ON THE BASEBAND COMPENSATION OF IQ IMBALANCES IN OFDM SYSTEMS

Alireza Tarighat and Ali H. Sayed

Department of Electrical Engineering
University of California, Los Angeles, CA 90095

ABSTRACT

OFDM is a widely recognized and standardized modulation scheme for future high bit rate communications. Implementation of OFDM-based systems suffers from Inphase-Quadrature phase (IQ) imbalances in the front-end analog processing. The IQ imbalances can severely limit the operating SNR and, consequently, the supported constellation sizes. In this paper, the effect of IQ imbalances on OFDM receivers is analyzed and system level algorithms to compensate for these distortions are proposed. The algorithms include different post and pre-FFT estimation and correction techniques.

1. INTRODUCTION

OFDM is a widely used modulation technique in high-speed wireless systems. The OFDM-based physical layer has been already chosen for several such systems, such as the IEEE 802.11a wireless local area network (WLAN), the recently adopted IEEE 802.11g wireless local area network (WLAN), and the European digital video broadcasting system (DVB-T). It is also under consideration as the high rate alternate physical layer to the draft IEEE P802.15.3 wireless personal area network (WPAN), the IEEE 802.20 mobile broadband wireless access (MBWA) and the IEEE 802.16 wireless metropolitan area networks (WirelessMAN). Considering the expected large demand for such systems, a low-cost, low-power and fully-integrated implementation of these standards is challenging in the front-end RF/analog processing although still possible by direct conversion architecture [1]. However, a dominating problem with direct conversion receivers compared to heterodyne receivers is that the baseband signals are more severely distorted by imbalances within the I and Q branches. Such distortions are likely to increase in future systems, when higher silicon integration is desired as well as higher carrier frequencies. This paper proposes digital baseband signal processing techniques to compensate for such distortions in OFDM systems.

2. PROBLEM FORMULATION

Let $b(t)$ represent the received complex signal before being distorted by the IQ imbalance caused by the analog signal processing. The distorted signal in the time domain can be written as [2]-[3]:

$$b'(t) = \mu b(t) + \nu b^*(t) \tag{1}$$

where the distortion parameters, μ and ν, are related to the amplitude and phase imbalances between the I and Q branches in the RF/analog demodulation process. A simplified model for the parameters μ and ν is given as [3]:

$$\begin{aligned} \mu &= \cos(\theta/2) + j\alpha \sin(\theta/2) \\ \nu &= \alpha \cos(\theta/2) - j \sin(\theta/2) \end{aligned} \tag{2}$$

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where θ and α are respectively the phase and amplitude mismatch between the I and Q branches. These parameters are not known at the receiver since they are caused by manufacturing inaccuracies in the analog components. The effect of the IQ imbalances on an OFDM system and the resulting distortion on the received OFDM signal have been modelled and discussed in [4]-[6]. A derivation of the OFDM signals in the presence of IQ imbalances using the formulation of [7] is presented below. This formulation will help us extend the results to the MIMO case. The formulation will also be used to develop and evaluate several baseband compensation techniques.

In OFDM systems, a block of data is transmitted as an OFDM symbol. The ith transmitted data block of size N and its IDFT are

$$\mathbf{s}_i \triangleq \text{col}\{\mathbf{s}_i(1), \mathbf{s}_i(2), \ldots, \mathbf{s}_i(N)\} \tag{3}$$

$$\bar{\mathbf{s}}_i = \mathbf{F}^* \mathbf{s}_i \tag{4}$$

where $\mathbf{F}$ is the unitary discrete Fourier transform (DFT) matrix. A cyclic prefix of length P is added to each transformed block of data and then transmitted through the channel. An FIR model with $L+1$ taps is assumed for the channel, i.e., $\mathbf{h} = \text{col}\{h_0, h_1, \ldots, h_L\}$. At the receiver, the received samples corresponding to the transmitted block $\bar{\mathbf{s}}_i$ are collected into a vector, after discarding the received cyclic prefix samples. The received block of data before being distorted by IQ imbalances can be written as [7]:

$$\bar{\mathbf{y}}_i = \mathbf{H}^c \bar{\mathbf{s}}_i + \bar{\mathbf{v}}_i \tag{5}$$

