ADAPTIVE FREQUENCY-DOMAIN EQUALIZATION OF SPACE-TIME BLOCK-CODED TRANSMISSIONS*

Waleed M. Younis

Electrical Engineering Dept.
University of California
Los Angeles, CA 90095
waleed@ee.ucla.edu

Naofal Al-Dhahir

AT&T Shannon Laboratory
Florham Park, NJ 07932
naofal@research.att.com

Ali H. Sayed

Electrical Engineering Dept.
University of California
Los Angeles, CA 90095
sayed@ee.ucla.edu

ABSTRACT

We develop an adaptive equalization scheme for space–time block-coded (STBC) transmissions. The scheme is based on a modified low–complexity version of the fast–converging RLS algorithm. Complexity reduction is achieved by exploiting the rich structure of STBC.

1. INTRODUCTION

Transmit diversity signaling using the Alamouti scheme [1] has been adopted in several wireless standards such as WCDMA and CDMA2000 due to its many attractive features including the following:

- It achieves full spatial diversity at full transmission rate for any (real or complex) signal constellation.
- It does not require channel state information (CSI) at the transmitter (i.e., open loop).
- Maximum likelihood decoding involves only *linear* processing at the receiver (due to the orthogonal code structure), thus keeping user terminals simple.

When implemented over frequency–selective channels, the Alamouti scheme should be implemented at a *block* not *symbol* level and combined with effective equalization schemes to realize additional multipath diversity gains without sacrificing full spatial diversity. A low–complexity scheme that achieves this goal is the single–carrier frequency–domain–equalized (SC FDE) STBC described in [2]. This scheme combines the above–mentioned advantages of the Alamouti scheme with those of SC FDE [3], namely, low complexity (due to use of the FFT) and reduced sensitivity, compared with orthogonal frequency division multiplexing (OFDM), to carrier frequency offsets and nonlinear distortion (due to reduced peak to average ratio).

Joint equalization and decoding of SC FDE–STBC transmissions require channel state information (CSI) at the receiver, which can be estimated using training sequences embedded in each block. Then, the optimum equalizer/decoder settings are computed from the estimated CSI. An alternative to this two–step channel–estimate–based approach is *adaptive* equalization/decoding that does not require explicit CSI estimation, which can be both challenging and costly especially for transmit diversity. When the channel varies within a transmission block, adaptive techniques can also *track* these variations. Otherwise, unreliable CSI results in *mismatched* equalizer/decoder settings which degrade performance. In this paper, we propose effective low–complexity adaptive SC FDE–STBC equalization/decoding algorithms for both training and tracking modes. Adaptive algorithms for SC FDE were developed in [4] for receive diversity only. There does not seem to exist previous work on adaptive SC FDE–STBC. The rest of this paper is organized as follows. In Section 2, we review the non–adaptive SC FDE–STBC. The adaptive version is developed in Section 3 for training and tracking modes. Simulation results for the EDGE environment are presented in Section 4 and the paper is concluded in Section 5.

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2. STBC FOR BROADBAND CHANNELS

2.1. Alamouti Scheme

For STBCs to achieve multi–path (in addition to spatial) diversity gains on frequency–selective channels, the Alamouti scheme [1] should be implemented at a *block not symbol* level. Several schemes have been proposed [5]. Here, we consider the transmission scheme depicted in Figure 1, which is known as SC FDE–STBC [2]. Denote the n^{th} symbol of the k^{th} transmitted block from antenna i by $\mathbf{x}_i^{(k)}(n)$. At times $k = 0, 2, 4, \cdots$, pairs of length–N blocks $\mathbf{x}_1^{(k)}(n)$ and $\mathbf{x}_2^{(k)}(n)$ (for $0 \leq n \leq N-1$) are generated by an information source according to:

$$\begin{aligned} \mathbf{x}_1^{(k+1)}(n) &= -\mathbf{x}_2^{*(k)}((-n)_N) \\ \mathbf{x}_2^{(k+1)}(n) &= \mathbf{x}_1^{*(k)}((-n)_N) \end{aligned} \quad (1)$$

for $n = 0, 1, \cdots, N-1$ and $k = 0, 2, 4, \cdots$, where $(\cdot)^*$ and $(\cdot)_N$ denote complex conjugation and modulo–N operations, respectively. In addition, a cyclic prefix of length ν is added to each transmitted block to eliminate IBI and make all channel matrices *circulant*. Finally, the transmitted power from each antenna is half its value in the single–transmit case so that total transmitted power is fixed.

