An Optimum OFDM Receiver Exploiting Cyclic Prefix for Improved Data Estimation

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) is a promising technique for multi-antenna broadband systems, since it significantly reduces receiver complexity by providing orthogonal sub-channels. A drawback of OFDM systems is the performance/rate reduction due to the cyclic prefix overhead. In this paper, we propose receiver structures that exploit the cyclic prefix to increase the performance of the link. The proposed structures use the standard OFDM transmitter, and the modifications are only made at the receiver. Optimum receivers in both the least-mean-squares and least-squares senses are presented, and they do not result in any extra processing complexity compared to the standard OFDM receiver. The proposed architecture is further extended to a MIMO OFDM structure. Simulation results validate the improved performance of the proposed receiver.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) significantly reduces receiver complexity in wireless broadband systems, and it has recently been proposed for use in wireless broadband multi-antenna systems [1]. It is also a promising modulation technique to be used in multiple-input multiple-output (MIMO) systems [2],[3]. The orthogonal and independent sub-channels in OFDM are achieved by appending a cyclic prefix to each transmitted block of data, which is then discarded at the receiver. The cyclic extension results in overhead transmission time and reduces the rate efficiency of the communications link. This overhead in time is always present, since the length of the cyclic prefix is usually chosen to cover the longest channel spread. The contribution of this paper is to introduce a procedure at the receiver to exploit the cyclic prefix to improve data estimation. The procedure is formulated as a linear optimization problem and is studied in the mean and least-squares error senses. The least-squares solution results in a simple receiver structure that outperforms the standard OFDM receiver with the same complexity. Both single-input-single-output (SISO) and MIMO cases are addressed in the paper.

The paper is organized as follows. The next section describes an OFDM system and formulates the problem to be solved. In Sec. 3, the optimum solution in the least-mean-squares sense is derived. Sec. 4 presents the least-squares (LS) solution and a simple architecture for it. In Sec. 5, the proposed technique is generalized

to an OFDM-MIMO system. Sec. 6 includes simulation and compares them with current OFDM receivers. Conclusions are given in Sec. 7.

2. PROBLEM FORMULATION

In OFDM systems, a block of data is transmitted as an OFDM symbol. Assuming a block size equal to N, the ith transmitted block of data is

$$\mathbf{s}_i = \text{col}\{\mathbf{s}_i(N), \mathbf{s}_i(N-1), \ldots, \mathbf{s}_i(1)\} \qquad (1)$$

Each block is passed through the IDFT operation

$$\bar{\mathbf{s}}_i = \mathbf{F}^* \mathbf{s}_i \qquad (2)$$

where $\mathbf{F}$ is the unitary discrete Fourier transform (DFT) matrix. A cyclic prefix of length P is added to each transformed block of data, and then transmitted through the channel. An FIR model with $L + 1$ taps is assumed for the channel, i.e.,

$$\mathbf{h} = \text{col}\{h_0, h_1, \ldots, h_L\} \qquad (3)$$

with $L \leq P$ in order to preserve the orthogonality between tones[1]. At the receiver, the received samples corresponding to the transmitted block $\bar{\mathbf{s}}_i$ are collected into a vector $\bar{\mathbf{r}}_i$, as indicated in Figure 1. However, in contrast to current OFDM architectures, the received cyclic prefix samples will not be discarded. In other words, all $(N + P)$ received samples are collected into the vector $\bar{\mathbf{r}}_i$, which is related to the transmitted symbols according to equation (4) on the top of the next page.

(a) ith transmitted symbol

(b) corresponding received symbol
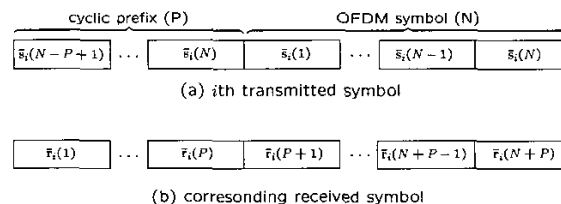
Fig. 1. Transmitted and received OFDM symbols.

