ON THE PERFORMANCE OF CLUSTERED ENERGY-AWARE WIRELESS NETWORKS

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ABSTRACT

We propose a robust energy-aware clustering architecture for large-scale wireless sensor networks and analyze its performance in terms of throughput capacity and power consumption. The results show that clustered networks can achieve performance improvement by exploiting traffic locality and spatial separation.

1. INTRODUCTION

Wireless sensor networks (WSNs) consist of many spatially distributed sensor devices with sensing, communications, and computation capabilities. The unique characteristics of the sensors, such as limited bandwidth and energy constraints, make the design of such networks more challenging. One critical design issue is to utilize bandwidth and energy efficiently, while sustaining the system lifetime as long as possible.

In recent work [1], it has been shown that the per node throughput capacity of a general-purpose wireless ad hoc network[1] is $\Theta\left(R/\sqrt{N_t \log N_t}\right)$, where R is the common transmission rate of each node and N_t is the total number of nodes. The result implies that the per node throughput capacity vanishes as the network size approaches infinity. Therefore, it is preferable to split the network into multiple clusters, with each cluster containing nodes that are geographically close and strongly correlated.

To enable WSNs to efficiently utilize the available bandwidth and energy, we develop a clustered network architecture based on [2, 3]. We analyze its performance and show how the performance scales with the network size. The results will show that if the number of clusters is medium or large with respect to the network size, clustered networks achieve improved scaling performance in terms of throughput capacity and power consumption when compared to general purpose ad-hoc networks.

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[1]The notation $y = \Theta(f(N))$ is used to signify that there exist positive constants κ_1 and κ_2 such that $\kappa_1 f(N) \le y \le \kappa_2 f(N)$.

2. NETWORK ARCHITECTURE

We consider a space covered by M predetermined clusters, each of which has an area A, as shown in Fig. 1. Each cluster contains $N+1$ stationary nodes and the total number of nodes in the network is $N_t = (N+1) \times M$. During each cycle, a cluster selects one of its nodes as the master node and the terminal nodes send data packets to it. A master node performs important tasks, such as signal processing, sending data packets to the base station (BS), or relaying packets to other nodes. Since the master node sends only the decision to the BS according to a certain data aggregation function, this amount of traffic is assumed to be ignorable. Moreover, although the master node is more power-intensive than terminal nodes, the fraction of time it functions as a master node is only about $1/N$.

To maximize the system lifetime, each cluster rotates the master role evenly among its nodes based on their energy. We apply an energy-aware strategy and always select the node with most energy to serve as the master node from the candidates. This scheme is robust in that failure of some nodes will not prevent the network from operating. Within a cluster, routes of packets are established either through a single-hop direct transmission or through multi-hop routing.

In the media access control (MAC) layer, we adopt time division multiple access (TDMA) for bandwidth and power-efficiency considerations. The master node divides the transmission time into Q time slots, and in each cycle at most Q terminal nodes can transmit packets to the master node of a cluster. At any time t, the transmission from a terminal node i to the master node in its cluster would be successfully received if the signal-to-interference and noise ratio (SINR) at the master node satisfies

$$\frac{P_i(t)G_{ii}(t)}{\sum_{j\in\mathcal{T}, j\neq i} P_j(t)G_{ji}(t) + \sigma_i^2} \geq \gamma \tag{1}$$

where G_{ji} denotes the channel gain from the j-th transmitting node to the master node of terminal node i, σ_i^2 is thermal noise at the master node, P_i is the transmission power of the terminal node i, and γ is a certain threshold. Moreover, $\mathcal{T}$ denotes the set of all transmitting nodes in the network. For a fading and shadowing channel, the channel gain is modelled as

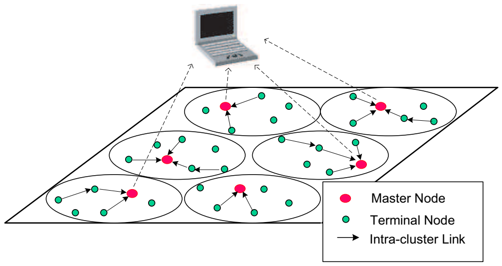

Fig. 1. A generic model for clustered networks.

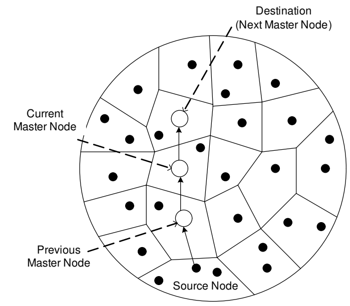

Fig. 2. Example of a multi-hop route within a cluster.

