A DYNAMIC ANTENNA SCHEDULING STRATEGY FOR MULTI-USER MIMO COMMUNICATIONS

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ABSTRACT

The paper develops a dynamic antenna scheduling strategy for downlink MIMO communications, where the transmitted signal for each user is beamformed towards a selected subset of receive antennas at this user. The proposed method removes the condition on the number of transmit-receive antennas in comparison to traditional zero-forcing and time-scheduling strategies. By characterizing the probability distribution of the so-called signal-to-leakage-plus-noise (SLNR) ratio, we show that there is an optimal set of receive antennas that maximizes the system performance for each channel realization. This fact is used to propose an antenna scheduling scheme that leads to improvements in terms of SINR outage probabilities.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) schemes can provide a substantial gain in network downlink throughput by allowing multiple users to communicate in the same frequency and time slots. However, the multiplicity of users causes co-channel interference (CCI) among users. Several works in the literature have proposed schemes for perfectly canceling the CCI for each user, such as using zero forcing (ZF) solutions [1],[2],[3], [4]. Unfortunately, ZF designs tend to impose an impractical condition on the number of transmit-receive antennas (essentially requiring more transmit antennas than the combined number of all receive antennas). One way to alleviate this condition is to resort to time-scheduling [5], where only a subset of the users are allowed to communicate at each time slot.

In this paper, we motivate and propose an antenna scheduling strategy where the transmitted signal vector for each user is beamformed towards a selected subset of the receive antennas at this user. Although the non-selected receive antennas are ignored when designing the beamforming vectors, they are still used at the receiver side to improve the decoding of the signal. The proposed method relaxes the condition on the number of transmit-receive antennas and is shown to lead to improvements in terms of SINR outage probabilities.

2. SYSTEM MODEL

Consider a downlink multi-user environment with a base station communicating with K users. The base station employs N transmit antennas and each user could be equipped with multiple antennas as well. Let M_i denote the number of receive antennas at the ith user. A block diagram of the system is shown in Figure 1, where $s_i(n)$ denotes the transmitted data intended for user i at time n. The scalar data $s_i(n)$ is multiplied by an $N \times 1$ beamforming vector $\mathbf{w}_i$ before being transmitted over the channel. In this way, the overall $N \times 1$ transmitted vector at time n is given by

$$\mathbf{x}(n) = \sum_{k=1}^{K} \mathbf{w}_k s_k(n) \qquad (N \times 1) \tag{1}$$

The data $s_i(n)$ and the beamforming coefficients $\mathbf{w}_i$ are assumed to be normalized as follows:

$$\mathsf{E}|s_k(n)|^2 = 1, \quad \|\mathbf{w}_k\|^2 = 1$$

for $k = \{1, \ldots, K\}$.

The $N \times 1$ vector $\mathbf{x}(n)$ is then broadcast over the channel. Assuming a narrow-band (single-path) channel, the received vector of size $M_i \times 1$ at the ith user at time n is given by

$$\mathbf{y}_i(n) = \mathbf{H}_i \sum_{k=1}^{K} \mathbf{w}_k s_k(n) + \mathbf{v}_i(n) \qquad (M_i \times 1) \tag{2}$$

where the elements of $\mathbf{H}_i$ are complex Gaussian variables with zero-mean and unit-variance. Furthermore, the additive noise $\mathbf{v}_i(n)$ satisfies

$$\mathsf{E}\mathbf{v}_i(n)\mathbf{v}_i^*(n) = \sigma_i^2 \mathbf{I}_{M_i}$$

where $\mathbf{I}_{M_i}$ is the $M_i \times M_i$ identity matrix. It is assumed that each channel matrix $\mathbf{H}_i$ is available at the base station and at the corresponding user. Dropping the time index n for notational simplicity we rewrite (2) as

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{w}_i s_i + \sum_{k=1, k \neq i}^{K} \mathbf{H}_i \mathbf{w}_k s_k + \mathbf{v}_i \qquad (M_i \times 1) \tag{3}$$

where the second term is the co-channel interference (CCI) caused by the multi-user nature of the system. The signal-to-interference-plus-noise ratio (SINR) at the *input* of the receiver is given by

$$\text{SINR}_i = \frac{\|\mathbf{H}_i \mathbf{w}_i\|^2}{M_i \sigma_i^2 + \sum_{k=1, k \neq i}^{K} \|\mathbf{H}_i \mathbf{w}_k\|^2} \tag{4}$$

Using (4) as an optimization criterion and maximizing it to select the $\{\mathbf{w}_i\}$ would generally result in a challenging optimization problem to solve with K coupled variables $\{\mathbf{w}_i\}$.