In equation (4), $\bar{\mathbf{v}}_i$ is additive white Gaussian noise. In order to remove the effect of the $(i-1)$th transmitted block on the ith received block, only the last L samples, $\{\bar{\mathbf{r}}_i(L), \ldots, \bar{\mathbf{r}}_i(1)\}$, will be

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[1]The problem of finding the channel length L, as well as timing synchronization in OFDM systems, has been studied in the literature [1],[4].

$$\underbrace{\begin{bmatrix} \bar{r}_i(N+P) \\ \bar{r}_i(N+P-1) \\ \vdots \\ \bar{r}_i(P+1) \\ \hline \bar{r}_i(P) \\ \vdots \\ \bar{r}_i(1) \end{bmatrix}}_{(N+P)\times 1} = \underbrace{\begin{bmatrix} h_0 & h_1 & \cdots & h_L & & & \\ & h_0 & h_1 & \cdots & h_L & & \\ & & \ddots & & \ddots & \ddots & \\ & & & h_0 & h_1 & \cdots & h_L \end{bmatrix}}_{H_{(N+P)\times(N+P+L+1)}} \underbrace{\begin{bmatrix} \bar{s}_i(N) \\ \bar{s}_i(N-1) \\ \vdots \\ \bar{s}_i(1) \\ \hline \bar{s}_i(N) \\ \vdots \\ \bar{s}_i(N-P+1) \\ \hline \bar{s}_{i-1}(N) \\ \vdots \\ \bar{s}_{i-1}(N-L+1) \end{bmatrix}}_{(N+P+L+1)\times 1} + \bar{v}_i \qquad (4)$$

discarded, rather than all last P samples. Collecting the remaining $\{\bar{r}_i(N+P), \ldots, \bar{r}_i(L+1)\}$ samples into a vector $\bar{y}_i$, and using the structure of the transmitted symbol, equation (4) gives

$$\bar{y}_i = \mathbf{H}_c \bar{s}_i + \bar{v}_i \qquad (5)$$

where

$$\mathbf{H}_c = \begin{bmatrix} h_0 & h_1 & \cdots & h_L & & & & \\ & h_0 & h_1 & \cdots & h_L & & & \\ & & \ddots & & & \ddots & & \\ & & & h_0 & h_1 & \cdots & h_L & \\ \vdots & & & & & \ddots & & \vdots \\ h_2 & \cdots h_L & & & & & h_0 & h_1 \\ h_1 & \cdots & & h_L & & & & h_0 \\ \hline h_0 & h_1 & \cdots & h_L & & & & \\ & \ddots & & & & \ddots & & \\ & & & h_0 & h_1 & \cdots & h_L & \end{bmatrix} \qquad (6)$$

The matrix $\mathbf{H}_c$ has size $(N+P-L) \times N$ and is circulant (but not square). A receiver using the above structure is expected to outperform the conventional structure, where only N received samples are used to estimate the transmitted symbol. We introduce the square circulant matrix $\mathbf{H}_0$ of size $N \times N$,

$$\mathbf{H}_0 = \begin{bmatrix} h_0 & h_1 & \cdots & h_L & & & & \\ & h_0 & h_1 & \cdots & h_L & & & \\ & & \ddots & & & \ddots & & \\ & & & h_0 & h_1 & \cdots & h_L & \\ \vdots & & & & & \ddots & & \vdots \\ h_2 & \cdots h_L & & & & & h_0 & h_1 \\ h_1 & \cdots & & h_L & . & & & h_0 \end{bmatrix} \qquad (7)$$

and note that $\mathbf{H}_c = \mathbf{K}\mathbf{H}_0$, where

$$\mathbf{K} = \left[\begin{array}{c|c} \mathbf{I}_N & \\ \hline \mathbf{I}_{P-L} & \mathbf{0}_{N-(P-L)} \end{array} \right] \qquad (8)$$

and $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. It is known that the square circulant matrix $\mathbf{H}_0$ can be diagonalized by the DFT matrix as $\mathbf{H}_0 = \mathbf{F}^*\mathbf{\Lambda}\mathbf{F}$, where

$$\mathbf{\Lambda} = \text{diag}\left(\mathbf{F}^* \begin{bmatrix} \mathbf{h} \\ \mathbf{0}_{(N-(L+1))\times 1} \end{bmatrix} \right)$$