2.2. SC FDE-STBC

With 2 transmit and 1 receive antenna, received blocks k and $k+1$ are given by

$$\mathbf{y}^{(j)} = \mathbf{H}_1^{(j)}\mathbf{x}_1^{(j)} + \mathbf{H}_2^{(j)}\mathbf{x}_2^{(j)} + \mathbf{n}^{(j)} \ : \ \text{for } j = k, k+1 \quad (2)$$

where $\mathbf{H}_1^{(j)}$ and $\mathbf{H}_2^{(j)}$ are the *circulant* channel matrices from transmit antennas 1 and 2, respectively, over block j, to the receive an-

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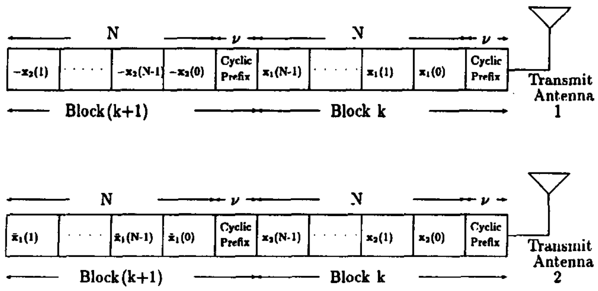

Fig. 1. Block format for SC FDE–STBC Transmission scheme.

tenna. Applying the DFT matrix to $\mathbf{y}^{(j)}$, we get (for $j = k, k+1$)

$$\begin{aligned}\mathbf{Y}^{(j)} &\triangleq \mathbf{Q}\mathbf{y}^{(j)} \\ &\triangleq \Lambda_1^{(j)}\mathbf{X}_1^{(j)} + \Lambda_2^{(j)}\mathbf{X}_2^{(j)} + \mathbf{N}^{(j)}\end{aligned} \quad (3)$$

where $\mathbf{X}_i^{(j)} \triangleq \mathbf{Q}\mathbf{x}_i^{(j)}$ and $\mathbf{N}^{(j)} \triangleq \mathbf{Q}\mathbf{n}^{(j)}$. Using the encoding rule in (1) and properties of the DFT [6], and assuming the 2 channels are fixed over 2 consecutive blocks, we get

$$\begin{aligned}\mathbf{X}_1^{(k+1)}(m) &= -\mathbf{X}_2^{*(k)}(m) \\ \mathbf{X}_2^{(k+1)}(m) &= \mathbf{X}_1^{*(k)}(m)\end{aligned} \quad (4)$$

for $m = 0, 1, \cdots, N-1$ and $k = 0, 2, 4, \cdots$. Combining (3) and (4), we arrive at

$$\begin{aligned}\mathbf{Y} = \begin{bmatrix} \mathbf{Y}^{(k)} \\ \bar{\mathbf{Y}}^{(k+1)} \end{bmatrix} &= \begin{bmatrix} \Lambda_1 & \Lambda_2 \\ \Lambda_2^* & -\Lambda_1^* \end{bmatrix} \begin{bmatrix} \mathbf{X}_1^{(k)} \\ \mathbf{X}_2^{(k)} \end{bmatrix} + \begin{bmatrix} \mathbf{N}^{(k)} \\ \bar{\mathbf{N}}^{(k+1)} \end{bmatrix} \\ &\triangleq \Lambda\mathbf{X} + \mathbf{N}\end{aligned} \quad (5)$$

where $\bar{(\cdot)}$ denotes complex conjugation of the entries of the vector. Since Λ is an orthogonal matrix, we can multiply both sides of (5) by Λ^* to decouple the two signals $\mathbf{X}_1^{(k)}$ and $\mathbf{X}_2^{(k)}$ resulting in

$$\tilde{\mathbf{Y}} = \Lambda^*\mathbf{Y} = \begin{bmatrix} \tilde{\Lambda} & \mathbf{0} \\ \mathbf{0} & \tilde{\Lambda} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1^{(k)} \\ \mathbf{X}_2^{(k)} \end{bmatrix} + \tilde{\mathbf{N}} \quad (6)$$