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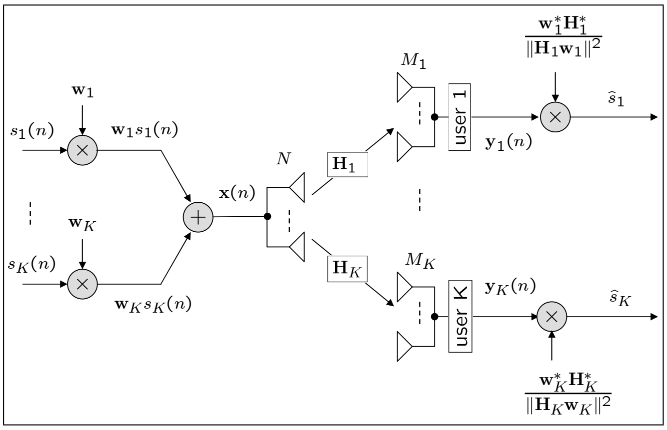

Figure 1: Block diagram of the multi-user beamforming system.

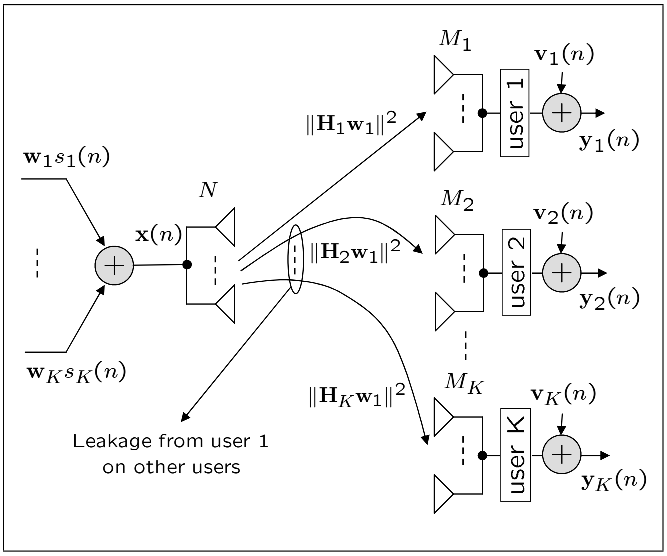

Figure 2: A block diagram depicting the leakage from user 1 on other users.

An alternative criterion was introduced in [6] to select the beamforming coefficients $\{\mathbf{w}_i\}$, and which is based on maximizing instead a signal-to-leakage-plus-noise ratio (SLNR). In this formulation, the *leakage* for user i is defined as the total power leaked from this user to all other users — Fig. 2. The leakage power is given by

$$\sum_{k=1,k\neq i}^{K} \|\mathbf{H}_k\mathbf{w}_i\|^2$$

and the beamforming vectors $\{\mathbf{w}_i\}_{i=1}^K$ are obtained by solving the optimization problem:

$$\mathbf{w}_i^o = \arg \max_{\mathbf{w}_i\in\mathbb{C}^{N\times 1}} \underbrace{\frac{\|\mathbf{H}_i\mathbf{w}_i\|^2}{M_i\sigma_i^2 + \sum_{k=1,k\neq i}^K \|\mathbf{H}_k\mathbf{w}_i\|^2}}_{\text{SLNR for user } i} \tag{5}$$

subject to $\|\mathbf{w}_i\|^2 = 1, \quad i = \{1,\ldots,K\}$. Compared with (4), the term $\|\mathbf{H}_i\mathbf{w}_k\|^2$ in the denominator has been replaced by $\|\mathbf{H}_k\mathbf{w}_i\|^2$.