We shall drop the time index i for notation simplicity and rewrite (5) as

$$\boxed{\begin{array}{c} \bar{y} = \mathbf{K}\mathbf{H}_0\mathbf{F}^*\mathbf{s} + \bar{v} = \mathbf{K}\mathbf{F}^*\mathbf{\Lambda}\mathbf{s} + \bar{v} \\ \mathbf{K} = \left[\begin{array}{c|c} \mathbf{I}_N & \\ \hline \mathbf{I}_{P-L} & \mathbf{0}_{N-(P-L)} \end{array} \right] \end{array}} \qquad (9)$$

We are interested in estimating s from $\bar{y}$.

3. MMSE SOLUTION

For a linear model of the form $\mathbf{w} = H\mathbf{x}+\mathbf{u}$, where u is zero-mean and uncorrelated with x, the linear least-mean-squares estimator of x given w is [5]

$$\begin{aligned} \hat{\mathbf{x}} &= R_{xw}R_w^{-1}\mathbf{w} \\ &= [R_x^{-1} + H^*R_u^{-1}H]^{-1}H^*R_u^{-1}\mathbf{w} \end{aligned} \qquad (10)$$

Applying this solution to (9), and using $R_x = \sigma_x^2\mathbf{I}_N$ and $R_u = \sigma_u^2\mathbf{I}_{N+P-L}$, the optimum estimator for s from $\bar{y}$ is

$$\hat{s} = \left[\frac{\sigma_v^2}{\sigma_s^2}\mathbf{I}_N + \mathbf{\Lambda}^*\mathbf{F}\mathbf{K}^*\mathbf{K}\mathbf{F}^*\mathbf{\Lambda} \right]^{-1} \mathbf{\Lambda}^*\mathbf{F}\mathbf{K}^*\bar{y} \qquad (11)$$

Now since $\mathbf{K}^*\mathbf{K} = \mathbf{I}_N + \mathbf{E}$, where

$$\mathbf{E} = \begin{bmatrix} \mathbf{I}_{P-L} & \\ & \mathbf{0}_{N-(P-L)} \end{bmatrix} \qquad (12)$$

Eq. (11) becomes

$$\hat{s} = \left[\left(\frac{\sigma_v^2}{\sigma_s^2}\mathbf{I}_N + \mathbf{\Lambda}^*\mathbf{\Lambda} \right) + \mathbf{\Lambda}^*\mathbf{F}\mathbf{E}\mathbf{F}^*\mathbf{\Lambda} \right]^{-1} \mathbf{\Lambda}^*\mathbf{F}\mathbf{K}^*\bar{y} \qquad (13)$$

where the first part inside the matrix inversion is diagonal. Applying the matrix inversion lemma,[2] and defining

$$\mathbf{\Gamma} = \left(\frac{\sigma_v^2}{\sigma_s^2}\mathbf{I}_N + \mathbf{\Lambda}^*\mathbf{\Lambda} \right)^{-1} \qquad (14)$$

the solution (13) can be rewritten as

$$\hat{s} = \mathbf{\Gamma}\mathbf{\Lambda}^*\mathbf{F}\left[\mathbf{I} - (\mathbf{I} + \mathbf{E}\mathbf{F}^*\mathbf{\Lambda}\mathbf{\Gamma}\mathbf{\Lambda}^*\mathbf{F})^{-1}\mathbf{E}\mathbf{F}^*\mathbf{\Lambda}\mathbf{\Gamma}\mathbf{\Lambda}^*\mathbf{F} \right] \mathbf{K}^*\bar{y}$$

