{a}$ , we can estimate  $\mathbf{c}'$ , which reduces the number of unknowns from  $N$  to  $M$ .

Moreover, we notice that in (5), there exists an ambiguity of a scaling factor among the estimates of  $\mu$ ,  $\mathbf{A}$  and  $\mathbf{H}$ . To resolve this ambiguity, we add the following constraints to the original problem<sup>4</sup>:

$$\mu = 1, \gamma = 0 \text{ and } c[0] = 1,$$

where  $c[0]$  stands for the first element of  $\mathbf{c}$ . By (6),  $c[0] = 1$  gives  $\mathbf{p}_1^* \mathbf{c}' = 1$ , where  $\mathbf{p}_1^*$  is the first row of  $\mathbf{P}$ .

Consequently, knowing  $\mathbf{x}$  and  $\mathbf{y}$ , we can estimate  $\mathbf{H}$  by solving

$$\min_{\nu, \epsilon, \mathbf{c}', \mathbf{h}'} \|\mathbf{y} - \mathbf{A}\mathbf{H}\mathbf{x} - \nu\tilde{\mathbf{A}}\tilde{\mathbf{H}}\tilde{\mathbf{x}}\|^2, \text{ s.t. } \gamma = 0, \mathbf{p}_1^* \mathbf{c}' = 1.$$

The optimization problem is nonlinear and nonconvex. A sub-optimal solution can be found by the following algorithm.

**Initialization:**

1)  $\nu_0 = 0$ .

2) Find the initial  $\mathbf{h}'_0$  by solving the following least-squares problem [12]:

$$\mathbf{h}'_0 = \arg \min_{\mathbf{h}'} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$$

where  $\mathbf{H} = \text{diag}\{\mathbf{h}\}$  and  $\mathbf{h} = \mathbf{F}_h \mathbf{h}'$ .

3) Let  $\mathbf{h}_0 = \mathbf{F}_h \mathbf{h}'_0$  and  $\mathbf{H}_0 = \text{diag}\{\mathbf{h}_0\}$ . Find the initial  $\epsilon_0$  by solving the following problem:

$$\epsilon_0 = \arg \min_{\epsilon} \|\mathbf{y} - \mathbf{A}\mathbf{H}_0\mathbf{x}\|^2.$$

Here,  $\mathbf{A}$  is determined by  $\mathbf{a} = \frac{1}{N} \mathbf{F}_a \mathbf{f}$ , where we let  $\mathbf{c} = [1 \ 1 \ \dots \ 1]^T$ . This can be done through a one-dimensional search for  $\epsilon_0$ , and an approximate  $\epsilon_0$  is sufficient for the iteration.

4) Let  $\mathbf{c}_0 = [1 \ 1 \ \dots \ 1]^T$ . Find the initial  $\mathbf{c}'_0$  by solving the following constrained least-squares problem:

$$\min_{\mathbf{c}'} \|\mathbf{c}_0 - \mathbf{P}\mathbf{c}'\|^2, \text{ s.t. } \mathbf{p}_1^* \mathbf{c}' = 1.$$

**Iterations:**

For  $i = 1, 2, \dots$ , find  $\Delta\nu_i$ ,  $\Delta\epsilon_i$ ,  $\Delta\mathbf{c}'_i$  and  $\Delta\mathbf{h}'_i$  by solving the following optimization problem:

$$\min_{\Delta\nu_i, \Delta\epsilon_i, \Delta\mathbf{c}'_i, \Delta\mathbf{h}'_i} \|\mathbf{y} - (\mathbf{A}_{i-1}\mathbf{H}_{i-1}\mathbf{x} + \nu_{i-1}\tilde{\mathbf{A}}_{i-1}\tilde{\mathbf{H}}_{i-1}\tilde{\mathbf{x}}) - [(\Delta\mathbf{A})\mathbf{H}_{i-1}\mathbf{x} + \mathbf{A}_{i-1}(\Delta\mathbf{H})\mathbf{x}] - [(\Delta\nu)\tilde{\mathbf{A}}_{i-1}\tilde{\mathbf{H}}_{i-1}\tilde{\mathbf{x}} + \nu_{i-1}(\Delta\tilde{\mathbf{A}})\tilde{\mathbf{H}}_{i-1}\tilde{\mathbf{x}} + \nu_{i-1}\tilde{\mathbf{A}}_{i-1}(\Delta\tilde{\mathbf{H}})\tilde{\mathbf{x}}]\|^2, \text{ s.t. } \mathbf{p}_1^* (\Delta\mathbf{c}') = 0, \quad (8)$$

where  $\Delta\mathbf{A}$ ,  $\Delta\mathbf{H}$ ,  $\Delta\tilde{\mathbf{A}}$  and  $\Delta\tilde{\mathbf{H}}$  are the first-order perturbation terms of  $\mathbf{A}$ ,  $\mathbf{H}$ ,  $\tilde{\mathbf{A}}$  and  $\tilde{\mathbf{H}}$  due to  $\Delta\nu_i$ ,  $\Delta\epsilon_i$ ,  $\Delta\mathbf{c}'_i$  and  $\Delta\mathbf{h}'_i$ . Then, update the estimates of  $\nu$ ,  $\epsilon$ ,  $\mathbf{c}'$  and  $\mathbf{h}'$  as follows:

$$\nu_i = \nu_{i-1} + \Delta\nu_i, \quad \epsilon_i = \epsilon_{i-1} + \Delta\epsilon_i,$$

$$\mathbf{c}'_i = \mathbf{c}'_{i-1} + \Delta\mathbf{c}'_i, \quad \mathbf{h}'_i = \mathbf{h}'_{i-1} + \Delta\mathbf{h}'_i.$$

The optimization problem given by (8) can be solved efficiently as a standard least-squares problem.