Problem (5) can be rewritten as

$$\mathbf{w}_i^o = \arg \max_{\mathbf{w}_i\in\mathbb{C}^{N\times 1}} \frac{\mathbf{w}_i^*\mathbf{H}_i^*\mathbf{H}_i\mathbf{w}_i}{M_i\sigma_i^2 + \mathbf{w}_i^*\tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\mathbf{w}_i} \tag{6}$$

subject to $\|\mathbf{w}_i\|^2 = 1$, where

$$\tilde{\mathbf{H}}_i = \left[\mathbf{H}_1^T \cdots \mathbf{H}_{i-1}^T\mathbf{H}_{i+1}^T \cdots \mathbf{H}_K^T\right]^T \qquad \left(\sum_{\substack{j=1\\ j\neq i}}^{K} M_j \times N\right) \tag{7}$$

is an extended channel matrix that excludes $\mathbf{H}_i$ only. The solution to (6) is given by

$$\mathbf{w}_i^o \propto \max.\ \text{eigenvector}\left(\left(M_i\sigma_i^2\mathbf{I} + \tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\right)^{-1}\mathbf{H}_i^*\mathbf{H}_i\right) \tag{8}$$

in terms of the eigenvector that corresponds to the maximum eigenvalue ($\lambda_{\max}$) of the matrix in (8). The norm of $\mathbf{w}_i^o$ is scaled to $\|\mathbf{w}_i^o\|^2 = 1$. It was indicated in [6] that this solution outperforms classical zero-forcing (ZF) solutions in terms of BER; in addition it does not impose a condition on the number of transmit and receive antennas. Figure 3 is a plot of the output SINR outage curves using three different schemes, namely, the proposed SLNR-based scheme (using the full number of receive antennas), the ZF scheme [4] and a single-user beamforming scheme which ignores the CCI when selecting the beamforming vectors [7]. These three schemes are compared to an interference-free scenario, which is added in the plot only for comparison purposes. The figure shows that the SLNR-based scheme outperforms the ZF scheme.[1] Choosing $\mathbf{w}_i^o$ according to (8) results in a maximum SLNR given by

$$\boxed{\text{SLNR}_i = \lambda_{\max}} \tag{9}$$

where $\lambda_{\max}$ is the maximum eigenvalue of $\left(M_i\sigma_i^2\mathbf{I} + \tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i\right)^{-1}\mathbf{H}_i^*\mathbf{H}_i$.

In order to examine how $\lambda_{\max}$, and hence SLNR, vary with the system parameters $\{N, K, M_i\}$, we have derived the joint probability density function (pdf) of the eigenvalues of the matrix expression $(M_i\sigma_i^2\mathbf{I} + \tilde{\mathbf{H}}_i^*\tilde{\mathbf{H}}_i)^{-1}\mathbf{H}_i^*\mathbf{H}_i)$; details omitted for brevity:

$$f(\lambda_1,\ldots,\lambda_N; N, \{M_i\}) \propto \prod_{\substack{l,j=1\\ l<j}}^{l,j=m} |\lambda_l - \lambda_j|^2 \cdot \prod_{j=1}^{N} \frac{|\lambda_j|^{M_i-N} e^{-M_i\sigma_i^2(\lambda_j+1)}}{|M_i\sigma^2(\lambda_j+1)|^{(\sum_{j=1}^K M_j)}}$$

Integrating over all $\lambda_i < \lambda_{\max}$, we obtain the pdf of $\lambda_{\max}$. Figures 4 and 5 show how the pdf of $\lambda_{\max}$ of a user depends on the the number of receive antennas of this user and on the sum of receive antennas of all other users, respectively. In the figures, $t = \sum_{\substack{j=1\\ j\neq i}}^K M_j$.

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[1] In comparison, zero forcing solutions require the number of transmit antennas (N) to be essentially at least equal to the combined number of receive antennas by all users ($\sum_{\substack{j=1\\ j\neq i}}^K M_j$) [4].

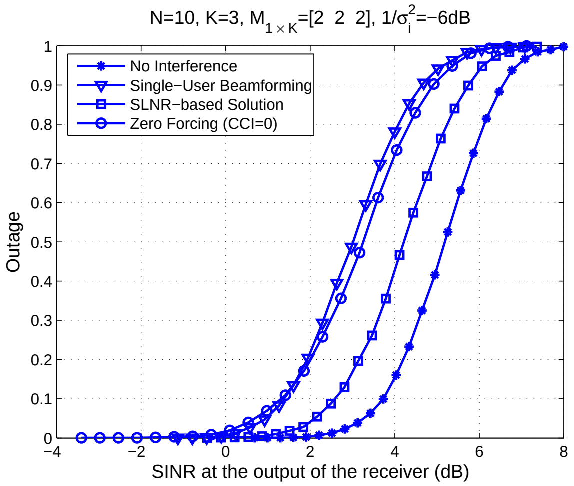

Figure 3: SINR outage probability for one user comparing 3 different schemes with the ideal interference-free case.