$$\overline{^{2}(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}}$$
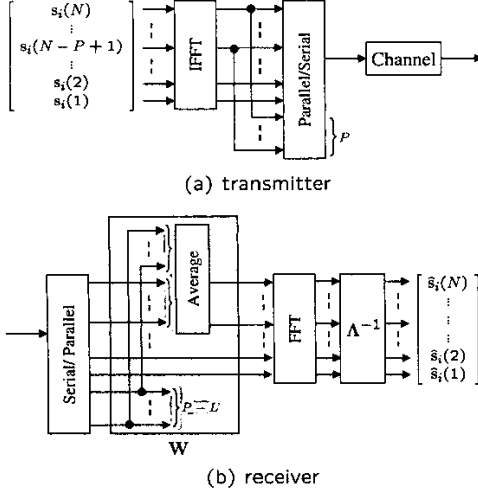
(a) transmitter

(b) receiver

Fig. 2. The proposed OFDM receiver for the SISO case. The block $\mathbf{W}$ is what distinguishes this structure from a conventional OFDM receiver.

or, equivalently,

$$\hat{\mathbf{s}} = \mathbf{\Gamma}\mathbf{\Lambda}^*\mathbf{F}\left[\mathbf{I} + \mathbf{E}(\mathbf{F}^*\mathbf{\Lambda}\mathbf{\Gamma}\mathbf{\Lambda}^*\mathbf{F})\right]^{-1}\mathbf{K}^*\bar{\mathbf{y}} \qquad (15)$$

Note that replacing the matrix $\mathbf{E}$ with $\mathbf{0}$ in the above solution results in the classical OFDM receiver using only N samples, which is $\hat{\mathbf{s}} = \mathbf{\Gamma}\mathbf{\Lambda}^*\mathbf{F}\mathbf{K}^*\bar{\mathbf{y}}$. Therefore, the only modification in the receiver is the matrix inversion term in the final solution. Recalling the value of $\mathbf{E}$ from (12), the term $\mathbf{E}(\mathbf{F}^*\mathbf{\Lambda}\mathbf{\Gamma}\mathbf{\Lambda}^*\mathbf{F})$ simply amounts to selecting the first $(P - L)$ rows of the circulant matrix $(\mathbf{F}^*\mathbf{\Lambda}\mathbf{\Gamma}\mathbf{\Lambda}^*\mathbf{F})$ and replacing the remaining rows with zeros. Next, we formulate the data recovery problem (9) as a least-squares problem. Its solution will perform very closely to the MMSE receiver and will lead to a simpler receiver structure.

4. LEAST-SQUARES SOLUTION

The least-squares solution can be obtained from the least-mean-squares solution by replacing σ_v^2/σ_s^2 with zero. In this case, the matrix $\mathbf{\Gamma}$ defined in (14) becomes $(\mathbf{\Lambda}^*\mathbf{\Lambda})^{-1}$. Substituting into (15) gives

$$\hat{\mathbf{s}} = (\mathbf{\Lambda}^*\mathbf{\Lambda})^{-1}\mathbf{\Lambda}^*\mathbf{F}\,(\mathbf{I} + \mathbf{E})^{-1}\mathbf{K}^*\bar{\mathbf{y}} \qquad (16)$$

where

$$(\mathbf{I} + \mathbf{E})^{-1} = \begin{bmatrix} \frac{1}{2}\mathbf{I}_{P-L} & \\ & \mathbf{I}_{N-(P-L)} \end{bmatrix} \qquad (17)$$