where

$$\mathbf{H}^c = \begin{bmatrix} h_0 & h_1 & \cdots & h_L & & & \\ & h_0 & h_1 & \cdots & h_L & & \\ & & \ddots & & & \ddots & \\ & & & h_0 & h_1 & \cdots & h_L \\ \vdots & & & & \ddots & & \vdots \\ h_2 & \cdots h_L & & & & h_0 & h_1 \\ h_1 & \cdots & h_L & & & & h_0 \end{bmatrix} \tag{6}$$

is an $N \times N$ circulant matrix. It is known that $\mathbf{H}^c$ can be diagonalized by the DFT matrix as $\mathbf{H}^c = \mathbf{F}^* \mathbf{\Lambda} \mathbf{F}$, where

$$\mathbf{\Lambda} = \text{diag}\{\lambda\}, \quad \lambda = \mathbf{F}^* \begin{bmatrix} \mathbf{h} \\ \mathbf{0}_{(N-(L+1))\times 1} \end{bmatrix} \tag{7}$$

and λ is a vector. We shall drop the time index i and rewrite (5) as

$$\bar{\mathbf{y}} = \mathbf{F}^* \mathbf{\Lambda} \mathbf{F} \bar{\mathbf{s}} + \bar{\mathbf{v}} = \mathbf{F}^* \text{diag}\,(\lambda)\, \mathbf{F} \bar{\mathbf{s}} + \bar{\mathbf{v}} \tag{8}$$

The received block of data $\bar{\mathbf{y}}$ after being distorted by IQ imbalances becomes

$$\bar{\mathbf{z}} = \mu \bar{\mathbf{y}} + \nu \text{conj}(\bar{\mathbf{y}}) \tag{9}$$

where the notation $\text{conj}(\bar{\mathbf{y}})$ denotes a column vector whose entries are the complex conjugates of the entries of $\bar{\mathbf{y}}$. Now remember that the DFT of the complex conjugate of a sequence is related to the DFT of the original sequence as:

$$\mathbf{F}x = X \quad \text{then} \quad \mathbf{F}\text{conj}(x) = X^{\#} \tag{10}$$

where

$$X = \begin{bmatrix} X(1) \\ X(2) \\ \vdots \\ X(N/2) \\ X(N/2+1) \\ X(N/2+2) \\ \vdots \\ X(N) \end{bmatrix} \Longrightarrow X^{\#} \triangleq \begin{bmatrix} X^*(1) \\ X^*(N) \\ \vdots \\ X^*(N/2+2) \\ X^*(N/2+1) \\ X^*(N/2) \\ \vdots \\ X^*(2) \end{bmatrix} \tag{11}$$

That is, the elements 1 and $N/2+1$ are conjugated, while the others are both conjugated and mirrored. Now we use (5) to write

$$\text{conj}(\bar{\mathbf{y}}) = \text{conj}(\mathbf{H}_c)\text{conj}(\bar{\mathbf{s}}) + \text{conj}(\bar{\mathbf{v}}) \tag{12}$$

where $\text{conj}(\mathbf{H}_c)$ is composed from $\text{conj}(\mathbf{h})$ as in (6). From (10) we have

$$\mathbf{F}^* \begin{bmatrix} \text{conj}(\mathbf{h}) \\ \mathbf{0}_{(N-(L+1))\times 1} \end{bmatrix} = \lambda^{\#} \tag{13}$$

As a result, $\text{conj}(\mathbf{H}_c)$ can be written as $\text{conj}(\mathbf{H}_c) = \mathbf{F}^*\text{diag}\left(\lambda^{\#}\right)\mathbf{F}$. Substituting into (12) results in

$$\text{conj}(\bar{\mathbf{y}}) = \mathbf{F}^*\text{diag}\left(\lambda^{\#}\right)\mathbf{s}^{\#} + \text{conj}(\bar{\mathbf{v}}) \tag{14}$$

where $\mathbf{F}\text{conj}(\bar{\mathbf{s}})$ was substituted by $\mathbf{s}^{\#}$. Now let us consider a receiver that applies the FFT operation to the received block of data $\bar{\mathbf{z}}$, as is done in a standard OFDM receiver. Applying the DFT matrix to (9), i.e., computing $\mathbf{z} = \mathbf{F}\bar{\mathbf{z}}$, and substituting (8) and (14) into (9) lead to