where $\tilde{\Lambda} \overset{def}{=} |\Lambda_1|^2 + |\Lambda_2|^2$ is an $N \times N$ diagonal matrix with (i,i) element equal to $|\Lambda_1(i,i)|^2 + |\Lambda_2(i,i)|^2$, which is also equal to the sum of the squared i^{th} DFT coefficients of first and second CIRs. This quantifies the transmit diversity gain achieved by this scheme. The filtered noise vector $\tilde{\mathbf{N}}$ has a diagonal auto–correlation matrix equal to diag$(\tilde{\Lambda}, \tilde{\Lambda})$. Therefore, the i^{th} coefficient of the MMSE–FDE in this case is equal to $\frac{1}{(\tilde{\Lambda}(i,i) + \frac{1}{SNR})}$ for $0 \le i \le N-1$. Note that the same N SC MMSE–FDE taps are applied to blocks $\tilde{\mathbf{Y}}^{(k)}$ and $\tilde{\mathbf{Y}}^{(k+1)}$ (for $k = 0, 2, 4, \cdots$) since their equivalent channel gain matrix and SNR vector are the same. The SC MMSE–FDE output is transformed back to time–domain where decisions are made. The receiver block diagram is shown in Figure 2.

3. ADAPTIVE SCHEME

In this section, we develop an adaptive technique for equalization of the SC FDE–STBC presented in Section 2. The equalization technique described in Section 2 requires the channels to be known

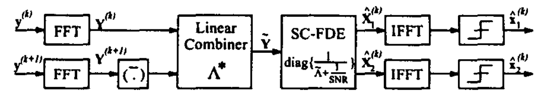

Fig. 2. Receiver block diagram

at the receiver. Channel estimation is done by adding a training sequence to each block and using this training sequence to estimate the channel followed by equalization. Adding a training sequence increases the system overhead. Minimization of the system overhead requires using longer blocks, which is not viable for wireless channels with fast variations. The proposed adaptive equalizer uses a few training blocks during initialization, then it tracks the channel variations without additional training sequences. From Figure 2, we note that the MMSE equalizer consists of a linear combiner (Λ^*) followed by scaling by a diagonal matrix. We denote the combined matrix by $\mathbf{A}$,

$$\mathbf{A} = \Lambda^* \begin{bmatrix} \text{diag}\left\{\frac{1}{\tilde{\Lambda}(i,i) + \frac{1}{SNR}}\right\} & \mathbf{0} \\ \mathbf{0} & \text{diag}\left\{\frac{1}{\tilde{\Lambda}(i,i) + \frac{1}{SNR}}\right\} \end{bmatrix} \quad (7)$$

which can be seen to have the structure

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_2^* & -\mathbf{A}_1^* \end{bmatrix}$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ are diagonal matrices given by

$$\mathbf{A}_1 = \Lambda_1^* \cdot \text{diag}\left\{\frac{1}{\tilde{\Lambda}(i,i) + \frac{1}{SNR}}\right\}_{i=0}^{N-1}$$

$$\mathbf{A}_2 = \Lambda_2 \cdot \text{diag}\left\{\frac{1}{\tilde{\Lambda}(i,i) + \frac{1}{SNR}}\right\}_{i=0}^{N-1}$$

Therefore, from Figure 2,

$$\begin{bmatrix} \hat{\mathbf{X}}_1^{(k)} \\ \hat{\mathbf{X}}_2^{(k)} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_2^* & -\mathbf{A}_1^* \end{bmatrix} \mathbf{Y} \quad (8)$$

which can be written alternatively as

$$\begin{aligned}\begin{bmatrix} \hat{\mathbf{X}}_1^{(k)} \\ \hat{\mathbf{X}}_2^{(k)} \end{bmatrix} &= \begin{bmatrix} \text{diag}(\mathbf{Y}^k) & -\text{diag}(\bar{\mathbf{Y}}^{k+1}) \\ \text{diag}(\mathbf{Y}^{k+1}) & \text{diag}(\bar{\mathbf{Y}}^k) \end{bmatrix} \begin{bmatrix} \bar{\mathbf{W}}_1 \\ \mathbf{W}_2 \end{bmatrix} \\ &\triangleq \mathbf{U}_k \mathcal{W}\end{aligned} \quad (9)$$

