An Optimal Strategy for Cooperative Spectrum Sensing in Cognitive Radio Networks

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***Abstract*— Spectrum sensing is a key enabling functionality in cognitive radio (CR) networks, where the CRs act as secondary users that opportunistically access free frequency bands. Due to the effects of channel fading, individual CRs may not be able to reliably detect the existence of a primary radio, who is a licensed user for the particular band. In this paper, we present optimal cooperation strategies for spectrum sensing to combat the effects of destructive channels and malfunctioning devices. Our approach conducts spectrum sensing based on the linear combination of local test statistics from individual secondary users. We propose two optimization schemes to control the combining weights, and compare their performance. Our first approach is to optimize the probability distribution function of the global test statistics at the fusion center. For the second scheme, we maximize the global detection sensitivity under constraints on the false alarm probability. Simulation results illustrate the significant cooperative gain achieved by the proposed strategies.**

I. Introduction

Cognitive radios [1] have emerged as a potential technology to revolutionize spectrum utilization. According to the Federal Communications Commission (FCC), cognitive radios are defined as radio systems that continuously perform spectrum sensing, dynamically identify unused spectrum, and then operate in those spectrum holes where the licensed (primary) radio systems are idle. In this way, spectrum utilization efficiency is dramatically enhanced. Spectrum sensing should also monitor for the activation of primary users in order for the secondary users to stop their transmission and vacate spectrum segments.

Spectrum sensing requires the detection of possibly-weak signals of unknown types with high reliability [2]. However, such detection performance is usually compromised by fading channel conditions between the target-under-detection and the CRs, since it is hard to distinguish between a white spectrum and a weak signal attenuated by deep fading.

In order to improve the reliability of spectrum sensing, cooperation among secondary users has been recently proposed [2] [3]. In such scenarios, a network of cooperative cognitive radios experiencing different fading states from the target, would have a better chance of detecting the primary user if they exchange sensing information among themselves. In other words, cooperative spectrum sensing can alleviate the problem of corrupted detection by exploiting spatial diversity, and thus reduces the probability of interfering with primary users. Since cooperative sensing is generally coordinated over a control channel, efficient cooperation schemes should be designed to reduce bandwidth requirements while maximizing the sensing reliability.

Although distributed detection has a rich literature (see [4] and the references therein), the study of cooperative spectrum sensing for cognitive radios is very limited. In [5], a simple fusion rule known as the OR logic operation was used to combine decisions from several secondary users. In [6], two decision-combining approaches were studied: hard decision with the AND logic operation and soft decision using the Neyman-Pearson criteria [4]. It was shown that the soft decision combination of spectrum sensing results yields gains over hard decision combining. In [7], the authors exploited the fact that adding up signals at two secondary users can increase the signal-to-noise ratio (SNR) and detection reliability if the received signals are correlated. This cooperative method is different from those discussed in [5] [6] in that it requires a wide-band control channel.

In this paper, we present an optimal cooperation strategy for spectrum sensing, where the final decision is based on a linear combination of the local test statistics from individual secondary users. The combining weight for each user's signal indicates its contribution to the global decision making. For example, if a secondary user generates a high-SNR signal and frequently makes its local decision consistent with the real hypothesis, then it is assigned a larger weighting coefficient. For those secondary users experiencing deep fading, their weights are decreased in order to reduce their negative contribution to the decision fusion. To achieve this goal, we formulate two optimization schemes to control the combining weights. The first approach optimizes a particular probability distribution function (PDF) at the fusion center in order to improve the detection performance. The second approach maximizes the probability of detection provided that the probability of false alarm is constrained. The optimized cooperation schemes improve the sensing reliability while relaxing the harsh requirements on the RF front-end sensitivity and signal processing gain at individual CR nodes. Simulation studies illustrate that the proposed cooperation schemes achieve superior sensing performance.