$$\mathbf{z} = \mu\text{diag}\left(\lambda\right)\mathbf{s} + \nu\text{diag}\left(\lambda^{\#}\right)\mathbf{s}^{\#} + \mathbf{v} \tag{15}$$

where $\mathbf{v}$ is a transformed version of the original noise vector $\bar{\mathbf{v}}$. As seen from (15), the vector $\mathbf{z}$ is no longer related to the transmitted block $\mathbf{s}$ through a diagonal matrix, as is the case in an OFDM system with ideal I and Q branches. In the ideal case we would have obtained $\mathbf{z} = \text{diag}\left(\lambda\right)\mathbf{s} + \mathbf{v}$. Discarding the samples corresponding to tones 1 and $N/2+1$, i.e., $\mathbf{z}(1)$ and $\mathbf{z}(N/2+1)$, and defining two new vectors:

$$\begin{aligned} \tilde{\mathbf{z}} &= \text{col}\{\mathbf{z}(2), \ldots, \mathbf{z}(N/2), \mathbf{z}^*(N/2+2), \ldots, \mathbf{z}^*(N)\} \\ \tilde{\mathbf{s}} &= \text{col}\{\mathbf{s}(2), \ldots, \mathbf{s}(N/2), \mathbf{s}^*(N/2+2), \ldots, \mathbf{s}^*(N)\} \end{aligned} \tag{16}$$

then equation (15) gives

$$\tilde{\mathbf{z}} = \underbrace{\begin{bmatrix} \mu\lambda(2) & & & \nu\lambda^*(N) \\ & \ddots & ⋰ & \\ & \mu\lambda(N/2) & \nu\lambda^*(N/2+2) & \\ & \nu^*\lambda(N/2) & \mu^*\lambda^*(N/2+2) & \\ & ⋰ & \ddots & \\ \nu^*\lambda(2) & & & \mu^*\lambda^*(N) \end{bmatrix}}_{\tilde{\mathbf{\Lambda}}} \tilde{\mathbf{s}} + \tilde{\mathbf{v}} \tag{17}$$

where $\tilde{\mathbf{v}}$ is related to $\mathbf{v}$ in a similar manner to (16). Note that the matrix $\tilde{\mathbf{\Lambda}}$ in the above equation is not a diagonal matrix, as is the case for $\mathbf{\Lambda}$ in (8), although it collapses to a diagonal matrix by setting ν equal to zero. Equation (17) can be reduced to 2×2 de-coupled sub-equations, for $k = \{2, \ldots, N/2\}$, each written as

$$\tilde{\mathbf{z}}_k = \tilde{\mathbf{\Gamma}}_k\tilde{\mathbf{s}}_k + \tilde{\mathbf{v}}_k \tag{18}$$

where

$$\tilde{\mathbf{z}}_k = \begin{bmatrix} \mathbf{z}(k) \\ \mathbf{z}^*(N-k+2) \end{bmatrix}, \; \tilde{\mathbf{s}}_k = \begin{bmatrix} \mathbf{s}(k) \\ \mathbf{s}^*(N-k+2) \end{bmatrix} \tag{19}$$

$$\tilde{\mathbf{\Gamma}}_k = \begin{bmatrix} \mu\lambda(k) & \nu\lambda^*(N-k+2) \\ \nu^*\lambda(k) & \mu^*\lambda^*(N-k+2) \end{bmatrix} \tag{20}$$

The objective is to recover $\tilde{\mathbf{s}}_k$ from $\tilde{\mathbf{z}}_k$ in (18) for $k = \{2, \ldots, N/2\}$.

3. COMPENSATION ALGORITHMS

The estimation problem posed by (18)-(20) can be solved by different algorithms and approaches.

