A STABLE ADAPTIVE STRUCTURE FOR DELTA MODULATION WITH IMPROVED PERFORMANCE

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ABSTRACT

In this paper,we propose and study an adaptive delta modulator that has improved SNR performance and better robustness in tracking highly varying signals. The step-size adaptation used in this modulator is based on information about the absolute value of the quantizer input. The modulator is shown to be free of zero-input limit cycles and is BIBO stable.

1. INTRODUCTION

Adaptive delta modulation (ADM) is used widely in coding TV and speech signals. ADM attempts to increase the dynamic range and the tracking capabilities of fixed step-size delta modulation. In feedforward adaptation, the step size of the quantizer is adapted in proportion to the input signal strength [1]. In feedback adaptation, the adaptation is made based on the history of the quantizer's output. More details on ADM and its applications can be found in [2]-[5].

In this study we develop an adaptive delta modulator that is based on estimating the strength of the input to the quantizer. In the next section, we explain the structure of the proposed modulator. In Section 3, we perform limit cycle and stability analysis of the modulator. In Section 4, we show some simulation results.

2. MODULATOR STRUCTURE

For the sake of motivation, consider the plot shown in Figure 1 and assume that we want to construct a signal $v(n)$ that tracks a signal $x(n)$ (e.g., a step signal). This can be achieved according to the following construction. At each instant of time, we start with the value $v(n-1)$ and update it to $v(n)$ so that this new value is closer to $x(n)$ than its old value. The update is based on the difference between $x(n)$ and $v(n-1)$, defined by

$$e_a(n) = x(n) - v(n-1) \tag{1}$$

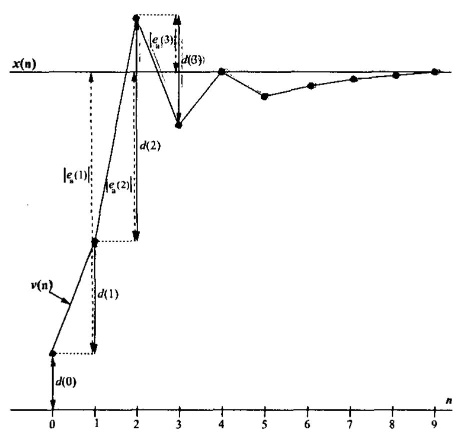

Fig. 1. Response of an output signal $v(n)$ tracking a step input $x(n)$.

The signal $v(n-1)$ is increased or decreased by a positive amount $d(n)$ depending on the size and sign of this error. Intuitively, if the error is 'large' we employ a large value for the correction term $d(n)$ and if the error is 'small' we employ a smaller correction term. More specifically, in our construction, the value of $d(n)$ is made to change by a constant factor of α or $1/\alpha$, where $\alpha > 1$. The law by which $d(n)$ varies is chosen as

$$d(n) = \begin{cases} \alpha d(n-1), & \text{if } |e_a(n)| > d(n-1) \\ \frac{1}{\alpha} d(n-1), & \text{otherwise.} \end{cases} \tag{2}$$

The sign of the error, $e_a(n)$, decides whether $v(n-1)$ increases or decreases at each time instant. Thus, the signal

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$v(n)$ is varied according to the adaptation rule

$$\boxed{v(n) = v(n-1) + \text{sign}\,[e_a(n)]\,d(n)} \qquad (3)$$

Observe that the correction term $d(n)$,also called step-size, can be expressed in the equivalent form

$$d(n) = \alpha^{w(n)} d(0) \qquad (4)$$

where

$$\boxed{w(n) = w(n-1) + q(n)} \qquad (5)$$

and

$$\boxed{q(n) = \text{sign}\,[|e_a(n)| - d(n-1)]} \qquad (6)$$

This alternative representation allows us to describe the scheme for updating $v(n)$ in block diagram form, as shown in Figure 2. The top and bottom parts of the figure implement equations (4) and (3), respectively.

