# A MINIMUM CO-USER INTERFERENCE APPROACH FOR MULTI-USER MIMO DOWNLINK PRECODING

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## ABSTRACT

This paper proposes multi-user precoding schemes for MIMO wireless networks. We design precoders that minimize the interference power caused by a user on all other users, as opposed to forcefully nulling the interference. The resulting scheme relaxes the traditional constraint on the number of transmit and receive antennas.

## 1. INTRODUCTION

Exploiting the spatial dimension in MIMO wireless communication helps improve the performance and capacity of wireless links [1]. One conventional way to deal with the resulting MIMO channel distortions is through receiver optimization. However, it has been recently noted that by using transmit diversity optimization and precoding, the system performance can be improved as well.

Precoding strategies for single user systems have been studied under a variety of system objectives [2]-[5]. For MIMO multi-user systems, the available downlink transmission strategies may be categorized into three broad groups. The first group uses time-division multiple access (TDMA) schemes, where the base-station serves one user at a time; in this case the system throughput does not increase linearly with the number of transmit antennas [5]. The second group uses dirty paper coding (DPC) [6], where the base station transmits to multiple users simultaneously and the channel state information (CSI) is assumed to be available at the receiver. It is known that the sum rate capacity of the Gaussian broadcast channel can be achieved using DPC [7]. However, due to the computational complexity of the successive encodings and decodings, it may be difficult to implement DPC. The third group of multi-user transmission schemes uses zero-forcing beamforming, which is a suboptimal strategy that can serve multiple users simultaneously with less complexity and with performance close to the DPC scheme [7, 8, 9].

Motivated by [10], this paper proposes multi-user precoding schemes that minimize the interference power caused by one user on all other users, as opposed to forcefully nulling the interference. The resulting scheme will relax the traditional constraint on the number of transmit and receive antennas in conventional ZF beamforming and will also lead to improved BER performance.

## 2. PROBLEM FORMULATION

Consider a downlink MIMO wireless network with $K$ users and one access-point (AP) with $M_t$ transmit antennas – see Fig. 1. It is assumed that each user $k$ has $M_{r,k}$ receive antennas. Let $H_k$ denote the $M_{r,k} \times M_t$ channel matrix between the AP and the $k$th user. A quasi-static fading condition is assumed for each channel so that the channel realizations stay fixed for the duration of a single frame. Let $h_{i,j,k}$ denote the $(i,j)$ element of $H_k$, which stands for the channel coefficient from the $j$th transmit antenna to the $i$th receive antenna of the $k$th user. Each $h_{i,j,k}$ is assumed to have a Rayleigh distributed amplitude with variance 1 and a uniformly distributed phase between 0 and $2\pi$. Moreover,

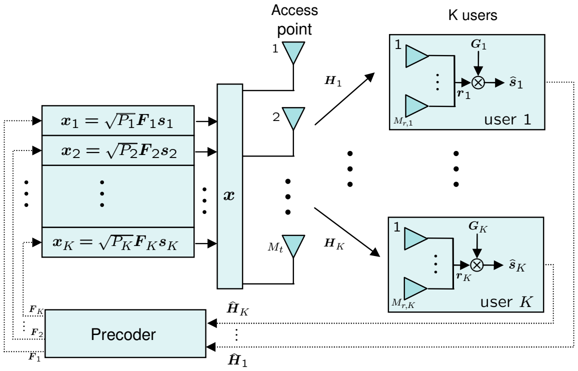

**Fig. 1**. Multi-user precoding scheme for a MIMO wireless network .

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the $h_{i,j,k}$ are i.i.d. random variables. The transmitter feeds the $k$th user bit stream into a vector encoder and modulator. The result is a vector $\boldsymbol{s}_k$ with $M_{s,k}$ symbols to be transmitted to user $k$:

$$\begin{aligned} \boldsymbol{s}_k &= \frac{1}{\sqrt{M_{s,k}}}[s_{k,1}, s_{k,2}, ..., s_{k,M_{s,k}}]^T \qquad (1) \\ E[\boldsymbol{s}_k\boldsymbol{s}_k^*] &= \frac{1}{M_{s,k}}\boldsymbol{I}_{M_{s,k}} \end{aligned}$$

where $T$ and $*$ denote matrix transportation and complex conjugate transposition, respectively, and $\boldsymbol{I}_{M_{s,k}}$ is the $M_{s,k} \times M_{s,k}$ identity matrix. Each of the $M_{s,k}$ bit streams is modulated independently using the same constellation.