3.1. Least-Squares Equalization

The least-squares estimate of $\tilde{\mathbf{s}}_k$ is given by [8]:

$$\hat{\tilde{\mathbf{s}}}_k = (\tilde{\mathbf{\Gamma}}_k^*\tilde{\mathbf{\Gamma}}_k)^{-1}\tilde{\mathbf{\Gamma}}_k^*\tilde{\mathbf{z}}_k \tag{21}$$

In order to implement this solution, the channel information $\{\lambda\}$ and the distortion parameters (μ,ν) are required. Training symbols are required to enable the receiver to estimate those values, and then implement the least-squares estimator. We may also use equation (18) for channel estimation, by first rewriting it as:

$$\tilde{\mathbf{z}}_k = \begin{bmatrix} \mathbf{s}(k) & 0 & \mathbf{s}^*(N-k+2) & 0 \\ 0 & \mathbf{s}(k) & 0 & \mathbf{s}^*(N-k+2) \end{bmatrix} \times \begin{bmatrix} \mu\lambda(k) \\ \nu^*\lambda(k) \\ \nu\lambda^*(N-k+2) \\ \mu^*\lambda^*(N-k+2) \end{bmatrix} + \tilde{\mathbf{v}}_k \tag{22}$$

Assuming n_{Tr} OFDM symbols are transmitted for training, then n_{Tr} realizations of the above equation can be collected to perform the least-squares estimation of the channel taps $\{\lambda(k), \lambda(N-k+2)\}$ and the distortion parameters $\{\mu, \nu\}$ in $\tilde{\mathbf{\Gamma}}_k$. The estimated $\tilde{\mathbf{\Gamma}}_k$ can then be substituted into (21) for data estimation. This estimator

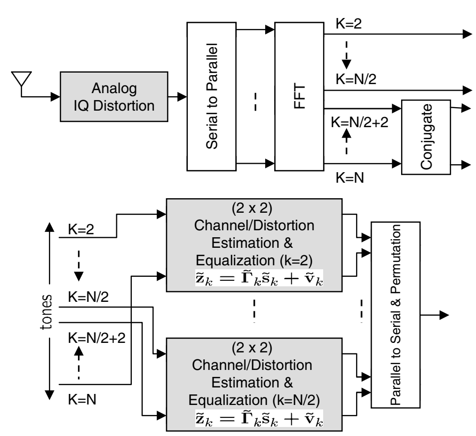

Fig. 1. An OFDM receiver with post-FFT compensation scheme.

is optimum in the least-squares sense, and will be referred to as post-FFT least-squares equalization. A receiver block diagram for this subsection and the following subsection is shown in Figure 1.

3.2. Adaptive Equalization

The adaptive estimation of $\mathbf{s}(k)$ and $\mathbf{s}^*(N-k+2)$ in (19) can be attained as follows:

$$\hat{\mathbf{s}}(k) = \mathbf{w}_k \tilde{\mathbf{z}}_k \quad (23)$$

$$\hat{\mathbf{s}}^*(N-k+2) = \mathbf{w}_{N-k+2} \tilde{\mathbf{z}}_k \quad (24)$$

where $\mathbf{w}_k$ and $\mathbf{w}_{N-k+2}$ are 1×2 equalization vectors updated according to the LMS rules [8]:

$$\mathbf{w}_k(i+1) = \mathbf{w}_k(i) + \mu_{\text{LMS}} \tilde{\mathbf{z}}_k^* e_k(i) \quad (25)$$

$$\mathbf{w}_{N-k+2}(i+1) = \mathbf{w}_{N-k+2}(i) + \mu_{\text{LMS}} \tilde{\mathbf{z}}_k^* e_{N-k+2}(i) \quad (26)$$

where $e_k(i) = d(i) - \mathbf{w}_k(i)\tilde{\mathbf{z}}_k(i)$ is the error signal generated at iteration instance i using a training symbol $d(i)$. A similar relation holds for $e_{N-k+2}(i)$. Moreover, μ_{LMS} is the LMS step-size parameter. Although LMS is the simplest adaptive implementation in terms of complexity, it suffers from a slow convergence rate [8]. This problem is severe for the application at hand, since current OFDM systems usually deploy a short length for training symbols in order to reduce training overhead in packet-based data transmission. However, in the presence of IQ imbalances, there is cross-coupling between every tone and its mirrored tone, which makes the convergence rate slower due to coupling. In LMS, the coefficients in (25) and (26) are usually initiated with zero as their initial value. We propose a different initialization that enhances the convergence rate of the algorithm significantly: the equalizer coefficients are initialized to values calculated as if the receiver assumes ideal I and Q branches, similar to a standard OFDM receiver. This algorithm is referred to as post-FFT LMS equalization.