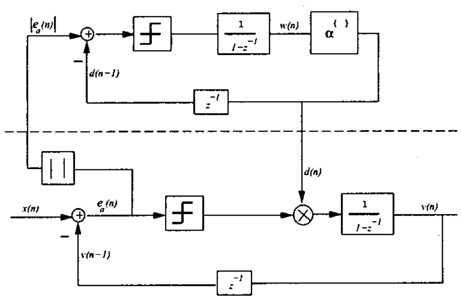

Fig. 2. Adaptive Delta Modulator.

The diagram shown in Figure 2 is the proposed adaptive delta modulator, with the upper part being the adaptation scheme of the quantizer step-size. The adaptation signal $d(n)$ is tracking the absolute value of the error signal $e_a(n)$.

3. ANALYSIS

In this section, we show that the proposed modulator is both free of zero-input limit cycles for a proper range of α. Furthermore, the Bounded Input Bounded Output (BIBO) stability of the modulator is proven with an upper bound on α.

3.1. Limits Cycles

The initial condition response of the modulator described by equations (1)–(3) is obtained by setting the input signal $x(n)$ to zero. The initial condition response of $|v(n)|$ and $d(n)$ is shown in Figure 3. Note that the signal $d(n)$ goes through two stages, increasing and then decreasing stages. We will prove that in both stages, the energy of the output signal $v(n)$ decays monotonically to zero.

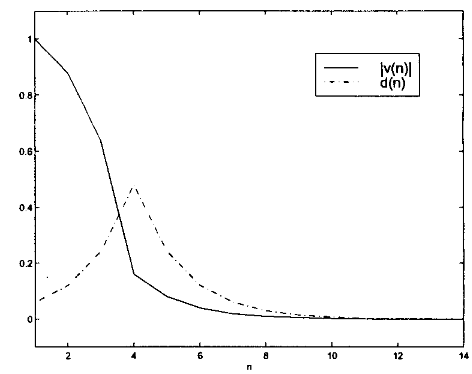

Fig. 3. Zero-input response and corresponding step size trajectory of the modulator.

Consider first the case when

$$|v(n-1)| > d(n-1).$$

Using (6), we have

$$q(n) = +1 \qquad (7)$$

and the step size $d(n)$ will increase by a factor of α. The output signal will be

$$v(n) = v(n-1) - \alpha d(n-1)\text{sign}\,[v(n-1)]\,. \qquad (8)$$

Squaring both terms, we get

$$v^2(n) = v^2(n-1) - 2\alpha d(n-1)|v(n-1)| + \alpha^2 d^2(n-1). \qquad (9)$$

If $\alpha \leq 2$ then

$$2\alpha d(n-1)|v(n-1)| > \alpha^2 d^2(n-1).$$

Therefore,

$$\boxed{v^2(n) < v^2(n-1)} \qquad (10)$$

In other words, the signal $v(n)$ will decrease in amplitude while $d(n)$ still increases until the other case is satisfied, namely,

$$d(n-1) \geq |v(n-1)|.$$

In this case,

$$q(n) = -1$$

causing the step size $d(n)$ to start decreasing by $1/\alpha$. The output signal is then given by

$$v(n) = v(n-1) - \frac{1}{\alpha} d(n-1)\text{sign}\,[v(n-1)]\,. \qquad (11)$$

The energy function is

$$v^2(n) = v^2(n-1) - \frac{2}{\alpha} d(n-1)|v(n-1)| + \frac{1}{\alpha^2} d^2(n-1). \qquad (12)$$

Since $d(n) = \frac{1}{\alpha}d(n-1)$ we have

$$v^2(n-1) - \frac{2}{\alpha}d(n-1)|v(n-1)| = v^2(n) - d^2(n). \quad (13)$$

Since $d(n-1) \geq |v(n-1)|$, it is also true that

$$v^2(n-1) \leq d(n-1)|v(n-1)|. \quad (14)$$

Moreover, if $\alpha \leq 2$ then we can write

$$v^2(n-1) \leq \frac{2}{\alpha}d(n-1)|v(n-1)|.$$

From equation (13), we conclude that

$$\boxed{v^2(n) \leq d^2(n)} \quad (15)$$

or

$$|v(n)| \leq d(n). \quad (16)$$

Thus,

$$q(n+1) = -1$$

and equations (11)-(16) will repeat. This indicates that the signal $|v(n)|$ is trapped under the positive quantity $d(n)$ which is decaying exponentially to zero.