$G_{ji} = S_0 10^{\eta/10}/d_{ji}^{\beta}$, where d_{ji} denotes the distance between the terminal node j and the master node of terminal node i, S_0 is a function of the carrier frequency, β denotes the path loss exponent, and η is a zero mean Gaussian random variable with variance σ_η^2 (i.e., $10^{\eta/10}$ represents the shadowing factor with a lognormal distribution). In practice, the values of β and σ_η depend on the physical environment. We usually have $2 < \beta < 6$ and $6 < \sigma_\eta < 12$. In the proposed protocol, the symbols d_0 and d_M will be used to denote, respectively, the minimum and maximum distances for each transmission in a cluster.

3. THROUGHPUT CAPACITY

The throughput capacity is critical when evaluating a network architecture. Without loss of generality, it is defined as the time average of the number of bits per second that can be transmitted from every source node to its destination. We assume that the transceiver is able to adaptively control the transmit power level so that the SINR can maintain a value of at least γ. In view of Shannon's capacity formula, the transmission rate for each node, denoted by R, is bounded by $W \log_2(1+\gamma)$ bits/sec, where W is the bandwidth.

3.1. Packet Routing

In order to derive the throughput capacity, we first investigate the routing behavior of packets within a cluster. If a source node is far away from its destination, it sends data packets to the destination through relay nodes. To secure efficiency, routes should be over nearly straight-line paths. An example is illustrated in Fig. 2. We partition the cluster into many cells with an equal area D. Assuming that the nodes are independently and uniformly distributed in the cluster, it can be shown, when N is large enough, that there exists a positive constant μ, such that if $D > \mu \log N/N$, then each cell contains at least one node with high probability (*w.h.p*).

Consider a source-destination (S-D) pair j at any time t. The number of hops needed for the packet to move from the source to the destination is denoted by h_j. The distance between the two ends of the S-D pair j is l_j. Since routes are nearly straight lines, the number of hops for S-D pair j can be written as $h_j = \Theta\left(l_j/\sqrt{D}\right)$. Because $d_0 \le l_j \le d_M$, we have

$$\mathbb{E}(h_j) = \mathbb{E}(l_j)\Theta\left(\frac{1}{\sqrt{D}}\right) = \Theta\left(\sqrt{\frac{N}{\log N}}\right). \tag{2}$$

The result shows that the number of hops per route is an increasing function of the number of nodes in the cluster. From the view point of the whole network, when the network is divided into many non-overlapping clusters, the number of hops within the cluster will decrease because routing is restricted within individual clusters.

3.2. Throughput Capacity

It can be shown that each cluster can attain a common transmission rate of R bits/sec if the transmit power is large enough (see Sec. 4.1). This rate should be shared by the N nodes, each of which generates traffic at a rate of $\lambda(N)$, i.e.,

$$N\lambda(N)\mathbb{E}(h) = R. \tag{3}$$

Denoting by h the generic form of h_j and substituting (2) into (3), we obtain that the per node throughput capacity

$$\lambda(N) = \Theta\left(R\sqrt{\frac{\log N}{N^3}}\right) \quad \text{bits/sec} \tag{4}$$

is feasible *w.h.p.* Considering the $N_t \approx MN$ nodes in the whole network, we have the following result for the asymptotic per node throughput capacity

$$\boxed{\lambda(N) = \Theta\left(R\sqrt{M^3(\log N_t - \log M)/N_t^3}\right) \quad \text{w.h.p.}} \tag{5}$$

The result implies that the throughput capacity improves as M increases. Specifically, if M is proportional to N_t, the network can achieve a constant throughput capacity of $\Theta(1)$ bits/sec *w.h.p*. The improvement over general purpose ad-hoc networks is mainly due to the fact that clustering limits the number of hops in routing and thus reduces the relaying burden carried by each node. Another important observation is that the short-range communication imposed by the clustered structure reduces the interference and allows for more simultaneous transmissions in the whole network.

Compared to general-purpose wireless ad-hoc networks in [1], which has a throughput capacity of $\Theta\left(R/\sqrt{N_t \log N_t}\right)$, the clustered architecture can achieve a better throughput capacity if

$$\sqrt[3]{\frac{N_t^2}{\log N_t}} < M < \frac{N_t}{2} \tag{6}$$

for large N_t. The improvement is achieved by taking advantage of traffic locality. Nevertheless, if the degree of traffic locality is low, the general-purpose wireless network may outperform the clustered network.

4. POWER CONSUMPTION

In this section, we derive the per node power consumption for both clustered and non-clustered networks.

4.1. Clustered Networks

For a large-scale wireless network consisting of many non-overlapping clusters with the same area A, we will show that there exists $P_0 > 0$, such that if all the transmitting nodes transmit at an equal power level $P > P_0$, then each cluster can obtain a transmission rate of at least R bits/sec. Therefore, there exists a power control scheme that guarantees a constant transmission rate of R bits/sec in each cluster [4].