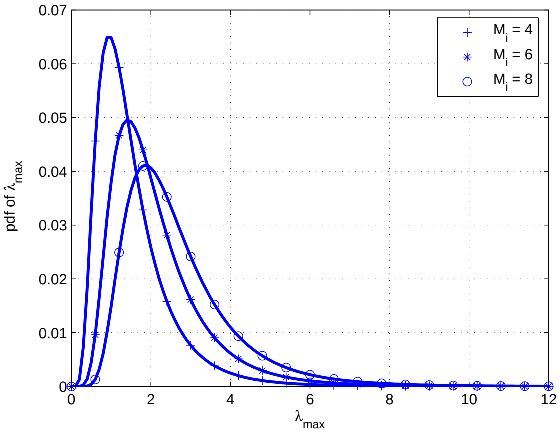

Figure 4: The derived pdf of $\lambda_{\max}$ for $N=2$, $t=6$, $\sigma^2=0.2$, and three values of $M_i=\{4,6,8\}$.

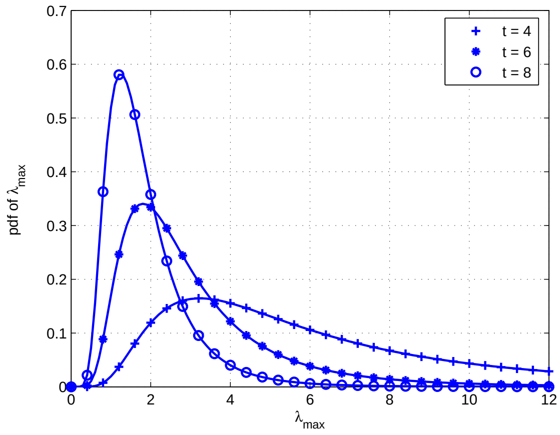

Figure 5: The derived pdf of $\lambda_{\max}$ for $N=2$, $M_i=4$, $\sigma^2=0.2$, and three values of $t=\{4,6,8\}$.

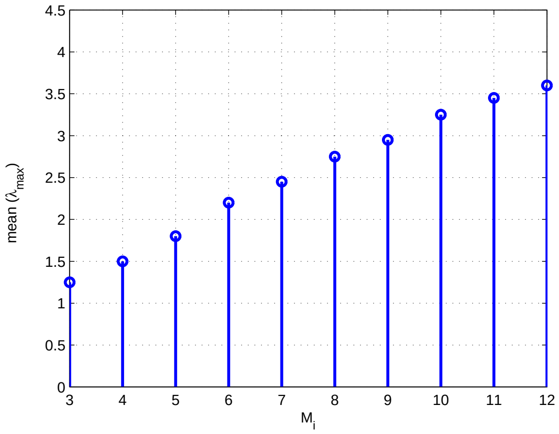

Figure 6: Mean value of $\lambda_{\max}$ versus M_i for fixed values of N, t, and $\sigma^2=0.2$.

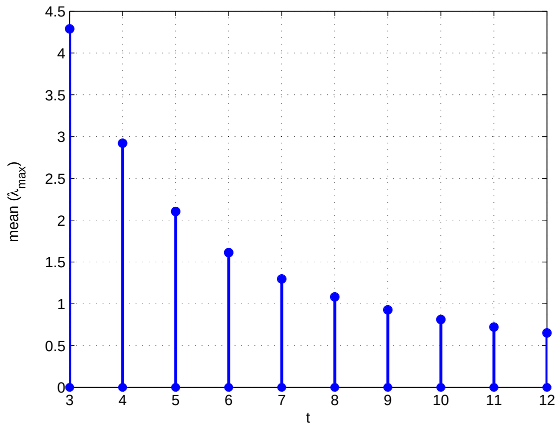

Figure 7: Mean value of $\lambda_{\max}$ versus t for fixed values of N, M_i, and $\sigma^2=0.2$.

The average SLNR of a user (given by the mean value of $\lambda_{\max}$), increases as the number of antennas of this user M_i increases. This is illustrated in Figure 6. An interesting result, however, is that as t (the total number of antennas by all other users) increases, the average SLNR decreases. The intuition here is that as the number of channel links to other users increases, the less the degrees of freedom we have in the choice of the beamforming vector $\mathbf{w}_i$.