<sup>4</sup>The constraints, although may be different from their actual values, ensure that the original problem has a unique solution. As a result, the obtained channel estimate is a scaled version of the true channel response, which is still useful in data symbol detection after the correction of the common phase error. The common phase error term can be estimated by using the pilot tones inserted into OFDM symbols.

#### 4. CRAMER-RAO LOWER BOUND (CRLB)

To evaluate the proposed algorithm, we compare its performance with the Cramer-Rao lower bound (CRLB) that gives a lower bound on the covariance matrix of any unbiased estimator of unknown parameters. In the following derivation, it is assumed that 1) all pilot symbols  $x[k]$  have the same power and let  $\sigma_p^2 = \mathbf{E}\{|x[k]|^2\}$ ; 2) the pilot symbols, the phase noise, the channel coefficients and the additive noise are independent of each other; 3) the channel tap coefficients  $h[n]$  are independently identically distributed and circularly symmetric Gaussian with mean zero, and let  $\sigma_H^2 = \mathbf{E}\{|H[k]|^2\}$ ; 4) the additive noise  $\mathbf{w}$  is circularly symmetric Gaussian with covariance matrix  $\sigma_w^2 \mathbf{I}_N$ .

*Scenario 1: No Impairment*

In this scenario, we consider two cases: one tries to estimate  $\mathbf{h}$  and the other tries to estimate  $\mathbf{h}'$ . If  $\mathbf{h}$  is directly estimated, the CRLB for estimating  $H[k]$  is

$$\mathbf{E}\{|\hat{H}[k] - H[k]|^2\} \geq \frac{\sigma_w^2}{\sigma_p^2}.$$

If  $\mathbf{h}'$  is estimated instead, the CRLB for estimating  $H[k]$  is

$$\mathbf{E}\{|\hat{H}[k] - H[k]|^2\} \geq \frac{L\sigma_w^2}{N\sigma_p^2}.$$

*Scenario 2: the Proposed Algorithm*

In this case, the data model can be formulated as

$$\mathbf{y} = \mu \mathbf{A}_{\text{PCA}} \mathbf{H} \mathbf{x} + \nu \tilde{\mathbf{A}}_{\text{PCA}} \tilde{\mathbf{H}} \tilde{\mathbf{x}} + \mu (\mathbf{A} - \mathbf{A}_{\text{PCA}}) \mathbf{H} \mathbf{x} + \nu (\tilde{\mathbf{A}} - \tilde{\mathbf{A}}_{\text{PCA}}) \tilde{\mathbf{H}} \tilde{\mathbf{x}} + \mathbf{w}, \quad (9)$$

where  $\mathbf{h} = \mathbf{F}_h \mathbf{h}'$  and  $\mathbf{A}_{\text{PCA}}$  is determined by the vector  $\mathbf{a}_{\text{PCA}} = \frac{1}{N} e^{j\gamma} \mathbf{F}_a [\mathbf{f} \odot (\mathbf{P}\mathbf{c}')] ]$  according to the construction of  $\mathbf{A}$ . Note that  $\mathbf{A} - \mathbf{A}_{\text{PCA}}$  represents the modeling error existing in the approximation given by (7). The parameter vector to be estimated is

$$\boldsymbol{\theta} = [\nu \ \epsilon \ \mathbf{c}'^T \ \mathbf{h}'^T]^T,$$

where  $\mathbf{c}''$  is the vector consisting of all elements of  $\mathbf{c}'$  except its first element  $c'[0]$ <sup>5</sup>. The desired signal component in (9) is

$$\mathbf{s}_{\boldsymbol{\theta}} = \mu \mathbf{A}_{\text{PCA}} \mathbf{H} \mathbf{x} + \nu \tilde{\mathbf{A}}_{\text{PCA}} \tilde{\mathbf{H}} \tilde{\mathbf{x}},$$

and the noise component is

$$\mathbf{w}' = \mu (\mathbf{A} - \mathbf{A}_{\text{PCA}}) \mathbf{H} \mathbf{x} + \nu (\tilde{\mathbf{A}} - \tilde{\mathbf{A}}_{\text{PCA}}) \tilde{\mathbf{H}} \tilde{\mathbf{x}} + \mathbf{w}.$$