The paper is organized as follows. In Section II, we describe the system model. Section III introduces the weighting cooperation for spectrum sensing in cognitive radio networks. To maximize the sensing performance, we propose two op-

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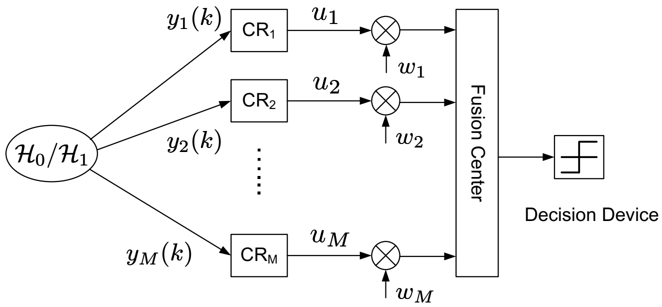

Fig. 1. A schematic representation of weighting cooperation for spectrum sensing in cognitive radio networks.

timization schemes based on different criteria in Section IV. Simulation results illustrating the effectiveness of the proposed approaches are given in Section V. Section VI concludes the paper with discussions on extensions of the proposed work.

II. System Model

We consider a binary hypothesis test for spectrum sensing at the kth time instant as follows

$$\begin{aligned} \mathcal{H}_0 :\ & y(k) = v(k) \\ \mathcal{H}_1 :\ & y(k) = hs(k) + v(k) \end{aligned}$$

where $y(k)$ is the received signal by a secondary user, $s(k)$ denotes the signal transmitted by the primary user, and $v(k)$ represents the zero-mean additive white Gaussian noise (AWGN), i.e., $v(k) \sim \mathcal{CN}(0, \sigma_v^2)$. The scalar h is the channel gain, which can be assumed to be fixed during a detection interval. Without loss of generality, $v(k)$, $s(k)$, and h are assumed to be independent of each other.

As illustrated in Fig. 1, each secondary user calculates its summary statistics u_i over a decision interval of $2n$ samples, where $2n$ is determined from the time-bandwidth product. It then sends the result to the fusion center through a control channel. The fusion center computes the global test statistics, u_c from the outputs of the individual secondary users, $\mathbf{u} = (u_1, u_2, \ldots, u_M)^T$. In this paper, we assume perfect control channels, while non-perfect cases will be considered in future work.

III. Cooperative Spectrum Sensing

In this section, we present a cooperative strategy for spectrum sensing. Since the transmitted signal of the primary user is unknown, we adopt energy detection (i.e., radiometry) as the local sensing rule, which is discussed as follows.

A. Local Sensing

We first consider local spectrum sensing at individual secondary users. For a sequence of $2n$ samples over each detection interval, the quantity

$$E_s = \sum_{k=0}^{2n-1} |s(k)|^2 \qquad (1)$$

represents the transmitted signal energy. The test statistics of the i-th secondary user using energy detector are given by

$$u_i = \sum_{k=0}^{2n-1} |y_i(k)|^2 \qquad i = 1, 2, \ldots, M \qquad (2)$$

Since u_i is the sum of the squares of $2n$ Gaussian random variables, it can be shown that u_i/σ_v^2 follows a central chi-square χ^2 distribution with $2n$ degrees of freedom if $\mathcal{H}_0$ is true; otherwise, it would follow a noncentral χ^2 distribution with $2n$ degrees of freedom. That is,

$$\frac{u_i}{\sigma_v^2} \sim \begin{cases} \chi_{2n}^2 & \mathcal{H}_0 \\ \chi_{2n}^2(\eta_i) & \mathcal{H}_1 \end{cases} \qquad (3)$$

where $\eta_i = |h_i|^2 E_s / \sigma_v^2$ is the SNR at the i-th secondary user. According to Lyapunov's central limit theorem [8], if the number of samples is large, the test statistics u_i are asymptotically normally distributed with mean

$$\bar{u}_i = \begin{cases} 2n\sigma_v^2 & \mathcal{H}_0 \\ (2n + \eta_i)\sigma_v^2 & \mathcal{H}_1 \end{cases} \qquad (4)$$

and variance

$$\sigma_i^2 = \begin{cases} 4n\sigma_v^4 & \mathcal{H}_0 \\ 4(n + \eta_i)\sigma_v^4 & \mathcal{H}_1 \end{cases} \qquad (5)$$

This can be compactly represented as $u_i \sim \mathcal{N}\left(\bar{u}_i, \sigma_i^2\right)$.

