A PHASE NOISE COMPENSATION SCHEME FOR OFDM WIRELESS SYSTEMS

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ABSTRACT

Phase noise causes significant degradation in the performance of Orthogonal Frequency Division Multiplexing (OFDM) based wireless communication systems. In the proposed compensation scheme, the communication between the transmitter and receiver blocks consists of two stages. In the first stage, block-type pilot symbols are transmitted and the channel coefficients are jointly estimated with the phase noise in the time domain. In the second stage, comb-type OFDM symbols are transmitted such that the receiver can jointly estimate the data symbols and the phase noise. It is shown by computer simulations that the proposed scheme can effectively mitigate the inter-carrier interference caused by phase noise and improve the bit error rate of OFDM systems.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a widely recognized modulation technique for high data rate communications over wireless links [1]. Because of its capability to capture multipath energy and eliminate inter-symbol interference, OFDM has been chosen as the transmission method for several standards, including the IEEE 802.11a wireless local area network (WLAN) standard in the 5 GHz band, the IEEE 802.11g WLAN standard in the 2.4 GHz band, and the European digital video broadcasting system (DVB-T). Also, the OFDM-based physical layer is being considered by several standardization groups, such as the IEEE 802.15.3 wireless personal area network (WPAN) and the IEEE 802.20 mobile broadband wireless access (MBWA) groups. The heightened interest in OFDM has resulted in tremendous research activities in this field to make the real systems more reliable and less costly in practice.

One limitation of OFDM systems is that they are highly sensitive to the phase noise introduced by local oscillators [2, 3]. Phase noise is the phase difference between the phase of the carrier signal and the phase of the local oscillator. The distortion caused by phase noise is characterized by a common phase error (CPE) term and an inter-carrier interference (ICI) term. The CPE term represents the common rotation of all constellation points in the complex plane, while the ICI term behaves like additive Gaussian noise.

The high sensitivity of OFDM receivers to phase noise imposes a stringent constraint on the design and fabrication of oscillators and the supplementary circuitry. There have been works in the literature to mitigate the effects of phase noise in the digital domain. This approach provides an efficient, low-cost and reliable solution to the phase noise problem. Some authors have proposed methods to compensate for the CPE term, in which the constellation rotation is estimated using pilot tones embedded in OFDM symbols and then corrected by the demodulator [4]. Since the ICI effect is either ignored or treated as additive noise in these schemes, they perform poorly if the phase noise varies fast in comparison to the OFDM symbol rate. To overcome this difficulty, some ICI compensation schemes have been proposed. In the self-cancellation method presented in [5], each data symbol is transmitted using two adjacent subcarriers, and the received symbols are linearly combined to suppress ICI by exploiting the fact that the ICI coefficients change slowly over adjacent subcarriers. This technique has the advantage of low implementation complexity, but it reduces the spectral efficiency by one half. In [6], an FIR-type equalizer is employed to compensate for phase noise, and the filter coefficients are determined by the method of least squares. Since the filter length is limited by the number of pilot tones, it can only compensate for the ICI that is from adjacent subcarriers. In [7], the phase noise process is approximated by using a small number of sinusoidal components, and it is suggested to insert some pilot tones outside the spectrum occupied by data transmission and estimate the model parameters of phase noise using the received pilot signals. This scheme requires extra bandwidth and can only correct the ICI from adjacent subcarriers because of the approximation made in modeling. All these ICI compensation schemes assume that the receiver has perfect channel state information; however, in wireless communications, the channel is time-varying and the receiver has to estimate the channel in the presence of phase noise, which makes the scenario more complicated than what has been studied before.

In this paper, we propose a new phase noise compensation scheme for OFDM-based wireless communications with improved performance. The scheme consists of a channel estimation stage and a data transmission stage. In the channel estimation stage, block-type pilot symbols are transmitted so that the receiver can jointly estimate the channel coefficients and the phase noise. Instead of estimating the channel coefficients and phase noise in the frequency domain, we estimate them in the time domain by using interpolation techniques to reduce the number of unknowns. In the data transmission stage, comb-type pilot symbols, which contain both data symbols and pilot symbols, are transmitted, and the data symbols and the phase noise components are jointly estimated at the receiver in order to mitigate both the CPE and ICI effects of phase noise.