Since in both cases ($q(n) = +1$ and $q(n) = -1$), $v^2(n)$ is decreasing monotonically to zero, we conclude that the modulator is free from zero-input limit cycles if $\alpha \leq 2$. Notice that the above analysis is independent of the initial condition of $v(n)$.

3.2. BIBO Stability

In [6, 7], we showed that the quantizer with the adaptation scheme in Figure 2 can be replaced by a time varying gain $K(n)$. Then, the ADM can be redrawn as shown in Figure 4. The gain $K(n)$ is expressed as

$$K(n) = \alpha^{e_d(n)} \quad (17)$$

where $e_d(n)$ is the quantization error associated with the one-bit quantizer that appears in the adaptation block. We assume that $e_d(n)$ is uniformly distributed between $[-1, 1]$.

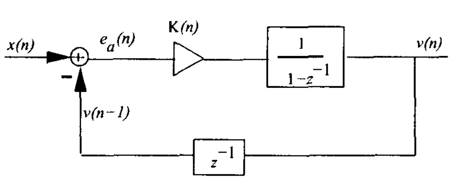

Fig. 4. Equivalent block diagram for the adaptive delta modulator.

The output of the modulator is now given by

$$v(n) = v(n-1) + K(n)e_a(n). \quad (18)$$

That is,

$$v(n) = (1 - K(n))v(n-1) + K(n)x(n) \quad (19)$$

or

$$v(n) = \sum_{i=1}^{n}\prod_{j=i}^{n}(1 - K(j))K(i)x(i). \quad (20)$$

If the input signal $x(n)$ has a bound Λ, then

$$|K(n)x(n)| \leq \max_{K(n)} \Lambda \quad (21)$$

where

$$\max_{K(n)} = \alpha. \quad (22)$$

Now, we can write

$$|v(n)| \leq \alpha\Lambda \sum_{i=1}^{n}\prod_{j=i}^{n}(1 - K(j)). \quad (23)$$

We have shown in [6] that if we choose α such that

$$2^{-1} < \alpha < 2 \quad (24)$$

then a bound L can be found that satisfies

$$|1 - K(n)| \leq L < 1. \quad (25)$$

In this case,

$$|v(n)| \leq \alpha\Lambda \sum_{i=1}^{n} L^j. \quad (26)$$

We conclude that the modulator output is bounded by

$$|v(n)| \leq \alpha\Lambda \frac{L}{1-L} \quad (27)$$

and therefore, the modulator is BIBO stable under the sufficient condition (24).

4. SIMULATION

A speech signal is sampled at the rate of 22kHz. Then, high information delta modulation (h.i.d.m), Constant Factor Delta Modulation (CFDM), and the new modulator are applied to code the speech signal. Both quantitative and subjective measures are used in the comparison process. The comparison is made without filtering the resulting voice signal. Figure 5 shows the original wave form together with the coded signals $v(n)$ using both the new ADM and h.i.d.m. (from top to bottom). We notice that h.i.d.m. failed to track the speech signal when high variations occur. On the other hand, the new ADM shows high robustness to signal variations. The signal-to-noise ratio (SNR) is used here to describe how well both modulators code the speech signal.

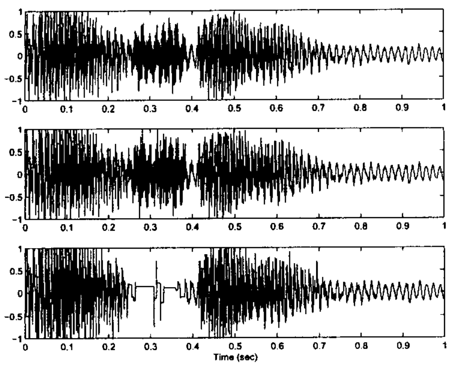

Fig. 5. Speech wave form coded by (from top to bottom): 8-bit PCM, New ADM, and h.i.d.m.