The AP transmitter precodes the $k$th user's symbol vector $\boldsymbol{s}_k$ by an $M_t \times M_{s,k}$ matrix $\boldsymbol{F}_k$ to be chosen as follows:

$$\boldsymbol{x}_k = \sqrt{P_k}\boldsymbol{F}_k\boldsymbol{s}_k \qquad (2)$$

where $\boldsymbol{x}_k$ is $M_t \times 1$ and $P_k$ is the transmit power for the $k$th user. In this paper we shall require $\boldsymbol{F}_k$ to satisfy the unitary condition

$$\boldsymbol{F}_k^*\boldsymbol{F}_k = \boldsymbol{I} \qquad (3)$$

This choice is motivated by the following considerations. It was shown in [3, 4] that matrices $\boldsymbol{F}_k$ that guarantee a certain MMSE performance, also maximize the link capacity and they are all unitary (i.e., $\boldsymbol{F}_k^*\boldsymbol{F}_k = \boldsymbol{I}$). The unitary property ensures constant transmission power for the $k$th user over all beams with uniform power allocation among different beams. In contrast, imposing a sum-power constraint (i.e., imposing $\|\boldsymbol{F}_k\boldsymbol{s}_k\|^2 \leq 1$) requires a numerical water-filling procedure in order to find the optimum $\boldsymbol{F}_k$ [3]. The unitary condition on $\boldsymbol{F}_k$ is already finding its way to applications. For example, the per user unitary rate control (PU2RC) scheme by Samsung [11] is used in the 3GPP standard as a unitary precoder for MIMO multi-user networks. Also, unitary precoders have been voted to be used in the 802.16e standard [12].

Now the transmitted signal to all $K$ users is given by

$$\boldsymbol{x} = \sum_{k=1}^{K}\boldsymbol{x}_k = \sum_{k=1}^{K}\sqrt{P_k}\boldsymbol{F}_k\boldsymbol{s}_k \qquad (4)$$

and the signal received by the $i$th user is

$$\begin{aligned} \boldsymbol{r}_i &= \boldsymbol{H}_i\boldsymbol{x} + \boldsymbol{v}_i \qquad (5) \\ &= \sqrt{P_i}\boldsymbol{H}_i\boldsymbol{F}_i\boldsymbol{s}_i + \boldsymbol{H}_i\left(\sum_{k=1,k\neq i}^{K}\sqrt{P_k}\boldsymbol{F}_k\boldsymbol{s}_k\right) + \boldsymbol{v}_i \end{aligned}$$

where $\boldsymbol{v}_i$ is an $\boldsymbol{M}_{r,i} \times 1$ complex Gaussian noise at the $i$th user's antenna array with covariance matrix $\sigma_v^2\boldsymbol{I}$. The first term in (5) denotes the desired signal for the $k$th user and the second term is the interference from other users. The above signaling scheme can be used to accommodate block space-time codes as well.

## 3. DOWNLINK ZERO-FORCING PRECODER DESIGN

Let the $M_{r,k} \times 1$ vector $\boldsymbol{y}_{i,k}$ denote the interference caused by the $k$th user on user $i$:

$$\boldsymbol{y}_{i,k} = \boldsymbol{H}_i\boldsymbol{x}_k$$

and collect the interferences caused by the $k$th user on all other users into a vector $\boldsymbol{y}_k$:

$$\boldsymbol{y}_k = \begin{pmatrix} \boldsymbol{y}_{1,k} \\ \vdots \\ \boldsymbol{y}_{k-1,k} \\ \boldsymbol{y}_{k+1,k} \\ \vdots \\ \boldsymbol{y}_{K,k} \end{pmatrix} = \underbrace{\begin{pmatrix} \boldsymbol{H}_1 \\ \vdots \\ \boldsymbol{H}_{k-1} \\ \boldsymbol{H}_{k+1} \\ \vdots \\ \boldsymbol{H}_K \end{pmatrix}}_{\Pi_k}\boldsymbol{x}_k \qquad (6)$$

where the $\left(\sum_{i=1,i\neq k}^{K} M_{r,i} \times M_t\right)$ matrix $\Pi_k$ is a collection of all channels expect for the $k$th user channel. For convenience, let $M_k = \sum_{i=1,i\neq k}^{K} M_{r,i}$. The ZF design chooses a precoder $\boldsymbol{F}_k$ that forces the interferences caused by the $k$th user to zero. In other words, it sets $\boldsymbol{y}_k = 0$, so that $\boldsymbol{F}_k$ should enforce the following condition:

$$\Pi_k\boldsymbol{x}_k = 0, \text{ i.e., } \boxed{\boldsymbol{F}_k \subset \mathcal{N}(\Pi_k)} \qquad (7)$$

where $\mathcal{N}(\Pi_k)$ denotes the nullspace of $\Pi_k$. In order for (7) to have a solution, the matrix $\Pi_k$ needs to have more columns that rows. This fact forces the following constraint on the number of transmit and receive antennas:

$$M_t - M_k \geq M_{s,k} \Rightarrow \boxed{M_t \geq M_{s,k} + \textstyle\sum_{k=1,k\neq i}^{K} M_{r,k}} \qquad (8)$$

This condition ensures that the subspace $\mathcal{N}(\Pi_k)$ will generally have at least dimension $M_{s,k}$. Condition (8) requires the number of transmit antennas $(M_t)$ to be essentially larger than the combined sum of all receive antennas $(M_{r,k}, k \neq i)$ and $M_{s,k}$. This condition is difficult to satisfy in practice. It will be relaxed as follows – see (17) further ahead.

## 4. DOWNLINK MINIMUM INTERFERENCE POWER DESIGN

We take another approach and choose the unitary precoder $\boldsymbol{F}_k$ in order to minimize the interference power caused by the $k$th user on the other users in the network. Using

$$\boldsymbol{r}_i = \sqrt{P_i}\boldsymbol{H}_i\boldsymbol{F}_i\boldsymbol{s}_i + \underbrace{\sum_{k=1,k\neq i}^{K}\sqrt{P_k}\boldsymbol{H}_i\boldsymbol{F}_k\boldsymbol{s}_k + \boldsymbol{v}_i}_{\text{intereference } \hat{v}_i}$$

we have that the $i$th user SINR before decoding may be expressed as

$$\text{SINR}_i \triangleq \frac{P_i \text{Tr}(\boldsymbol{H}_i \boldsymbol{F}_i \boldsymbol{F}_i^* \boldsymbol{H}_i^*)}{\sigma^2_{\hat{v}_i}} \tag{9}$$

where

$$\sigma^2_{\hat{v}_i} = \sigma^2_v + \frac{1}{M_{r,i}} \sum_{k=1, k\neq i}^{K} \frac{P_k}{M_{s,k}} \text{Tr}(\boldsymbol{H}_i \boldsymbol{F}_k \boldsymbol{F}_k^* \boldsymbol{H}_i^*) \tag{10}$$

In order to approximate the above interference power, we approximate the interference caused by the $k$th user on the $i$th user as the average of the interferences caused by the $k$th user on all users, i.e.,

$$\begin{aligned} & \frac{P_k}{M_{s,k}} \text{Tr}(\boldsymbol{H}_i \boldsymbol{F}_k \boldsymbol{F}_k^* \boldsymbol{H}_i^*) \\ = \; & \text{interference caused by } k\text{th user on the } i\text{th user} \\ \approx \; & \frac{\text{total interference caused by } k\text{th user on the other } K\text{-1 users}}{K-1} \\ = \; & \frac{E\|\boldsymbol{y}_k\|^2}{K-1} \end{aligned} \tag{11}$$

Figure 2 in the simulation section indicates that the approximation is reasonable. The above approximation for the interference motivates us to consider the following optimization problem for choosing $\boldsymbol{F}_k$:

$$\boldsymbol{F}_k = \arg \min_{\boldsymbol{F}_k^* \boldsymbol{F}_k = \boldsymbol{I}} E\|\boldsymbol{y}_k\|^2 \tag{12}$$

Due to the quasi static assumption of the channels, the expectation is only over the transmitted symbols $\boldsymbol{s}_k$. Using (2) and (6) we have

$$\begin{aligned} E\|\boldsymbol{y}_k\|^2 &= P_k E(\boldsymbol{x}_k^* \Pi_k^* \Pi_k \boldsymbol{x}_k) \\ &= P_k E(\boldsymbol{s}_k^* \boldsymbol{F}_k^* \Pi_k^* \Pi_k \boldsymbol{F}_k \boldsymbol{s}_k) \\ &= P_k E \text{Tr}(\boldsymbol{s}_k \boldsymbol{s}_k^* \boldsymbol{F}_k^* \Pi_k^* \Pi_k \boldsymbol{F}_k) \\ &= P_k \text{Tr} E(\boldsymbol{s}_k \boldsymbol{s}_k^* \boldsymbol{F}_k^* \Pi_k^* \Pi_k \boldsymbol{F}_k) \\ &= \frac{P_k}{M_{s,k}} \text{Tr}(\boldsymbol{F}_k^* \Pi_k^* \Pi_k \boldsymbol{F}_k) \end{aligned} \tag{13}$$