Given a coverage area, there might be several clusters that simultaneously transmit packets. These transmissions interfere with each other. Unlike the thermal noise that can be overcome by increasing the transmit power, the interference is difficult to combat because an increase in transmit power also increases interference to neighboring clusters. To reduce the interference, transmissions must be physically separated by some distance to provide sufficient isolation.

In a cluster, denote the distance between a transmitting node and its master node by d ($d_0 \le d \le d_M$). Within the distance of $(1+\varepsilon)d$ from the receiving master, where $\varepsilon > 0$ is a guard parameter for safety margin, we will assume that the number of potential interfering nodes from other clusters is upper bounded by some constant n_1. Therefore, we can enforce an appropriate spatial and temporal scheduling scheme that can ensure none of these close interfering nodes from neighboring clusters transmits simultaneously within the distance of $(1+\varepsilon)d$ from the master node. This can be accomplished by dividing the transmission phase into $1+n_1$ slots and each cluster gets one slot for transmitting packets (e.g., as in [1]). In this way, all the remaining interfering nodes would be from outside the distance of $(1+\varepsilon)d$ from the intended master of the node.

We now examine the interference among clusters. Consider the annulus of all points lying within a distance between a and b from the master node, as shown in Fig. 3. The area of the annulus is given by $\pi(b^2 - a^2) = \pi(2k+1)(1+\varepsilon)^2 d^2$, where $a = k(1+\varepsilon)d$ and $b = (k+1)(1+\varepsilon)d$ for $k = 1, 2, \cdots$. This area contains no more than $\pi(2k+1)(1+\varepsilon)^2 d^2/A$ clusters, each of which has an interfering transmitting node.

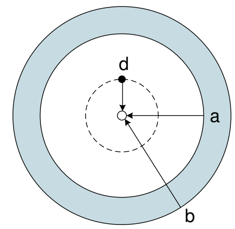

Fig. 3. Annulus between distances a and b from the master.

Consider a terminal node i transmitting to the master node in its cluster. By summing over all interfering nodes, i.e., over k, the interference at the master node can be bounded above as

$$\begin{aligned} I_i(t) &= \sum_{j \ne i} P_j(t) G_{ji}(t) \\ &\le \frac{\pi c' P}{A(1+\varepsilon)^{\beta-2} d^{\beta-2}} \left(\sum_{k=1}^{\infty} \frac{2k+1}{k^\beta} \right) \\ &\le \frac{\pi c' P}{A(1+\varepsilon)^{\beta-2} d^{\beta-2}} \left(3 + \frac{2}{\beta-2} + \frac{1}{\beta-1} \right) \end{aligned} \tag{7}$$

where

$$c' = S_0 10^{\eta/10}. \tag{8}$$

Therefore, for large enough power, the SINR at the master node of a cluster is lower bounded by

$$\begin{aligned} &\lim_{P\to\infty} \frac{\frac{c'P}{d^\beta}}{\frac{\pi c' P}{A(1+\varepsilon)^{\beta-2} d^{\beta-2}} \left(3 + \frac{2}{\beta-2} + \frac{1}{\beta-1}\right) + \sigma_i^2} \\ &= \frac{A(1+\varepsilon)^{\beta-2}}{\pi d^2 \left(3 + \frac{2}{\beta-2} + \frac{1}{\beta-1}\right)} = \gamma \end{aligned} \tag{9}$$

The result indicates that when the guard parameter ε is specified, there is a large enough power level P to transmit packets such that the SINR can attain a desired level γ. Subsequently, the transmission in the cluster would be successfully received by the master node.

Let us now investigate how the per node power consumption will scale with the total number of nodes N_t. From (1), we have

$$P_i(t) = \frac{\gamma}{G_{ii}(t)} \left(\sum_{j \ne i} P_j(t) G_{ji}(t) + \sigma_i^2 \right). \tag{10}$$

Since the channel gain $G_{ii}(t)$ depends only on the transmission distance and the physical environment, taking expectations of both sides of (10) gives

$$\begin{aligned} \mathbb{E}(P_i) &= \mathbb{E}\left(\frac{\gamma}{G_{ii}(t)}\right) \mathbb{E}\left(\sum_{j \ne i} P_j(t) G_{ji}(t) + \sigma_i^2 \right) \\ &\le \mathbb{E}\left(\frac{\gamma}{G_{ii}(t)}\right) \left(\frac{c'}{\gamma d^\beta} \mathbb{E}(P_i) + \sigma_i^2 \right) \\ &\le \frac{c' c''}{d^\beta} \mathbb{E}(P_i) + \gamma c'' \sigma_i^2 \end{aligned} \tag{11}$$

where (11) follows from

$$\mathbb{E}\left(\frac{\gamma}{G_{ii}(t)}\right) = \gamma \int_{d_0}^{d_M} \mathbb{E}\left[\frac{1}{G_{ii}(t)}\middle| d_i\right] f_{d_i}(r)\, dr = \gamma c'' \tag{12}$$

$f_{d_i}(r)$ is the probability density function of the distance between node i and the master node. It can be shown that the integral above is a constant and we denote it by c''.