Now the decision rule at each secondary user is decided by

$$u_i \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma_i \qquad i = 1, 2, \ldots, M \qquad (6)$$

where γ_i is the corresponding decision threshold. Therefore, secondary user i will have the following probabilities of false alarm and detection:

$$P_f^{(i)} = \Pr(u_i > \gamma_i | \mathcal{H}_0) = Q\left(\frac{\gamma_i - \bar{u}_{i,\mathcal{H}_0}}{\sigma_{i,\mathcal{H}_0}}\right) \qquad (7)$$

and

$$P_d^{(i)} = \Pr(u_i > \gamma_i | \mathcal{H}_1) = Q\left(\frac{\gamma_i - \bar{u}_{i,\mathcal{H}_1}}{\sigma_{i,\mathcal{H}_1}}\right) \qquad (8)$$

B. Global Decision

The test statistics $\{u_i\}$ of secondary users are transmitted through a control channel to the the fusion center. A global test statistic is calculated linearly as

$$u_c = \sum_{i=1}^{M} w_i u_i = \mathbf{w}^T \mathbf{u} \qquad (9)$$

where the weight vector $\mathbf{w} = (w_1, w_2, \ldots, w_M)^T$ satisfies $\|\mathbf{w}\|_2^2 = 1$ and $\|\cdot\|_2$ denotes the Euclidean norm. Since the $\{u_i\}_{i=1}^M$ are normal random variables, it follows that their linear combination is also normal. Consequently, u_c is normally distributed with mean

$$\bar{u}_c = \begin{cases} 2n\mathbf{1}^T \mathbf{w}\sigma_v^2 & \mathcal{H}_0 \\ (2n\mathbf{1} + \boldsymbol{\eta})^T \mathbf{w}\sigma_v^2 & \mathcal{H}_1 \end{cases} \qquad (10)$$

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where $\boldsymbol{\eta} = (\eta_1, \eta_2, \ldots, \eta_M)^T$, and variance

$$\begin{aligned}\sigma_c^2 &= \mathrm{E}\,(u_c - \bar{u}_c)^2 \\ &= \mathbf{w}^T \mathrm{E}\left[(\mathbf{u} - \bar{\mathbf{u}})(\mathbf{u} - \bar{\mathbf{u}})^T\right]\mathbf{w} \end{aligned} \tag{11}$$

In particular, the variances for different hypotheses are given by

$$\begin{aligned}\sigma_{c,\mathcal{H}_0}^2 &= \mathbf{w}^T \mathrm{E}\left[(\mathbf{u} - \bar{\mathbf{u}}_{\mathcal{H}_0})(\mathbf{u} - \bar{\mathbf{u}}_{\mathcal{H}_0})^T | \mathcal{H}_0\right]\mathbf{w} \\ &= \mathbf{w}^T\left(4n\sigma_v^4 \mathbf{I}\right)\mathbf{w} \\ &= 4n\sigma_v^4 \end{aligned} \tag{12}$$

and

$$\begin{aligned}\sigma_{c,\mathcal{H}_1}^2 &= \mathbf{w}^T \mathrm{E}\left[(\mathbf{u} - \bar{\mathbf{u}}_{\mathcal{H}_1})(\mathbf{u} - \bar{\mathbf{u}}_{\mathcal{H}_1})^T | \mathcal{H}_1\right]\mathbf{w} \\ &= 4\sigma_v^4 \mathbf{w}^T\left[n\mathbf{I} + \mathrm{diag}\,(\boldsymbol{\eta})\right]\mathbf{w} \end{aligned} \tag{13}$$

where $\mathbf{I}$ denotes the identity matrix. From (12) and (13), we observe that the global test statistic u_c has different variances under hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$. In particular, we have $\sigma_{c,\mathcal{H}_1}^2 > \sigma_{c,\mathcal{H}_0}^2$. Moreover, if $\eta_i \gg 1$, we have $\sigma_{c,\mathcal{H}_1}^2 \gg \sigma_{c,\mathcal{H}_0}^2$.