So we may rewrite (12) as

$$\begin{aligned} \boldsymbol{F}_k &= \arg \min_{\boldsymbol{F}_k^* \boldsymbol{F}_k = \boldsymbol{I}} \frac{P_k}{M_{s,k}} \text{Tr}(\boldsymbol{F}_k^* \Pi_k^* \Pi_k \boldsymbol{F}_k) \\ &= \arg \min_{\boldsymbol{F}_k^* \boldsymbol{F}_k = \boldsymbol{I}} \frac{P_k}{M_{s,k}} \text{Tr}(\boldsymbol{F}_k^* \boldsymbol{Y}_k \Lambda_k^2 \boldsymbol{Y}_k^* \boldsymbol{F}_k) \end{aligned} \tag{14}$$

where $\Pi_k = \boldsymbol{X}_k \Lambda_k \boldsymbol{Y}_k^*$ is the SVD of $\Pi_k$, with $\boldsymbol{X}_k$ and $\boldsymbol{Y}_k$ unitary. One solution of (14) is $\boldsymbol{F}_k = \Theta_k$ where

$$\boxed{\Theta_k = \text{last } M_{s,k} \text{ columns of } \boldsymbol{Y}_k} \tag{15}$$

The last $M_{s,k}$ columns of $\boldsymbol{Y}_k$ are the singular vectors corresponding to the $M_{s,k}$ smallest singular values of $\Pi_k$. By substituting (15) into (13), the minimum interference power is found to be

$$E\|\boldsymbol{y}_k\|^2 = \frac{P_k}{M_{s,k}} \sum_{m=M_t - M_{s,k}+1}^{M_t} \lambda^2_{m,k} \tag{16}$$

where $\lambda_{m,k}$ is the $m$th diagonal element of $\Lambda_k$ and corresponds to the $m$th singular value of $\Pi_k$. In order to be able to choose $M_{s,k}$ columns of $\boldsymbol{Y}_k$, we need to have

$$\boxed{M_t \geq M_{s,k}, \quad k = 1, ..., K} \tag{17}$$

which is a more relaxed condition than (8); it does not involve anymore the combined sum of all receive antennas. More generally, it is easy to verify that choosing $\boldsymbol{F}_k = \Theta_k \boldsymbol{E}_k$, for any $M_{s,k} \times M_{s,k}$ unitary matrix $\boldsymbol{E}_k$, also minimizes (14). We can use this degree of freedom and select $\boldsymbol{E}_k$ in order to maximize the $k$th user mutual information. Since we have chosen the $\boldsymbol{F}_k$'s in order to minimize the other user interferences, we may approximate the other users' interference as i.i.d with variance $\sigma^2_{\hat{v}_k}$, so that the throughput for the $k$th user becomes[1]

$$\boldsymbol{I}(\boldsymbol{H}_k \boldsymbol{F}_k) \approx \log_2 \det \left( \boldsymbol{I}_{M_{s,k}} + \frac{P_k}{M_{s,k} \sigma^2_{\hat{v}_k}} \boldsymbol{E}_k^* \Theta_k^* \boldsymbol{H}_k^* \boldsymbol{H}_k \Theta_k \boldsymbol{E}_k \right)$$

The optimal $\boldsymbol{E}_k$ that maximizes $\boldsymbol{I}(\boldsymbol{H}_k \boldsymbol{F}_k)$ is given by

$$\boxed{\boldsymbol{E}_k = \text{first } M_{s,k} \text{ columns of } \boldsymbol{V}_k.} \tag{18}$$

where $\boldsymbol{U}_k \Sigma_k \boldsymbol{V}_k$ is the SVD of $\bar{\boldsymbol{H}}_k = \boldsymbol{H}_k \Theta_k$. It can be verified that the above solution maximizes the resulting SNR and minimizes the MSE of the linear MMSE and ZF decoders [3],[4]. Substituting (18) into (4), the expression for $\boldsymbol{I}(\boldsymbol{H}_k \boldsymbol{F}_k)$ becomes

$$\boldsymbol{I}(\boldsymbol{H}_k \boldsymbol{F}_k) \approx \sum_{m=1}^{M_{s,k}} \log_2 \left( 1 + \frac{P_k \sigma^2_{m,k}}{M_{s,k} \sigma^2_{\hat{v}_k}} \right) \tag{19}$$

where $\sigma^2_{m,k}$ is the $m$th diagonal element of $\Sigma_k^2$.