Rearranging (11), we obtain

$$\mathbb{E}(P_i) \leq \frac{\gamma c'' \sigma_i^2 d^\beta}{d^\beta - c'c''} \leq \frac{\gamma c'' \sigma_i^2 d_0^\beta}{d_0^\beta - c'c''}. \tag{13}$$

It can be easily seen that $\mathbb{E}(P_i) = O(\gamma)$. Therefore, the average per node power consumption p is given by

$$\boxed{\mathbb{E}(p) = \mathbb{E}(P_i)/N = O\left(\gamma M/N_t\right)} \tag{14}$$

The result shows that the power consumed by the network depends on the number of clusters. Furthermore, if the number of clusters M is a constant, the per node power consumption is of the order of $1/N_t$.

4.2. Non-Clustered Wireless Networks

We now investigate the power consumption in a general-purpose multi-hop routing network without clustering, as described in [1]. Consider the protocol model from [1], in which all source nodes transmit with a common distance $r(N_t)$. Due to the requirement of network connectivity [5], the common transmission distance should satisfy

$$r(N_t) \geq \sqrt{\frac{\log N_t}{\pi N_t}} \tag{15}$$

which ensures that no node in the network will be isolated *w.h.p.* To successfully transmit packets from node X_i to another node X_j at a distance $|X_i - X_j|$, where $|X_i - X_j| \leq r(N_t)$, there should be no node simultaneously transmitting within the distance $(1+\varepsilon)r(N_t)$ from X_j. In this way, each transmission consumes an area of at least $\pi r^2(N_t)$ because $\varepsilon > 0$. Therefore, provided that the entire network is deployed in a disc of unit area in $\mathcal{R}^2$, the total number of simultaneous transmissions can be bounded above by

$$\frac{1}{\pi r^2(N_t)} \leq \frac{N_t}{\log N_t}. \tag{16}$$

Then the per node power consumption is given by

$$\mathbb{E}(p) \leq \frac{1}{N_t}\frac{N_t}{\log N_t}\mathbb{E}(P) = \frac{\mathbb{E}(P)}{\log N_t} \tag{17}$$

where $\mathbb{E}(P)$ is the average power consumption for each transmission. Using similar techniques to Section 4.1, it can be shown that $\mathbb{E}(P) = O(\gamma)$. Thus, we obtain a per node power consumption of $O\left(\gamma/\log N_t\right)$. For the physical model in [1], where each node transmits at a common power level, it can be shown that if ε is properly chosen then the above result holds for a large enough common power level P.

By comparing the above results, we find that the cluster-based structure is more power efficient than the non-clustered network provided that the number of clusters satisfies $M \leq N_t/\log N_t$. The reason is that clustering limits the number of hops in routing and the number of simultaneous transmissions nearby, and thus reduces energy dissipation. In addition, due to spatial separation, the transceivers of the clustered network can achieve the same SINR with less power compared with the non-clustered model.

5. CONCLUSION

We have proposed and analyzed a robust energy-aware clustering architecture for large-scale WSNs. It has been shown that clustered networks can take advantage of spatial separation and traffic locality to achieve improved performance. The proposed network is expected to utilize bandwidth and energy efficiently if $\sqrt[3]{\frac{N_t^2}{\log N_t}} < M < \frac{N_t}{\log N_t}$. In this paper, we focused on intra-cluster communications. An interesting extension is to optimally determine the link capacity between the clusters and BS according to a specific aggregation function so that the data traffic can fully exploit the available bandwidth and network architecture.

6. REFERENCES

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

[2] A. Subramanian and A. H. Sayed, "A probablistic power-aware routing algorithm for wireless ad-hoc networks," in *Proc. IEEE VTC*, vol. 4, pp. 2190-2194, Milan, Italy, May 2004.

[3] W. B. Heinzelman, A. P. Chandrakasan, and H. Balakrishnan, "An application specific protocol architecture for wireless microsensor networks," *IEEE Trans. Wireless Commun.*, vol. 1, no. 4, pp. 660–670, Oct. 2002.

[4] N. Bambos, S. C. Chen, and G. J. Pottie, "Channel access algorithms with active link protection for wireless communication networks with power control," *IEEE/ACM Trans. Networking*, vol. 8, no. 5, pp. 583–597, Oct. 2000.

[5] P. Gupta and P. R. Kumar, "Critical power for asymptotic connectivity in wireless networks," in *Stochastic Analysis, Control, Optimization, and Applications: A Volume in Honor of W. H. Fleming*, W. M. McEneay, G. Yin, and Q. Zhang, Eds., Boston: Birkhauser, pp. 547–566, 1998.