To make a decision on the presence of the primary signal, the global quantity u_c is compared with a threshold γ_c. The performance of spectrum detection at the fusion center can be evaluated as

$$P_f^{(c)} = Q\left(\frac{\gamma_c - \bar{u}_{c,\mathcal{H}_0}}{\sigma_{c,\mathcal{H}_0}}\right) \tag{14}$$

and

$$P_d^{(c)} = Q\left(\frac{\gamma_c - \bar{u}_{c,\mathcal{H}_1}}{\sigma_{c,\mathcal{H}_1}}\right) \tag{15}$$

IV. Performance Optimization

In the context of cognitive radio networks, the probabilities of false alarm and detection have unique implications. Specifically, $1 - P_d^{(c)}$ measures the interference from secondary users to the primary users. On the other hand, $P_f^{(c)}$ determines the spectrum efficiency, i.e., a large $P_f^{(c)}$ usually results in low spectrum utilization. In this section, we propose two methods to optimize the performance of spectrum sensing.

A. Optimization of the Probability Distribution Function

From (10) and (13), we observe that the weight vector $\mathbf{w}$ plays an important role in shaping the PDF of the global test statistic u_c. To measure the effect of the PDF on the detection performance, we introduce a modified *deflection* coefficient

$$\begin{aligned} d_m^2 &= \frac{(\bar{u}_{c,\mathcal{H}_1} - \bar{u}_{c,\mathcal{H}_0})^2}{\sigma_{c,\mathcal{H}_1}^2} \\ &= \frac{(\boldsymbol{\eta}^T\mathbf{w})^2}{4\mathbf{w}^T\left[n\mathbf{I} + \mathrm{diag}\,(\boldsymbol{\eta})\right]\mathbf{w}} \end{aligned} \tag{16}$$

For accurate inference, we would like to maximize d_m^2 under the unit norm constraint on the weight vector, i.e.,

$$\begin{aligned} &\text{maximize} \quad d_m^2(\mathbf{w}) \\ &\text{st.} \quad \|\mathbf{w}\|_2^2 = 1 \end{aligned} \tag{P1}$$

We solve this problem as follows. Since we have $n\mathbf{I} + \mathrm{diag}\,(\boldsymbol{\eta}) \succ 0$, its square root can be represented as

$$\begin{aligned}\mathbf{D} &= \left[n\mathbf{I} + \mathrm{diag}\,(\boldsymbol{\eta})\right]^{1/2} \\ &= \begin{bmatrix} \sqrt{n+\eta_1} & & & \\ & \sqrt{n+\eta_2} & & \\ & & \ddots & \\ & & & \sqrt{n+\eta_M} \end{bmatrix} \end{aligned} \tag{17}$$

Applying the linear transformation $\mathbf{q} = \mathbf{D}\mathbf{w}$ gives

$$\begin{aligned} d_m^2(\mathbf{w}) &= \frac{\mathbf{q}^T\mathbf{D}^{-1}\boldsymbol{\eta}\boldsymbol{\eta}^T\mathbf{D}^{-1}\mathbf{q}}{4\mathbf{q}^T\mathbf{q}} \\ &\overset{(a)}{\leq} \frac{1}{4}\lambda_{\max}\left(\mathbf{D}^{-1}\boldsymbol{\eta}\boldsymbol{\eta}^T\mathbf{D}^{-1}\right) \end{aligned} \tag{18}$$

where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of the matrix. Note that (a) follows the *Rayleigh Ritz* inequality [9] and the equality is achieved if $\mathbf{q} = \mathbf{q}^o$, which is the eigenvector of the positive definite matrix $\mathbf{D}^{-1}\boldsymbol{\eta}\boldsymbol{\eta}^T\mathbf{D}^{-1}$ corresponding to the maximum eigenvalue. Therefore, the optimal solution of (P1) is

$$\mathbf{w}_1 = \mathbf{D}^{-1}\mathbf{q}^o / \|\mathbf{D}^{-1}\mathbf{q}^o\|_2 \tag{19}$$

which maximizes the deflection coefficient d_m^2. To enforce $\bar{u}_{c,\mathcal{H}_1} > \bar{u}_{c,\mathcal{H}_0}$, we let $\mathbf{w}_1^o = \mathrm{sign}\left(\boldsymbol{\eta}^T\mathbf{w}_1\right)\mathbf{w}_1$. Intuitively, d_m^2 implies the SNR of the test statistic u_c. As confirmed by the simulation results below, a larger value of d_m^2 leads to a larger probability of detection. This approach performs closely to the one maximizing $P_d^{(c)}$ directly (which will be discussed in the next subsection), but with much less complexity.