Defining $\mathbf{W} = (\mathbf{I} + \mathbf{E})^{-1}\mathbf{K}^*$, we get

$$\hat{\mathbf{s}} = (\mathbf{\Lambda}^*\mathbf{\Lambda})^{-1}\mathbf{\Lambda}^*\mathbf{y} = \mathbf{\Lambda}^{-1}\mathbf{y}$$
$$\mathbf{y} = \mathbf{F}\mathbf{W}\bar{\mathbf{y}}$$
$$\mathbf{W} = \left[\begin{array}{c|c} \frac{1}{2}\mathbf{I}_{P-L} & \frac{1}{2}\mathbf{I}_{P-L} \\ \hline \multicolumn{2}{c}{\mathbf{I}_{N-(P-L)}} \end{array}\right] \qquad (18)$$

where the diagonal structure of the receiver is preserved since $\hat{\mathbf{s}}$ is related to $\mathbf{y}$ by the diagonal matrix $\mathbf{\Lambda}^{-1}$. A receiver implementing

the above solution is depicted in Figure 2. The main feature of this solution is that the receiver takes the average over the samples that are repeated in the cyclic prefix and not corrupted by the previous symbol, whereas the orthogonal structure is still preserved. Considering the equations in (18), one can verify that a standard OFDM receiver basically implements the same set of equations with $\mathbf{W}$ replaced by

$$\left[\begin{array}{c|c} \mathbf{I}_{P-L} & \mathbf{0}_{P-L} \\ \hline \multicolumn{2}{c}{\mathbf{I}_{N-(P-L)}} \end{array}\right]$$

Therefore, the processing complexity of the proposed receiver (18) is the same as that of current OFDM receivers.

5. GENERALIZATION TO MIMO SYSTEMS

The OFDM receiver structure proposed in the previous section can be expected to a MIMO OFDM system. First, we introduce the following parameters and notations for the MIMO case:

- n_T: number of transmit antennas.
- n_R: number of receive antennas.
- $L_{k,l} + 1$: number of channel taps from transmit antenna l to receive antenna k.
- $\mathbf{h}_{k,l}$: channel taps from transmit antenna l to receive antenna k as defined by (3).
- $\mathbf{s}_l$ and $\bar{\mathbf{s}}_l$: block of data transmitted by antenna l and its Fourier transform as defined by (1) and (2).
- $\bar{\mathbf{r}}_k$: received block of data by antenna k.
- $\bar{\mathbf{y}}_k$: received block of data by antenna k as defined by (5).
- $\mathbf{H}_c^{k,l}$ and $\mathbf{H}_0^{k,l}$: circulant matrices defined as in (6) and (7) corresponding to the channel from antenna l to antenna k.

Data blocks received on different antennas, $\bar{\mathbf{r}}_k$ for $k = \{1, \ldots, n_R\}$, are collected into a column vector of size $n_R \times (N + P)$. The same relation as described in Sec. 3 holds between $\bar{\mathbf{r}}_k$ and $\bar{\mathbf{s}}_l$ for $k = \{1 \ldots n_R\}$ and $l = \{1 \ldots n_T\}$. Defining $L_k = \max_l(L_{k,l})$, for each k, and discarding the last L_k samples of each $\bar{\mathbf{r}}_k$, we can verify that the following relation holds for the MIMO case (using the technique and notations that led to (5)):

$$\underbrace{\begin{bmatrix} \bar{\mathbf{y}}_{n_R} \\ \vdots \\ \bar{\mathbf{y}}_1 \end{bmatrix}}_{\bar{\mathbf{y}}^{(M)}} = \underbrace{\begin{bmatrix} \mathbf{H}_c^{n_R,n_T} & \cdots & \mathbf{H}_c^{n_R,1} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_c^{1,n_T} & \cdots & \mathbf{H}_c^{1,1} \end{bmatrix}}_{\mathbf{H}_c^{(M)}} \underbrace{\begin{bmatrix} \bar{\mathbf{s}}_{n_T} \\ \vdots \\ \bar{\mathbf{s}}_1 \end{bmatrix}}_{\bar{\mathbf{s}}^{(M)}} + \bar{\mathbf{v}}^{(M)} \qquad (19)$$