3.3. Pre-FFT Distortion Correction with Channel Estimation

In the previous two sub-sections, the distortion due to IQ imbalance is estimated and compensated for after the FFT operation at the receiver, i.e., in the frequency domain. The correction and compensation can be performed before the FFT operation, i.e., in the time domain as well. In fact, a correction in time with the exact values of μ and ν can completely remove the distortion caused by IQ imbalance, as will be shown in this sub-section. Recalling (1) as the model for the distorted signal, it can be verified that

$$c(t) \triangleq b'(t) - \left(\frac{\nu}{\mu^*}\right) b'^*(t) = \left(\mu - \frac{|\nu|^2}{\mu^*}\right) b(t) \quad (27)$$

Therefore, the IQ distortion can be removed by applying the above transformation to $b'(t)$ in (1) given that the value of $\frac{\nu}{\mu^*}$ is provided. Now let us consider the problem of estimating the parameter $\frac{\nu}{\mu^*}$ required for the operation defined by (27). Training sequences can be used for estimating this parameter. The channel estimates using the sets of equations defined by (22) can be used for $\frac{\nu}{\mu^*}$ estimation, either by direct least-squares or via an adaptive implementation. This set of equations can be used to estimate $\mu\lambda(k)$, $\nu\lambda^*(N-k+2)$, $\nu^*\lambda(k)$, and $\mu^*\lambda^*(N-k+2)$ in the least-squares sense; let us represent them as $\hat{\rho}_1(k)$, $\hat{\rho}_2(k)$, $\hat{\rho}_3(k)$, and $\hat{\rho}_4(k)$ respectively. The estimates of these parameters can be used to provide an estimate for $\frac{\nu}{\mu^*}$. Now two separate estimates for $\frac{\nu}{\mu^*}$ for $k = \{2, \ldots, N/2\}$ are given by

$$\left(\frac{\hat{\rho}_3(k)}{\hat{\rho}_1(k)}\right)^*, \; \frac{\hat{\rho}_2(k)}{\hat{\rho}_4(k)} \quad (28)$$

Assuming that the μ and ν parameters are constant over different tones, $k = \{2, \ldots, N/2\}$, we can average the above estimates over all the tones to obtain a final estimate for the parameter $\frac{\nu}{\mu^*}$:

$$\widehat{\left(\frac{\nu}{\mu^*}\right)} = \frac{1}{N-2} \sum_{k=2}^{N/2} \left[\left(\frac{\hat{\rho}_3(k)}{\hat{\rho}_1(k)}\right)^* + \left(\frac{\hat{\rho}_2(k)}{\hat{\rho}_4(k)}\right) \right] \quad (29)$$

Once the received signal is corrected before the FFT operation based on (27) and using the above estimate, a standard OFDM channel estimation and data decoding is conducted thereafter. The technique described here is referred to as pre-FFT correction with channel estimation.

The above solution to recover (28) from (22) relies on $(n_{Tr} \times 4)$ data matrices. The complexity and performance can be improved by designing a specially patterned pilot sequence. Using this sequence (in the next subsection), the least-squares solution will only require two data matrices that are $(\frac{n_{Tr}}{2} \times 1)$ each.

3.4. Pre-FFT Distortion Correction with a Special Pilot Pattern

Recalling (22), the system of equations can be reduced by transmitting zeros on tone $(N-k+2)$ during training while the known pilot values are transmitted on tone k—see Figure 2. Using this pattern, (22) collapses to

$$\mathbf{z}(k) = \mathbf{s}(k) \underbrace{[\mu\lambda(k)]}_{\rho_1(k)} + \mathbf{v}(k) \quad (30)$$

$$\mathbf{z}^*(N-k+2) = \mathbf{s}(k) \underbrace{[\nu^*\lambda(k)]}_{\rho_3(k)} + \mathbf{v}^*(N-k+2) \quad (31)$$

Now calculating the least-squares estimates for ρ_1 and ρ_3 assuming $(n_{Tr}/2)$ OFDM training symbols and substituting them into (28) results in the following estimate for $\frac{\nu}{\mu^*}$, $k = \{2, \ldots, N/2\}$,

$$\left(\frac{\hat{\rho}_3(k)}{\hat{\rho}_1(k)}\right)^* = \left(\frac{\sum_{i=1}^{n_{Tr}/2} \mathbf{s}_i^*(k)\mathbf{z}_i(k)}{\sum_{i=1}^{n_{Tr}/2} \mathbf{s}_i^*(k)\mathbf{z}_i^*(N-k+2)}\right)^* \quad (32)$$