B. Maximum Probability of Detection

In this subsection, we design the optimal spectrum sensor by maximizing the probability of detection for a given probability of false alarm. Substituting (10) and (12) into (14) leads to

$$P_f^{(c)} = Q\left(\frac{\gamma_c - 2n\mathbf{1}^T\mathbf{w}\sigma_v^2}{2\sigma_v^2\sqrt{n}}\right) = P_t \tag{20}$$

where

$$\gamma_c = 2\sigma_v^2\left[n\mathbf{1}^T\mathbf{w} + \sqrt{n}Q^{-1}(P_t)\right] \tag{21}$$

Substituting (10), (13), and (21) into (15), we get

$$P_d^{(c)} = Q\left(\frac{\gamma - \boldsymbol{\eta}^T\mathbf{w}}{2\sqrt{\mathbf{w}^T\left[n\mathbf{I} + \mathrm{diag}\,(\boldsymbol{\eta})\right]\mathbf{w}}}\right) \tag{22}$$

where

$$\gamma = \gamma_c/\sigma_v^2 - 2n\mathbf{1}^T\mathbf{w} = 2\sqrt{n}Q^{-1}(P_t) \tag{23}$$

Since $Q(x)$ is a non-increasing function with respect to x, maximizing $P_d^{(c)}$ is equivalent to

$$\begin{aligned} &\text{minimize} \quad \frac{\gamma - \boldsymbol{\eta}^T\mathbf{w}}{2\sqrt{\mathbf{w}^T\left[n\mathbf{I} + \mathrm{diag}\,(\boldsymbol{\eta})\right]\mathbf{w}}} \\ &\text{st.} \quad \|\mathbf{w}\|_2^2 = 1 \end{aligned} \tag{P2}$$

In the following, we will show how to minimize (P2) by solving the optimum $\mathbf{w}$.

1) Large Probability of False Alarm: To simplify the optimization, we assume a conservative cognitive radio system where the transmissions are important and $P_t \geq 0.5$. This implies that $\gamma = 2\sqrt{n}Q^{-1}(P_t) \leq 0$. Thus, (P2) can be transformed into the following optimization problem

$$\begin{aligned} &\text{maximize} \quad f(\mathbf{w}) = \frac{\left(\mathbf{w}^T\boldsymbol{\eta} - \gamma\right)^2}{4\mathbf{w}^T\left[n\mathbf{I} + \operatorname{diag}(\boldsymbol{\eta})\right]\mathbf{w}} \qquad \text{(P3)} \\ &\text{subject to} \quad \|\mathbf{w}\|_2^2 = 1 \end{aligned}$$

whose optimal solution is denoted by $\mathbf{w}_3^o$.

Finding the exact solution of (P3) is difficult since $f(\mathbf{w})$ is not a concave function. Nevertheless, we can bound the optimal value through some inequalities. Since $\boldsymbol{\eta}^T\mathbf{w} > 0$ and $\gamma \leq 0$, we have $\left(\boldsymbol{\eta}^T\mathbf{w} - \gamma\right)^2 \geq \mathbf{w}^T\boldsymbol{\eta}\boldsymbol{\eta}^T\mathbf{w}$. We find that the optimal value f^o of (P3) can be bounded below by

$$f^o \geq d_m^2\left(\mathbf{w}_1^o\right) \tag{24}$$

On the other hand, an upper bound for f^o can be derived as

$$f(\mathbf{w}) \overset{(b)}{\leq} \frac{1}{4n}\left(\boldsymbol{\eta}^T\mathbf{w} - \gamma\right)^2 \overset{(c)}{\leq} \frac{1}{4n}\left(\|\boldsymbol{\eta}\|_2 - \gamma\right)^2 \tag{25}$$

Therefore, we have

$$d_m^2\left(\mathbf{w}_1^o\right) \leq f^o \leq \frac{1}{4n}\left(\|\boldsymbol{\eta}\|_2 - \gamma\right)^2 \tag{26}$$