We treat  $\mathbf{w}'$  as circularly symmetric Gaussian with covariance matrix  $\sigma_{w'}^2 \mathbf{I}_N$ , where

$$\begin{aligned} \sigma_{w'}^2 &= \sigma_w^2 + (|\mu|^2 + |\nu|^2) \times \mathbf{E}\{\|\mathbf{a} - \mathbf{a}_{\text{PCA}}\|^2\} \times \sigma_H^2 \sigma_p^2 \\ &\approx \sigma_w^2 + \frac{1}{N} (|\mu|^2 + |\nu|^2) \times \mathbf{E}\{\|\mathbf{c} - \mathbf{P}\mathbf{c}'\|^2\} \times \sigma_H^2 \sigma_p^2 \\ &= \sigma_w^2 + \frac{1}{N} (|\mu|^2 + |\nu|^2) \left( \sum_{n=M+1}^N \sigma_{c,n}^2 \right) \sigma_H^2 \sigma_p^2. \end{aligned}$$

<sup>5</sup> $c'[0]$  can be inferred from the other elements of  $\mathbf{c}'$  because of the constraint  $\mathbf{p}_1^* \mathbf{c}' = 1$ .

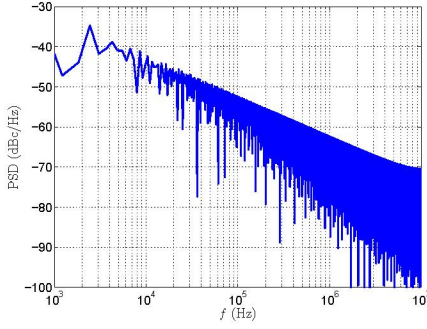


Fig. 2. Power spectral density (PSD) of simulated phase noise.

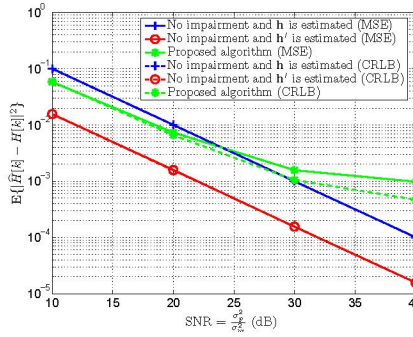


Fig. 3. MSEs and CRLBs of channel estimation.

Here,  $\sigma_{c,1}^2 \geq \sigma_{c,2}^2 \geq \dots \geq \sigma_{c,N}^2$  are the singular values of  $\mathbf{R}_c$ , i.e., the diagonal elements of  $\mathbf{\Sigma}$ . The Fisher information matrix  $\mathbf{I}_\theta$  can then be computed. By the CRLB, any unbiased estimator  $\hat{\theta}$  of  $\theta$  has a variance such that

$$\text{var}\{\hat{\theta}\} = \mathbf{E}\{(\hat{\theta} - \theta)(\hat{\theta} - \theta)^*\} \geq \mathbf{I}_\theta^{-1}.$$

Consequently, the CRLB for  $\mathbf{h}'$  can be computed. By using the relation  $\mathbf{h} = \mathbf{F}_h \mathbf{h}'$ , we have

$$\mathbf{E}\{|\hat{H}[k] - H[k]|^2\} = \mathbf{E}\{\|\hat{\mathbf{h}}' - \mathbf{h}'\|^2\}.$$

This implies that the lower bound for  $H[k]$  can be derived from the lower bound for  $\mathbf{h}'$ .

In the next section, the performance of the proposed algorithm is compared with the computed CRLBs.

## 5. COMPUTER SIMULATIONS

In the simulations, the system bandwidth is 20 MHz, i.e.,  $T_s = 0.05 \mu\text{s}$ , and the constellation used for symbol mapping is 64-QAM. The OFDM symbol size is  $N = 64$  and the prefix length is  $P = 20$ . The channel length is 6, and each tap is independently Rayleigh distributed with  $\sigma_H^2 = 1$ . We simulate an OFDM receiver with  $\epsilon = 0.5$  (i.e.,  $\Delta f = 156.25 \text{ kHz}$ ) and  $\alpha = 0.1$ ,  $\theta = 10^\circ$ . The phase noise spectrum is shown in Fig. 2 and measured in dB with respect to the carrier power, namely, dBc. Only one block-type pilot symbol is used for

each time of channel estimation. The assumed channel length in the time domain is  $L = 10$  and the length of the phase noise vector to be estimated is  $M = 8$ . The simulations show that the proposed algorithm converges within less than 10 iterations. Fig. 3 plots the mean-squared errors (MSE) of different channel estimation algorithms vs. the normalized signal-to-noise ratio at the receiver, i.e.,  $\text{SNR} = \sigma_p^2/\sigma_w^2$ . The associated CRLBs are also plotted by the dotted lines. We can see that the simulated performance consistently coincides with the derived lower bounds. Also, the performance of the proposed algorithm in the presence of all impairments is very close to that of the traditional estimation algorithm applied to systems with no impairments.

## 6. CONCLUSIONS

In the paper, OFDM channel estimation in the presence of frequency offset, IQ imbalance and phase noise is studied. The analysis and simulation results show that the proposed joint estimation scheme can effectively improve the system performance and reduce the sensitivity of OFDM receivers to the impairments. Since receivers with less analog impairments usually have the disadvantage of high implementation cost, our technique enables the use of low-cost receivers for OFDM communications.

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