where $\bar{\mathbf{v}}^{(M)}$ contains the additive noise on all receive antennas. Note that the block of data, $\bar{\mathbf{y}}^{(M)}$, has $(n_R \times N) + \sum_{k=1}^{n_R}(P - L_k)$ elements compared to the $(n_R \times N)$ samples used in a standard MIMO OFDM receiver. As in the SISO case, an improved optimum receiver based on (19) can be achieved with the same complexity as that of a standard receiver. Applying the decomposition $\mathbf{H}_c = \mathbf{K}\mathbf{H}_0$ to each sub-block in $\mathbf{H}_c^{(M)}$, we can write

$$\mathbf{H}_c^{(M)} = \underbrace{\begin{bmatrix} \mathbf{K}_{n_R} & & \\ & \ddots & \\ & & \mathbf{K}_1 \end{bmatrix}}_{\mathbf{K}^{(M)}} \underbrace{\begin{bmatrix} \mathbf{H}_0^{n_R,n_T} & \cdots & \mathbf{H}_0^{n_R,1} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_0^{1,n_T} & \cdots & \mathbf{H}_0^{1,1} \end{bmatrix}}_{\mathbf{H}_0^{(M)}} \qquad (20)$$
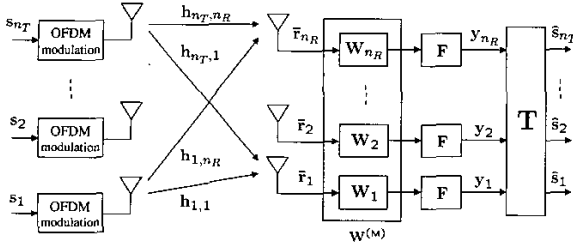
Fig. 3. Proposed structure applied to a MIMO system. The matrix $\mathbf{T}$ is defined in (24).

where each $\mathbf{K}_k$ is as in (8) with L replaced by L_k. As in (9) we can express $\mathbf{H}_0^{(M)}$ in the form

$$\mathbf{H}_0^{(M)} = \begin{bmatrix} \mathbf{F}^* & & \\ & \ddots & \\ & & \mathbf{F}^* \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{\Lambda}_{n_R,n_T} & \cdots & \mathbf{\Lambda}_{n_R,1} \\ \vdots & \ddots & \vdots \\ \mathbf{\Lambda}_{1,n_T} & \cdots & \mathbf{\Lambda}_{1,1} \end{bmatrix}}_{\mathbf{\Lambda}^{(M)}} \begin{bmatrix} \mathbf{F} & & \\ & \ddots & \\ & & \mathbf{F} \end{bmatrix}$$

where $\mathbf{\Lambda}^{k,l}$, $k = \{1, \cdots, n_R\}$, $l = \{1, \cdots, n_T\}$, are diagonal matrices. Taking into account that

$$\begin{bmatrix} \bar{\mathbf{s}}_{n_T} \\ \vdots \\ \bar{\mathbf{s}}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{F}^* \mathbf{s}_{n_T} \\ \vdots \\ \mathbf{F}^* \mathbf{s}_1 \end{bmatrix} \qquad (21)$$

Eq. (19) becomes

$$\bar{\mathbf{y}}^{(M)} = \mathbf{K}^{(M)} \begin{bmatrix} \mathbf{F}^* & & \\ & \ddots & \\ & & \mathbf{F}^* \end{bmatrix} \mathbf{\Lambda}^{(M)} \mathbf{s}^{(M)} + \bar{\mathbf{v}}^{(M)} \qquad (22)$$

where $\mathbf{s}^{(M)} = \mathrm{col}\{\mathbf{s}_l\}$. This equation plays the role of (9) in the MIMO case. However, the matrix $\mathbf{\Lambda}^{(M)}$ is no longer diagonal, but instead is composed of diagonal sub-blocks. We can make it into a block-diagonal matrix with blocks of size $n_R \times n_T$, by applying some suitable unitary permutations, $\mathbf{P}_{in}$ and $\mathbf{P}_{out}$, as in [2]:

$$\mathbf{\Gamma}^{(M)} = \mathbf{P}_{out} \mathbf{\Lambda}^{(M)} \mathbf{P}_{in}$$