Let $\mathbf{A} = 4\left[n\mathbf{I} + \operatorname{diag}(\boldsymbol{\eta})\right]$, which is positive definite, and hence, $\mathbf{w}^T\mathbf{A}\mathbf{w} > 0$ for any $\mathbf{w} \neq 0$. We would like to find a tighter bound for the optimal value f^o. Thus, note that for any $\alpha > 0$,

$$f(\mathbf{w}) \geq \alpha \Longleftrightarrow \phi_\alpha(\mathbf{w}) \leq 0 \tag{27}$$

where

$$\phi_\alpha(\mathbf{w}) = \mathbf{w}^T\left(\alpha\mathbf{A} - \boldsymbol{\eta}\boldsymbol{\eta}^T\right)\mathbf{w} + 2\gamma\boldsymbol{\eta}^T\mathbf{w} - \gamma^2 \tag{28}$$

Therefore, if the problem

$$\begin{aligned} &\text{find} \quad \mathbf{w} \qquad \text{(P4)} \\ &\text{subject to} \quad \phi_\alpha(\mathbf{w}) \leq 0 \\ &\qquad \|\mathbf{w}\|_2^2 = 1 \end{aligned}$$

is feasible, then we have $f^o \geq \alpha$. Conversely, if (P4) is not feasible, then we can conclude $f^o < \alpha$.

Furthermore, the feasibility problem (P4) can be transformed into a quadratic program

$$\begin{aligned} &\text{minimize} \quad \phi_\alpha(\mathbf{w}) \qquad \text{(P5)} \\ &\text{subject to} \quad \mathbf{w}^T\mathbf{w} = 1 \end{aligned}$$

Let ϕ_α^o denote the optimal value of (P5). If $\phi_\alpha^o \leq 0$, then (P4) is feasible; otherwise, (P4) is infeasible.

To solve problem (P5), we need to derive its Lagrangian dual problem, which is given by

$$\begin{aligned} L(\mathbf{w}, \nu) &= \mathbf{w}^T\left(\alpha\mathbf{A} - \boldsymbol{\eta}\boldsymbol{\eta}^T\right)\mathbf{w} + 2\gamma\boldsymbol{\eta}^T\mathbf{w} - \gamma^2 + \nu\left(\mathbf{w}^T\mathbf{w} - 1\right) \\ &= \mathbf{w}^T\left(\alpha\mathbf{A} + \nu\mathbf{I} - \boldsymbol{\eta}\boldsymbol{\eta}^T\right)\mathbf{w} + 2\gamma\boldsymbol{\eta}^T\mathbf{w} - \gamma^2 - \nu \end{aligned} \tag{29}$$

and its dual function is given by

$$g(\nu) = \inf_{\mathbf{w}} L(\mathbf{w}, \nu) \tag{30}$$

If the matrix $\alpha\mathbf{A} + \nu\mathbf{I} - \boldsymbol{\eta}\boldsymbol{\eta}^T \succeq 0$ and $\boldsymbol{\eta}$ is within the range of $\alpha\mathbf{A} + \nu\mathbf{I} - \boldsymbol{\eta}\boldsymbol{\eta}^T$, then

$$g(\nu) = -\gamma^2\boldsymbol{\eta}^T\left(\alpha\mathbf{A} + \nu\mathbf{I} - \boldsymbol{\eta}\boldsymbol{\eta}^T\right)^\dagger\boldsymbol{\eta} - \gamma^2 - \nu \tag{31}$$

where the superscript '†' represents the pseudo-inversion, and $g(\nu) = -\infty$ otherwise.

Consequently, the dual problem can be represented as

$$\begin{aligned} &\text{minimize} \quad \boldsymbol{\eta}^T\left(\alpha\mathbf{A} + \nu\mathbf{I} - \boldsymbol{\eta}\boldsymbol{\eta}^T\right)^\dagger\boldsymbol{\eta} + \nu/\gamma^2 \qquad \text{(P6)} \\ &\text{subject to} \quad \alpha\mathbf{A} + \nu\mathbf{I} - \boldsymbol{\eta}\boldsymbol{\eta}^T \succeq 0 \end{aligned}$$

which has the optimization variable $\nu \in \mathbb{R}$. Using Schur complemention, we can express the dual problem as a semi-define program (SDP) [10]

$$\begin{aligned} &\text{minimize} \quad \beta \qquad \text{(P7)} \\ &\text{subject to} \quad \begin{bmatrix} \alpha\mathbf{A} + \nu\mathbf{I} - \boldsymbol{\eta}\boldsymbol{\eta}^T & \boldsymbol{\eta} \\ \boldsymbol{\eta}^T & \beta - \nu/\gamma^2 \end{bmatrix} \succeq 0 \end{aligned}$$