where $\bar{\mathbf{W}}_1$ and $\mathbf{W}_2$ are the vectors containing the diagonal elements of $\bar{\mathbf{A}}_1$ and $\mathbf{A}_2$, respectively. Moreover, $\mathcal{W}$ is a $2N \times 1$ vector containing the elements of $\{\bar{\mathbf{W}}_1, \mathbf{W}_2\}$. $\mathbf{U}_k$ is an orthogonal matrix of size $2N \times 2N$ containing the received symbols from blocks k and $k+1$. Equation (9) reveals the special structure of the STBC problem. In the non-adaptive scenario, the coefficients of $\mathcal{W}$ are calculated from a channel estimate at every block. Eq. (9), suggests that $\mathcal{W}$ can be computed adaptively using a frequency domain block version of the RLS algorithm. The equalizer coefficients are updated every two blocks according to the following recursion:

$$\mathcal{W}_{k+2} = \mathcal{W}_k + \mathcal{P}_{k+2}\mathbf{U}_{k+2}\left[\mathbf{D}_{k+2} - \mathbf{U}_{k+2}\mathcal{W}_k\right] \quad (10)$$

where

$$\begin{aligned}\mathcal{P}_{k+2} &= \lambda^{-1}[\mathcal{P}_k - \lambda^{-1}\mathcal{P}_k \mathbf{U}_{k+2} \cdot \\ &\quad (\mathbf{I}_{2N} + \lambda^{-1}\mathbf{U}_{k+2}\mathcal{P}_k\mathbf{U}^*_{k+2})^{-1}\mathbf{U}^*_{k+2}\mathcal{P}_k]\end{aligned}$$

where λ is the RLS algorithm forgetting factor that is usually close to 1. The initial conditions are $\mathcal{W}_0 = 0$ and $\mathcal{P} = \delta\mathbf{I}_{2N}$, δ is a large number, and $\mathbf{I}_{2N}$ is the $2N \times 2N$ identity matrix. $\mathbf{D}_{k+2}$ is the desired response vector given by

$$\mathbf{D}_{k+2} = \begin{bmatrix} \mathbf{X}_1^{(k+2)} \\ \bar{\mathbf{X}}_2^{(k+2)} \end{bmatrix} \quad \text{for training}$$

and

$$\mathbf{D}_{k+2} = \begin{bmatrix} \check{\mathbf{X}}_1^{(k+2)} \\ \bar{\check{\mathbf{X}}}_2^{(k+2)} \end{bmatrix} \quad \text{for decision directed tracking}$$

It might seem that the computational complexity of the algorithm is higher since matrix inversion is needed. However, due to the special structure of the space–time block–code, no matrix inversion is needed and the complexity of the algorithm is actually similar to that of an LMS implementation. We thus get RLS performance at LMS cost! This is because of the following

It follows by an induction that $\mathcal{P}_{k+2}$ has a diagonal structure of the form

$$\mathcal{P}_{k+2} = \begin{bmatrix} \mathbf{P}_{k+2} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{k+2} \end{bmatrix} \tag{11}$$

where $\mathbf{P}_{k+2}$ is itself diagonal as well. This statement clearly holds at time $k+2=0$ since, by assumption, $\mathcal{P}_0 = \delta\mathbf{I}_{2N}$ (so that $\mathbf{P}_0 = \delta\mathbf{I}_N$). Now assume the statement holds at time k. Then it is easy to see that

$$(\mathbf{I}_{2N} + \lambda^{-1}\mathbf{U}_{k+2}\mathcal{P}_k\mathbf{U}^*_{k+2})^{-1} = \begin{bmatrix} \Delta_{k+2} & \mathbf{0} \\ \mathbf{0} & \Delta_{k+2} \end{bmatrix}$$

where Δ_{k+2} is diagonal and given by

$$\begin{aligned}\Delta_{k+2} = [\mathbf{I}_N + \lambda^{-1}(&\text{diag}(\mathbf{Y}^k)\mathbf{P}_k\text{diag}(\bar{\mathbf{Y}}^k) \\ &+ \text{diag}(\mathbf{Y}^{k+1})\mathbf{P}_k\text{diag}(\bar{\mathbf{Y}}^{k+1}))]^{-1}\end{aligned} \tag{12}$$