A similar expression can be derived for the case that zeros are transmitted on tone k and known pilot values are transmitted on tone $(N-k+2)$. The approach that led to (30) and (31) can be applied for this case to derive the least-squares estimates for $\rho_2(k)$ and $\rho_4(k)$, and consequently $\frac{\nu}{\mu^*} = \hat{\rho}_2(k)/\hat{\rho}_4(k)$ in a similar manner as (32). These two estimates are then averaged over different tones, $k = \{2, \ldots, N/2\}$, similar to (29), to obtain a final estimate of $\frac{\nu}{\mu^*}$. As implicitly considered in (32), from a total of n_{Tr} training OFDM symbols, the first $(n_{Tr}/2)$ training symbols only include pilot on tones $\{1, \ldots, N/2\}$ and zeros on the remaining

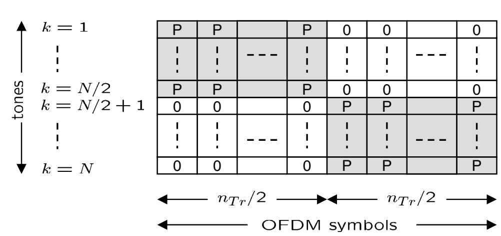

Fig. 2. A training scheme for distortion and channel estimation. Each column corresponds to an OFDM symbol. The letter P stands for pilot.

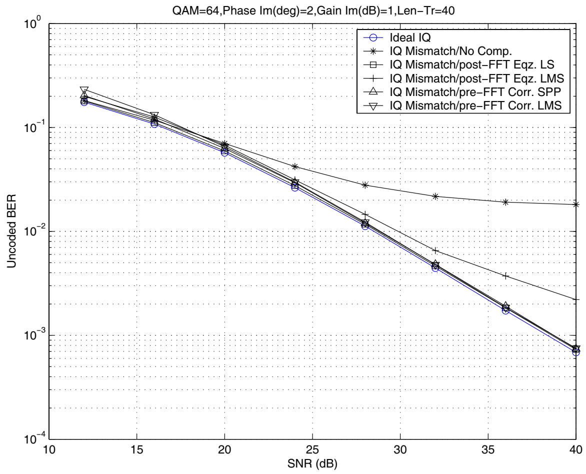

Fig. 3. BER vs. SNR for different compensation schemes.

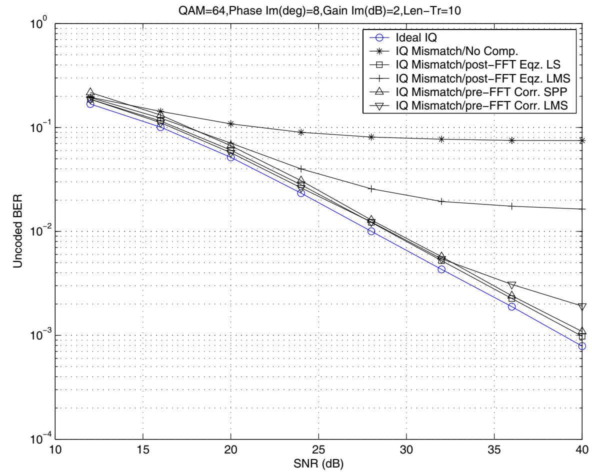

Fig. 4. BER vs. SNR for different compensation schemes.

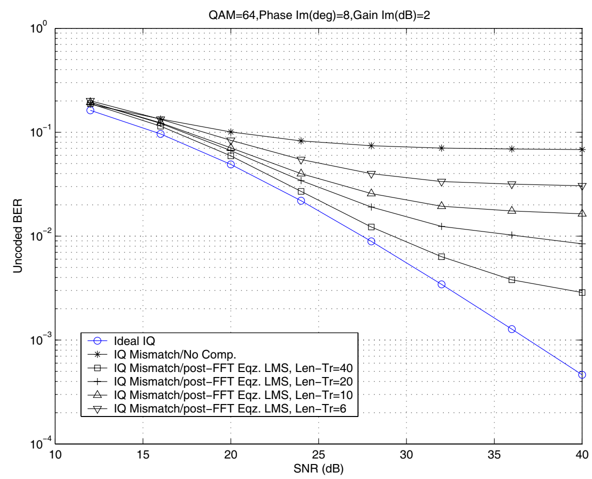

Fig. 5. BER vs. SNR for the post-FFT adaptive equalization with different lengths of training.

tones, and vice versa for the second $(n_{Tr}/2)$ training symbols as depicted in Figure 2. This switching is necessary since the training pilot tones have to cover all the tones in order to enable channel estimation on all the tones. The compensation scheme proposed here is referred to as pre-FFT correction with a special pilot pattern.