with variables $(\beta, \nu) \in \mathbb{R}^2$. This problem can be easily solved for the optimal solution ν^o and the optimal value β^o. It can be shown that the strong duality holds for problems (P5) and (P6). Moreover, if $\phi_\alpha^o \leq 0$, then

$$\mathbf{w}_4^o = -\gamma\left(\alpha\mathbf{A} + \nu^o\mathbf{I} - \boldsymbol{\eta}\boldsymbol{\eta}^T\right)^\dagger\boldsymbol{\eta} \tag{32}$$

is the solution of (P4).

To find the solution of (P3), we can use the *bisection* search method to solve a feasibility problem as (P4) at each step. Start from an interval $[L, U]$, which can be given by (24) and (25). We first solve the feasibility problem (P4) at its midpoint $(L+U)/2$, to determine whether the optimal value is in the lower or upper half of the interval. Update the interval and the optimal value $\mathbf{w}_3^o = \mathbf{w}_4^o$ accordingly. We then obtain a new interval containing the optimal value but with half the width of the initial interval. This is repeated until the width of the interval is small enough and $\mathbf{w}_3^o$ is a good approximation to the optimal solution.

2) Small Probability of False Alarm: Here we assume that $P_t < 0.5$ and $P_d^{(c)} \geq 0.5$. In this case, (P2) has the following equivalent form

$$\begin{aligned} &\text{minimize} \quad \left(\gamma - \boldsymbol{\eta}^T\mathbf{w}\right)/w_0 \qquad \text{(P8)} \\ &\text{st.} \quad 4\mathbf{w}^T\left[n\mathbf{I} + \operatorname{diag}(\boldsymbol{\eta})\right]\mathbf{w} \leq w_0^2 \\ &\qquad \|\mathbf{w}\|_2^2 = 1 \end{aligned}$$

By introducing a new variable $\mathbf{z} = \mathbf{w}/w_0$ where $w_0 > 0$, (P8) can be further transformed into a convex program

$$\begin{aligned} &\text{minimize} \quad \gamma\|\mathbf{z}\|_2 - \boldsymbol{\eta}^T\mathbf{z} \qquad \text{(P9)} \\ &\text{st.} \quad 4\mathbf{z}^T\left[n\mathbf{I} + \operatorname{diag}(\boldsymbol{\eta})\right]\mathbf{z} \leq 1 \end{aligned}$$

which can be easily solved.

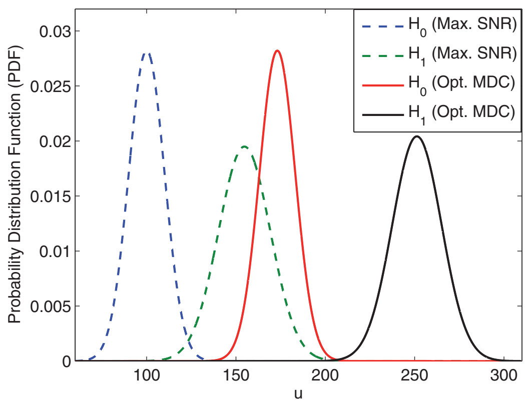

Fig. 2. The probability distribution functions (PDFs) of the test statistic (u) under different hypotheses, with $M = 3$, $n = 50$, and $\sigma_v^2 = 1$.

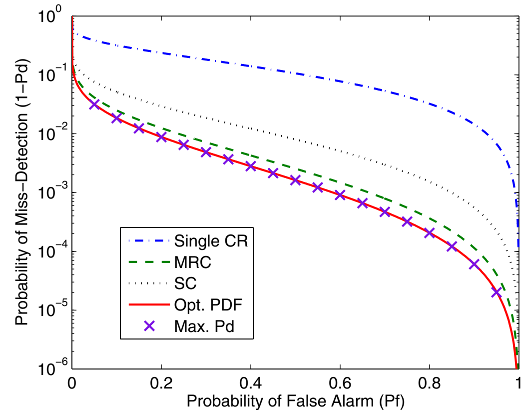

Fig. 3. The probability of miss-detection $(1 - P_d)$ vs. the probability of false alarm (P_f), with with $M = 3$, $n = 50$, and $\sigma_v^2 = 2$. The result is the average of 100 simulations.