Since diagonal matrices commute, Equation (12) simplifies to

$$\Delta_{k+2} = (\mathbf{I}_N + \lambda^{-1}P_k\text{diag}(|\mathbf{Y}^k|^2 + |\mathbf{Y}^{k+1}|^2))^{-1} \tag{13}$$

Then $\mathcal{P}_{k+2}$ has the desired structure with

$$\mathbf{P}_{k+2} = \lambda^{-1}\left[\mathbf{P}_k - \lambda^{-1}\mathbf{P}_k\Gamma_{k+2}\mathbf{P}_k\right] \tag{14}$$

where

$$\begin{aligned}\Gamma_{k+2} &= \text{diag}(\mathbf{Y}^k)\Delta_{k+2}\text{diag}(\bar{\mathbf{Y}}^k) \\ &\quad + \text{diag}(\mathbf{Y}^{k+1})\Delta_{k+2}\text{diag}(\bar{\mathbf{Y}}^{k+1}) \\ &= \Delta_{k+2}\text{diag}(|\mathbf{Y}^k|^2 + |\mathbf{Y}^{k+1}|^2)\end{aligned} \tag{15}$$

Substituting (13) in (15), we get

$$\Gamma_{k+2} = \left[\text{diag}(|\mathbf{Y}^k|^2 + |\mathbf{Y}^{k+1}|^2)^{-1} + \lambda^{-1}\mathbf{P}_k\right]^{-1} \tag{16}$$

Therefore, the algorithm collapses to

$$\mathcal{W}_{k+2} = \mathcal{W}_k + \begin{bmatrix} \mathbf{P}_{k+2} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{k+2} \end{bmatrix} \mathbf{U}_{k+2}\left[\mathbf{D}_{k+2} - \mathbf{U}_{k+2}\mathcal{W}_k\right] \tag{17}$$

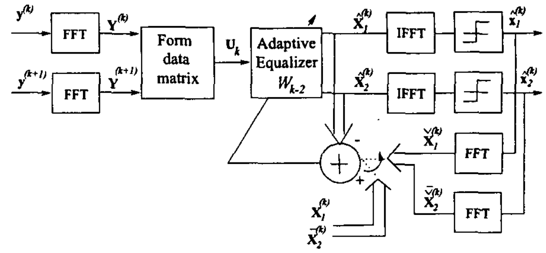

Fig. 3. Proposed receiver block diagram

where $\mathbf{P}_{k+2}$ is diagonal $N \times N$ computed via (14) and(16).

The block diagram of the adaptive receiver is depicted in Figure 3. The received signal is transformed to the frequency domain using FFT, then the data matrix $\mathbf{U}_k$ in (9) is formed. The filter output is the product of the data matrix $\mathbf{U}_k$ and the filter coefficients $\mathcal{W}_{k-2}$. The filter output $\mathbf{U}_k\mathcal{W}_{k-2}$ is transformed back to the time domain using IFFT and a decision device is used to generate the receiver output. The output of the equalizer is compared to the desired response to generate an error vector. The error vector is used to update the equalizer coefficients according to the RLS algorithm. The equalizer operates in a training mode until it converges, then it switches to a decision directed mode where previous decisions are used to update the equalizer coefficients for tracking. When tracking channels with fast variations, retraining blocks might be needed to prevent divergence of the adaptive algorithm.

Different parameters affect the performance of the adaptive equalizer. Careful selection of these parameters is needed to achieve best performance. The main factor affecting the parameter selection is how fast the channel changes, which can be measured by the doppler frequency. As the doppler frequency increases, shorter blocks are required to achieve better tracking. Using smaller forgetting factors to decrease the system memory, and retraining more frequently can prevent divergence. Using training blocks with smaller size than actual data blocks can help reduce the system overhead at the expense of a minor loss of performance. The effect of different parameters selections is shown in Section 4.