### 4.1. Comparison Discussion

In order to compare the performance of the minimum interference variance precoder of this section and the zero-forcing precoder of Sec. 3, we consider two different cases for an arbitrary user in the system:

$\underline{M_t - M_k \geq M_{s,k}}$: In this case, there are at least $M_{s,k}$ basis columns for the null space of $\Pi_k$ and the zero-forcing precoder cancels all the interferences caused by the $k$th user on the other users. As a result, the power of the interferences caused by the $k$th user is zero.

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[1] The exact expression for the mutual information considering a general noise and interference covariance matrix for multiuser MIMO networks is studied in [13].

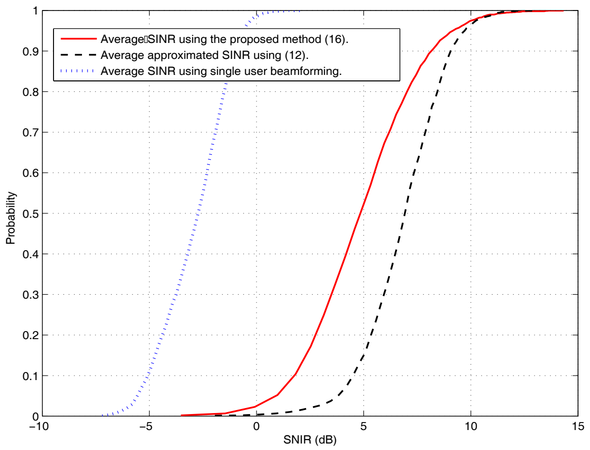

**Fig. 2**. CDF of SINR (9) and approximated SINR (11) at the receiver side using minimum interference variance precoding (15): 8PSK, gray coding, 6 transmit antennas, 4 users with (2,2,3,4) receive antennas.

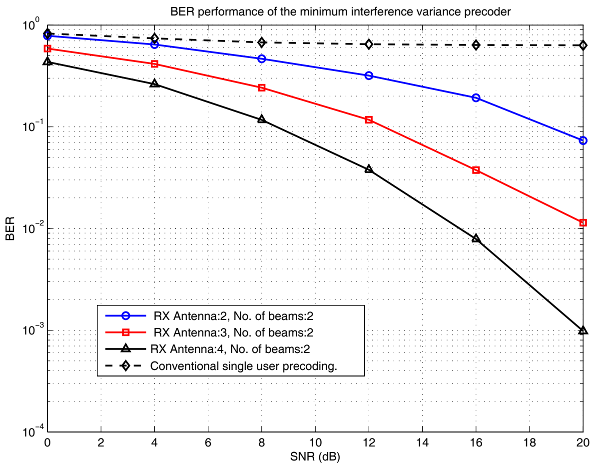

**Fig. 3**. BER performance of the minimum interference variance precoder method in (12): 32PSK, gray coding, 6 transmit antennas, 4 users with (2,2,3,4) receive antennas, (2,2,2,2) symbols per transmission.

$M_t - M_k < M_{s,k}$: In this case, the null space of $\Pi_k$ does not have enough basis columns to form a precoder $\boldsymbol{F}_k$ that cancels the interferences completely. So in order to use zero-forcing precoding, we have to decrease the number of active users in the system. However, the proposed method, which minimizes the interference power, still can be used when $M_t \geq M_{s,k}$.

## 5. SIMULATIONS

The uncoded BER performance and SINR performance of the proposed precoders are investigated. Figure 2 shows how the SINR improves when the minimum interference precoder (15) is used in comparison with conventional single user eigen beamforming. The figure also shows how good is the approximated SINR in (11) is in comparison with the actual SINR (9). Figure 3 shows the BER performance of the proposed multi-user precoding scheme when there are 3 users with (2,3,4) receiving antennas. All users use 2 symbols per transition ($M_{s,k} = 2$). The transmitter has 6 antennas and it uses 32PSK modulation along with gray coding. We have repeated the simulation using a conventional single user SVD precoder, when users pick their precoder to be the largest eigenvector of their channel matrix. It can be seen that in the case of conventional precoder, the inter-user interferences saturate the system for the entire range of SNR.

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