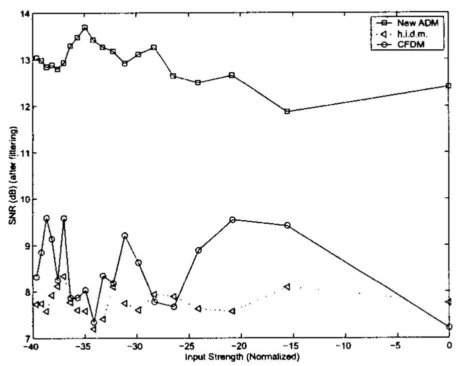

Fig. 6. Performance of the proposed ADM compared to h.i.d.m. and CFDM at the same bit rate.

Figure 6 shows a comparison between the performance of the new modulator and that of h.i.d.m. and CFDM at the same bit rate. The exponent term α is chosen to be 2.2 and an initial value for the step size adapter of 0.005 is used. From the figure, we notice the following. The new modulator outperforms the other two modulators. Moreover,th e new modulator shows more steady values of SNR with respect to change in the input strength. In a subjective test, the encoded speech signal using the proposed ADM modulator sounds more realistic than that encoded using h.i.d.m. and CFDM.

In another test, we investigated the effect of the exponent term α on the performance of the proposed ADM. Figure 7 shows the SNR performance of the proposed ADM with α ranging from 1 to 4.5. The result shows a concave behavior with an optimum value around $\alpha = 2.2$.

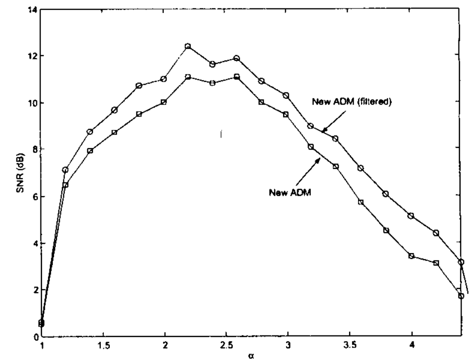

Fig. 7. SNR performance of the new ADM versus the exponent term α.

5. CONCLUSION

In this study, we proposed an adaptive delta modulator. Analytical results show that the proposed modulator is free of zero-input limit cycles and is BIBO stable. Simulation results of the modulator show superior performance compared to other schemes. Extensions to sigma-delta modulation appear in [6, 7].

6. REFERENCES

[1] L. Rabiner and R. Schafer, *Digital Processing of Speech Signals*, Prentice Hall, NJ, 1978.

[2] S. Zhang and G. Lockhart, "Design and simulation of an efficient adaptive delta modulation embedded coder," *IEE Proc.-Vis. Image Signal Process*, vol. 142, no. 3, pp. 155–160, Jun. 1995.

[3] Y. Zhu, S. Leung, and C. Wong, "A digital audio processing system based on nonuniform sampling delta modulation," *IEEE Trans. on Consumer Electronics*,v ol. 42, no. 1, pp. 80–86, Feb.1 996.

[4] P. Lallo, "Analysis and measurements of information transmission over an adaptive delta modulated voice channel," *Proc. MILCOM*, vol. 2, pp. 1057–1061, Monterey, CA, Nov. 1997.

[5] K. Balasubramanian, H. Camur, and H. Mahmutoglu, "A novel high performance adaptive delta modulator," *Proc. 16th IEEE Instrumentation and Measurement Technology Conference*, vol. 1, pp. 204-208, 1999.

[6] M. Aldajani and A. H. Sayed, "Stability analysis of an adaptive structure for sigma delta modulation," *Proc. 7th IEEE International Conference on Electronics, Circuits, and Systems*, vol. 1, pp. 129–132, Kaslik, Lebanon, Dec. 2000.

[7] M. Aldajani and A. H. Sayed, "Stability and performance analysis of an adaptive sigma delta modulator," to appear in *IEEE Trans. on Circuits and Systems II*, 2001.