3. ANTENNA SELECTION PROCEDURE

The above conclusions suggest that when designing the beamforming vectors $\mathbf{w}_i$ using the SLNR as the optimization criterion, there is a conflict among the users in the system. Increasing the number of antennas of a user improves the performance of this user but designing the other users beamformers based on all these antennas degrades the performance of other users. We propose an antenna selection scheme where the transmitter selects a subset of receive antennas at each user for the design of its beamforming vectors. We would like to emphasize here that although the non-selected receive antennas are ignored during the design of the beamforming vectors, they are not shutdown on the receiver side. In fact, using all the antennas at the receiver side to decode the signal improves the SINR of the user as opposed to using only the selected subset of antennas for which the beamforming vector is optimized. We thus suggest the following dynamic method for antenna selection that would improve system performance. According to the proposed scheme, we reduce the number of active antennas (taken into account when designing the beamforming vectors) for users not meeting an SINR threshold. The threshold value applies to the SINR at the output of the receiver. By lowering the number of selected antennas for these users, the other users in the network will have a higher probability to meet their SINR threshold. This procedure does not yield any degradation in the system performance in terms of outage values since those users not meeting the SINR threshold cannot establish a connection anyway. Thus for each channel realization, we perform a search over all possible receive antenna combinations ($2^{\sum_i M_i} - 1$) and choose the combination that results in a maximum number of users meeting an SINR threshold. In general, there may be more than one combination of receive antennas that fulfill this condition. Among all combinations, we choose the one that maximizes the SINR of the worst above-the-threshold user. This scheme does not require any change in the receivers since all the received antennas are used for decoding on the user side. In the cases that some users are meeting their SINR threshold by a large margin, their number of active antennas can be reduced in favor of other users in the system. As long as such users still meet their threshold, the reduction in their active antennas will help the other users in the system. Overall, this mechanism can statistically improve the outage results for all users in the network.

4. SIMULATIONS

The simulation results are shown in Figures 8 and 9. The plots compare the final SINR outage curves for the following two scenarios: 1) all antennas for all users are used, 2) the configuration suggested by the search scheme of Sec. 3 is used. The channel model described in Sec. 2 is used in all simulations. The following simulation results represent two different antenna configurations, 1) $t > N$: this is the case where the ZF scheme fails. 2) $t < N$: condition for ZF scheme is satisfied and the results show that our proposed scheme outperforms the ZF scheme.

4.1 $\mathbf{H}_i$ and $\tilde{\mathbf{H}}_i$ are tall matrices: $t > N$.

- Number of transmit antennas $N = 5$.
- Number of users $K = 3$.
- Number of available receive antennas $\{M_1, M_2, M_3\} = \{2, 2, 5\}$.
- Target SINR thresholds $\{T_1, T_2, T_3\} = \{7, 7, 10\}$dB.

The SNR per received antenna is defined as $1/\sigma_i^2$ and is assumed to be 0 dB. The simulation is conducted over 200 channel realizations.

Figure 8 shows the resulting outage curves for each of the 3 users in the system for the following 2 cases:

- Using all available antennas.
- Using the proposed antenna configuration.

The curves in Fig. 8 are SINR outage curves. That is, each curve is the cumulative density function (cdf) of the SINR

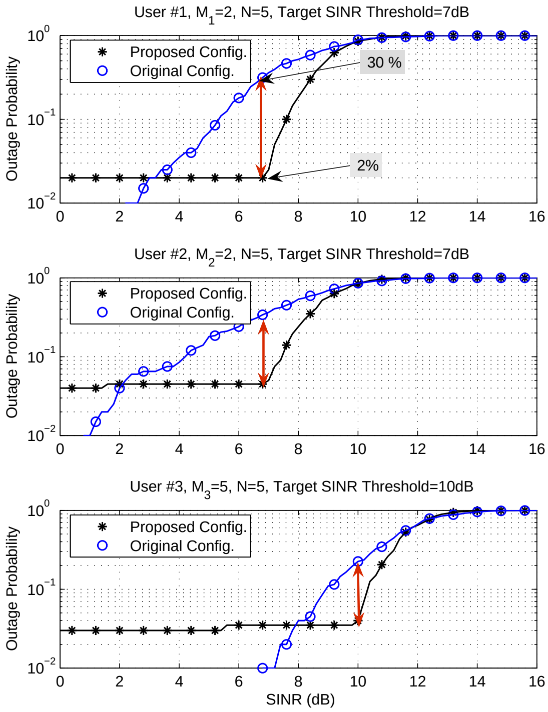

Figure 8: SINR outage probability for all the users.