Then Eq. (22) leads to

$$\mathbf{P}_{out} \mathbf{y}^{(M)} = \mathbf{\Gamma}^{(M)} \mathbf{P}_{in}^{-1} \mathbf{s}^{(M)} + \mathbf{v}^{(M)}$$

where

$$\mathbf{y}^{(M)} = \begin{bmatrix} \mathbf{F} \mathbf{W}_{n_R} & & \\ & \ddots & \\ & & \mathbf{F} \mathbf{W}_1 \end{bmatrix} \bar{\mathbf{y}}^{(M)} \qquad (23)$$

and $\mathbf{W}_k$ is defined as in (18) with L replaced by L_k. The least-squares solution of $\mathbf{s}^{(M)}$ from $\mathbf{P}_{out} \mathbf{y}^{(M)}$ is given by

$$\hat{\mathbf{s}}^{(M)} = \underbrace{\mathbf{P}_{in} \left[(\mathbf{\Gamma}^{(M)})^* \mathbf{\Gamma}^{(M)} \right]^{-1} (\mathbf{\Gamma}^{(M)})^* \mathbf{P}_{out}}_{\mathbf{T}} \mathbf{y}^{(M)} \qquad (24)$$

It can be verified that the solution (23)-(24) collapses to the one for a MIMO system with a standard OFDM receiver by replacing $\mathbf{K}_k$ and $\mathbf{W}_k$ in (20) and (23) with the identity matrix. The receiver for the MIMO case is depicted in Figure 3.
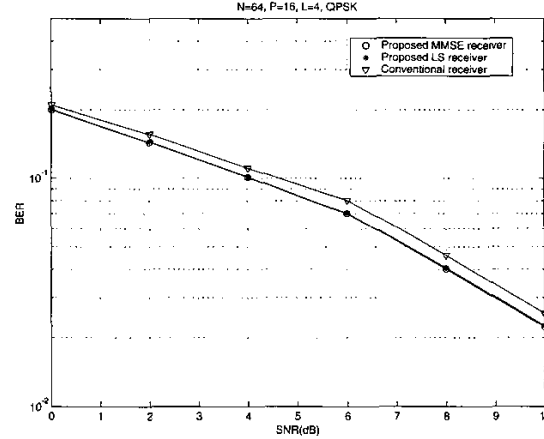

Fig. 4. BER vs. SNR for different receiver OFDM structures.

6. SIMULATION RESULTS

A typical SISO OFDM system is simulated to evaluate the performance of the proposed MMSE and LS solutions in comparison to a standard OFDM receiver. The parameters used in the simulation are $N = 64$, $P = 16$, $(L + 1) = 4$, and the channel taps are chosen independently with complex Gaussian distribution. The BER versus SNR for the proposed algorithms is simulated and shown in Figure 4. Although the MMSE solution is the optimum solution in the least-mean-squares sense, the least-squares solution closely achieves the same performance. Note further that the performance of the proposed receiver depends on the relative values of the cyclic prefix length and the channel spread.

7. CONCLUSION

We proposed a structure for OFDM receivers, for both SISO and MIMO cases, by exploiting the cyclic prefix samples to achieve a higher performance compared to the receivers that discard the entire cyclic prefix. This is an important issue in OFDM systems, since the length of the cyclic prefix is always over-designed for the worst-case channel delay, and this can introduce a large unnecessary overhead on the system for typical channel scenarios. The proposed least-squares architecture outperformed the current OFDM receivers, without any additional complexity. Furthermore, the concept introduced in this paper can be applied to other tasks performed by the receiver, such as timing and synchronization.

8. REFERENCES

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