Frequency-Flat vs. Frequency-Selective Distortions: The distortion parameters μ and ν have been considered constant throughout the derivations. This is a valid assumption for OFDM systems such as IEEE802.11a that occupy a total bandwidth of less than 20MHz. However, for systems with higher bandwidths this assumption is no longer realistic and the imbalances may vary with frequency. For frequency dependent imbalances, the system of equations given by (18)-(20) can be modified by using frequency-dependent μ and ν parameters; i.e., $\mu(k)$ and $\nu(k)$. This modification will not affect the post-FFT compensation schemes of Secs. 3.1-2, since they do not use the k-independency of μ and ν. However, the pre-FFT compensation schemes of Secs. 3.3-4 will not hold. Therefore, for frequency-selective IQ imbalances, the pos-FFT schemes presented in Secs. 3.1-2 should be used.

4. SIMULATION RESULTS

The parameters used in the simulation are OFDM symbol length of $N = 64$, cyclic prefix of $P = 16$, and channel length of $(L+1) = 4$. The channel taps are chosen independently with complex Gaussian distribution. In all the figures, 'Ideal IQ' legend refers to a receiver with no IQ imbalance and perfect channel knowledge and 'IQ Mismatch/No Comp.' refers to a receiver with IQ imbalance but no compensation scheme. 'IQ Mismatch/post-FFT Eqz. LS', 'IQ Mismatch/post-FFT Eqz. LMS', 'IQ Mismatch/pre-FFT Corr. LMS', and 'IQ Mismatch/pre-FFT Corr. SPP' refer to the schemes presented in Sec. 3.

5. CONCLUSION

A framework for studying and designing OFDM receivers with IQ imbalance correction was presented. Different algorithms to compensate for such distortion were discussed and compared, namely post-FFT equalization and pre-FFT correction schemes. Using training and a pilot pattern specifically designed for IQ imbalance estimation have also been discussed. The difference between frequency-flat and frequency-dependent IQ imbalance and its effect on the compensation scheme were addressed.

6. REFERENCES

[1] B. Razavi, *RF Microelectronics*, Prentice Hall, NJ, 1998.

[2] A. Baier, "Quadrature mixer imbalances in digital TDMA mobile radio receivers," *Proc. International Zurich Seminar on Digital Communications, Electronic Circuits and Systems for Communications*, pp. 147–162, Zurich, Germany, March 1990.

[3] C. L. Liu, "Impacts of I/Q imbalance on QPSK-OFDM-QAM detection," *IEEE Trans. on Consumer Electronics*, vol. 44, no. 3, pp. 984–989, Aug. 1998.

[4] A. Schuchert, R. Hasholzner, and P. Antoine, "A novel IQ imbalance compensation scheme for the reception of OFDM signals," *IEEE Trans. on Consumer Elecs.*, vol. 47, no. 3, pp. 313–318, Aug. 2001.

[5] S. Fouladifard and H. Shafiee, "Frequency offset estimation in OFDM systems in presence of IQ imbalance," *Proc. IEEE International Conference on Communications*, vol. 3, pp. 2071–2075, Alaska, May 2003.

[6] J. Tubbax, B. Come, L. Van der Perre, S. Donnay, M. Engels, M. Moonen, and H. De Man, "Joint compensation of IQ imbalance and frequency offset in OFDM systems," *Proc. Radio and Wireless Conference*, pp. 39–42, Boston, MA, Aug. 2003.

[7] A. Tarighat and A. H. Sayed, "An optimum OFDM receiver exploiting cyclic prefix for improved data estimation," *Proc. IEEE ICASSP*, vol. 4, pp. 217–220, Hong Kong, April 2003.

[8] A. H. Sayed, *Fundamentals of Adaptive Filtering*, Wiley, NY, 2003.