at the output of the receiver for the corresponding user. The outage curve represents $P(\text{SINR} \leq \mu)$ on the vertical axis for different values of μ on the horizontal axis. Consider the results for user 1 in the top plot of Fig. 8 . The SINR threshold for this user is 7 dB meaning that if the SINR value for this user falls below 7 dB, the package is dropped and it has to be re-transmitted. Thus, the probability $P(\text{SINR} \leq 7\text{ dB})$ measures the likelihood that this user will not establish communication with the transmitter. The figure shows that by using the original antenna configuration, user 1 achieves an outage of 30% while using the proposed scheme the outage reduces to 2%. Note that the curve for the proposed scheme is flat for SINR values up to the threshold (7 dB) and then it increases. This is because in our proposed scheme, the signal is transmitted to the user only if there is a reliable channel (i.e., if the SINR is above the threshold). This hard decision at the transmitter translates into the breakpoint in the curve. Thus the flat part of the curve corresponds to the case of no transmission and its value is the outage percentage for all the values of SINR below the threshold.

According to the results shown in Figure 8, the following outage improvements are achieved:

- User 1 (7dB outage): from 30% to 2%
- User 2 (7dB outage): from 40% to 5%
- User 3 (10dB outage): from 20% to 3%

Thus all three users experience a significant improvement in outage probability at their target SINR.

For this antenna configuration, it can be seen from Figure 8 that the SINR of the users are sacrificed in the region where SINR is below the target threshold. This yields no degradation in the target SINR outage since no reliable communica-

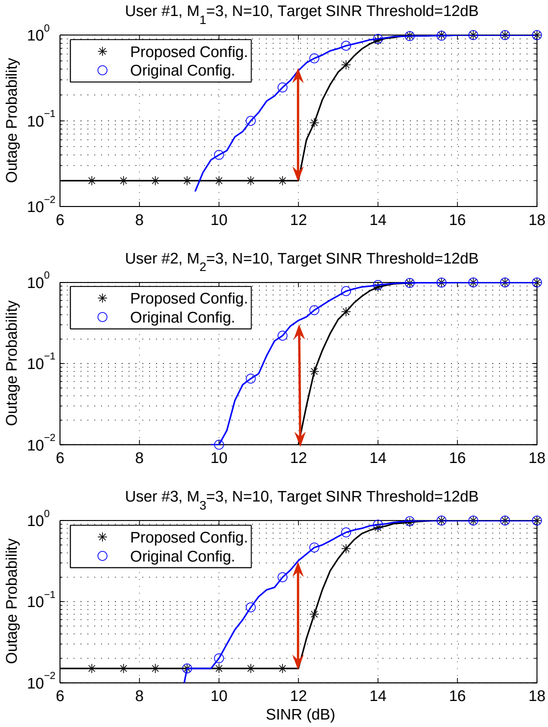

Figure 9: SINR outage probability for all the users.

tion is desired below this threshold anyways. However, by sacrificing the SINR of one users, the other users meet their thresholds with a greater probability, as was argued in Sec. 3.

4.2 $\mathbf{H}_i$ and $\tilde{\mathbf{H}}_i$ are fat matrices: $t < N$.

- Number of transmit antennas $N = 10$
- Number of users $K = 3$
- Number of available receive antennas $\{M_1, M_2, M_3\} = \{3, 3, 3\}$
- Target SINR thresholds $\{T_1, T_2, T_3\} = \{12, 12, 12\}$dB

According to the results shown in Figure 9, the following outage improvements are achieved:

- User 1 (12dB outage): from 40% to 2%
- User 2 (12dB outage): from 30% to 1%
- User 3 (12dB outage): from 30% to 1.6%

5. CONCLUSIONS

We proposed a dynamic antenna scheduling strategy for downlink MIMO communications that is based on characterizing and exploiting the dependence of the signal-to-leakage-plus-noise (SLNR) ratio on the system parameters. The SLNR strategy is found to relax the condition on the number of transmit-receive antennas in comparison to traditional zero-forcing and time-scheduling strategies. The dependence of the pdf of the SLNR on the system parameters was exploited to propose an antenna scheduling scheme that leads to significant improvement in terms of SINR outage probabilities. Simulation results illustrate the resulting system performance.

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