4. SIMULATION RESULTS

In this section, we provide simulation results for the performance of the proposed adaptive equalizer for space–time block–coding. Two transmit antennas with 8-PSK signal constellation are used. Data blocks of 32 symbols plus 3 symbols for the cyclic prefix are used. Typical Urban(TU) channel is considered with a linearized GMSK transmit pulse shape. The overall CIR memory of this channel is $\nu = 3$. In the following figures, we focus on the RLS algorithm and show how its performance is affected by different parameters.

Figure 4 shows the performance of the RLS algorithm at different doppler frequencies. It is obvious that the BER increases with doppler frequency as a result of the inability of the algorithm to track the faster channel variations. Since the equalizer coefficients are updated on block-by-block basis, it is useful to use smaller data blocks. Retraining more often can also be a good solution.

The effect of the FFT size and the number of blocks before

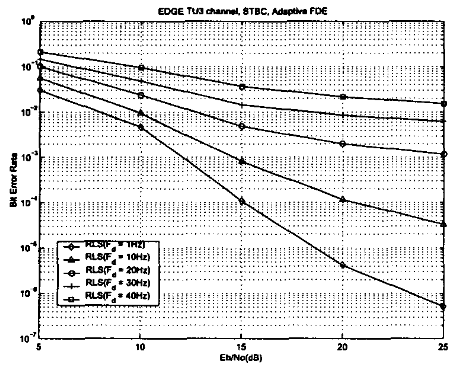

Fig. 4. Effect of doppler frequency on the performance of RLS algorithm

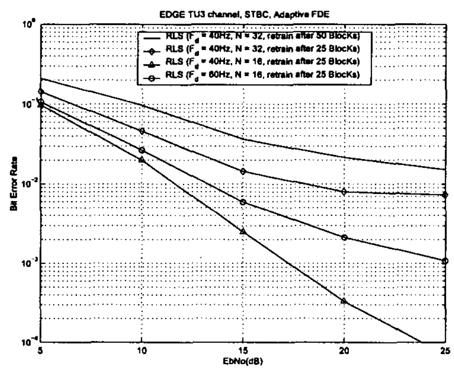

Fig. 5. Effect of FFT size and retraining frequency

retrainig on the BER performance is shown in Figure 5. It shows that retraining more often and using smaller data blocks can help improve the BER performance as the doppler frequency increases. Despite the increase of the system overhead, dramatic BER reduction was observed as we used 16-symbol data and retraining

Figure 6 shows that using smaller training blocks has a minor effect on the convergence speed, and steady state MSE of the algorithm. Figure 7 shows the steady state equalizer coefficients from 64-symbol blocks and 32-symbol blocks interpolated to produce the 64-symbol FFT. This means that smaller blocks can be used for training to reduce the overhead of training on the system without loss of performance. Using further smaller blocks can reduce the overhead more at the expense of performance loss.

5. CONCLUSIONS

An adaptive equalization scheme for space–time block–coded transmission is developed. The scheme is based on a modified low–complexity RLS algorithm that fully exploits the rich structure of STBC. Both training and tracking performance results of the scheme are presented. We are currently investigating the extension of the adaptive scheme to the multi–user case where joint interference cancellation and equalization must be performed.

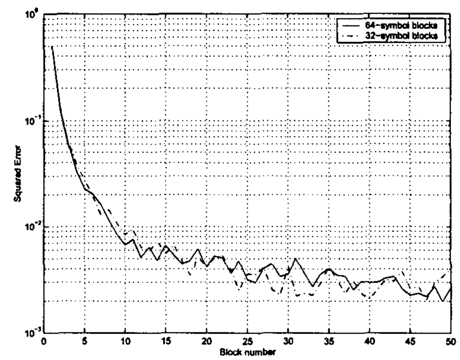

Fig. 6. Effect of block size on the convergence speed

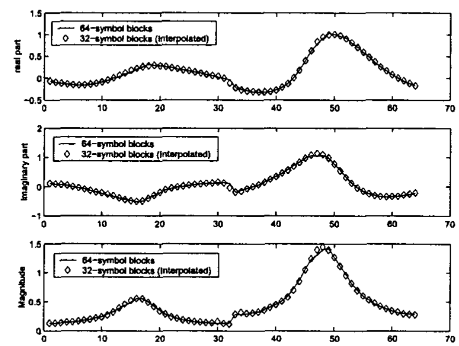

Fig. 7. Effect of block size on the equalizer coefficients

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