V. Numerical Results

In this section, the proposed cooperation schemes are evaluated numerically and compared with some existing methods. Consider $M = 3$ secondary users in the network, each of which independently senses the targeted spectrum band. The channel gain between each secondary user and the target primary user is generated according to a normal distribution $\mathcal{CN}(0, 1)$. For simplicity, we assume that the transmitted primary signal has unit power $|s(k)|^2 = 1$. The proposed schemes are compared with the maximum ratio combining (MRC) and selection combining (SC, i.e., selecting the user with maximum SNR) methods.

Figure 2 shows the probability distribution functions of the test statistics under different hypotheses. The optimized PDFs are compared with the PDFs of the secondary user with maximum SNR. It can be observed that the distance between $\bar{u}_{\text{opt.pdf},\mathcal{H}_0}$ and $\bar{u}_{\text{opt.pdf},\mathcal{H}_1}$ is larger than that of $\bar{u}_{\text{sc},\mathcal{H}_0}$ and $\bar{u}_{\text{sc},\mathcal{H}_1}$. Also, the spread of $u_{\text{opt.pdf},\mathcal{H}_1}$ is narrower than that of $u_{\text{sc},\mathcal{H}_1}$, which means that the optimized PDF would result in more accurate inference. These observations imply that the PDF optimization scheme outperforms any local spectrum sensing by individual secondary users.

Figure 3 plots the probability of miss-detection $(1 - P_d)$ against the probability of false alarm (P_f). The result shows that the proposed cooperation schemes lead to much less interference (much higher probability of detection) to the primary radio than MRC and SC based approaches. The cooperation gain is due to the optimization of the PDF of u_c under hypothesis $\mathcal{H}_1$. For a practical cognitive radio system that has a probability of detection greater than or equal to 50%, we observe that the probability of detection given by the PDF optimization method approximates closely to the exact maximum value obtained from (P2). Therefore, the PDF optimization scheme can be used as an efficient alternative choice for conservative opportunistic spectrum sharing.

VI. Conclusion

We have developed two optimization schemes for cooperative spectrum sensing in cognitive radio networks. The proposed schemes optimize the detection performance by operating over a linear combination of local test statistics from individual secondary users, which combats the destructive channel effects between the target and the secondary nodes. We conclude that the optimization of PDF would approximate the maximum-P_d approach for a fixed probability of false alarm. Some interesting extensions include studying finite-bit communications over non-ideal wireless channels. One can also consider fully distributed detection where each cognitive radio device can work as a fusion center.

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References

[1] J. Mitola III and G. Q. Maguire, "Cognitive radio: Making software radios more personal," *IEEE Pers. Commun.*, vol. 6, pp. 13–18, 1999.

[2] D. Cabric, S. M. Mishra, and R. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Proc. 38th Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, Nov. 2004, pp. 772–776.

[3] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Select. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[4] R. S. Blum, S. A. Kassam, and H. V. Poor, "Distributed detection with multiple sensors: Part II - Advanced topics," *Proc. IEEE*, vol. 85, no. 1, pp. 64–79, Jan. 1997.

[5] A. Ghasemi and E. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *Proc. IEEE DySPAN*, Baltimore, MD, Nov. 2005, pp. 131–136.

[6] E. Vistotsky, S. Kuffner, and R. Peterson, "On collaborative detection of TV transmissions in support of dynamic spectrum sharing," in *Proc. IEEE DySPAN*, Baltimore, MD, Nov. 2005, pp. 338–345.

[7] G. Ghurumuruhan and Y. Li, "Agility improvement through cooperative diversity in cognitive radio," in *Proc. IEEE GLOBECOM*, St. Louis, MO, Nov. 2005, pp. 2507–2509.

[8] B. V. Gendenko and A. N. Kolmogorov, *Limit Distributions for Sums of Independent Random Variables*. Addison-Wesley, MA, 1954.

[9] A. H. Sayed, *Fundamentals of Adaptive Filtering*. Wiley, NY, 2003.

[10] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge Univ. Press, 2003.