2. SYSTEM MODEL WITH PHASE NOISE

An OFDM system with phase noise is illustrated in Fig. 1. At the transmitter, the information bits are first mapped into constellation symbols, and then converted into a block of N symbols $x[k]$, $k = 0, 1, \ldots, N-1$, by a serial-to-parallel converter. The N symbols are the frequency components to be

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transmitted using the N subcarriers of the OFDM modulator, and are converted to OFDM symbols by the unitary inverse Fast Fourier Transform (IFFT). Then, a cyclic prefix of length P $(P \leq N)$ is added to the IFFT output, and the resulting $N+P$ symbols are converted to a continuous-time baseband signal $x(t)$ for transmission.

At the receiver, the received block after OFDM demodulation is $y[k]$, $k = 0,1,\ldots,N-1$, whose elements are related to the transmitted symbols $x[k]$, $k = 0,1,\ldots,N-1$, by

$$y[k] = \sum_{r=0}^{N-1} \alpha[r] H\left[(k-r)_N\right] x\left[(k-r)_N\right] + w[k], \qquad (1)$$

where $(k-r)_N$ stands for $((k-r) \bmod N)$, $H[k]$ is the channel response in the k^{th} subcarrier, $w[k]$ is the additive noise component in the k^{th} subcarrier, and $\alpha[r]$ is given by

$$\alpha[r] = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\phi(nT_s)} e^{-j\frac{2\pi rn}{N}}, \qquad (2)$$

where $\phi(t)$ is the phase noise at the local oscillator and T_s is the symbol time. Note that $H[k]$, $k = 0,1,\ldots,N-1$, are the Fourier transform coefficients of the discrete-time baseband channel impulse response $h[n]$, i.e.,

$$H[k] = \sum_{n=0}^{L-1} h[n] e^{-j\frac{2\pi kn}{N}},$$

where L is the length of $h[n]$ $(0 \leq n \leq L-1)$. OFDM modulation requires $P \geq L$ in order to eliminate the inter-symbol interference. Expression (1) can be rewritten as

$$y[k] = \alpha[0] H[k] x[k] + \sum_{r=1}^{N-1} \alpha[r] H\left[(k-r)_N\right] x\left[(k-r)_N\right] + w[k],$$

where the term $\alpha[0]H[k]x[k]$ is called the common phase error (CPE) and $\sum_{r=1}^{N-1} \alpha[r] H\left[(k-r)_N\right] x\left[(k-r)_N\right]$ is called the inter-carrier interference (ICI). In the absence of phase noise, we have the traditional relation

$$y[k] = H[k]x[k] + w[k],$$

which follows by setting $\alpha[0] = 1$ and $\alpha[r] = 0$ for $r \neq 0$. Using matrix notation, expression (1) can be represented as

$$\mathbf{y} = \mathbf{A}\mathbf{H}\mathbf{x} + \mathbf{w}, \qquad (3)$$

where

$$\mathbf{y} = \begin{bmatrix} y[0] & y[1] & \ldots & y[N-1] \end{bmatrix}^T,$$
$$\mathbf{x} = \begin{bmatrix} x[0] & x[1] & \ldots & x[N-1] \end{bmatrix}^T,$$
$$\mathbf{w} = \begin{bmatrix} w[0] & w[1] & \ldots & w[N-1] \end{bmatrix}^T,$$

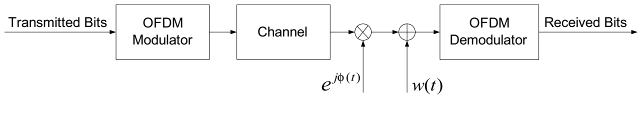


Figure 1: Model of the OFDM system with phase noise.

$$\mathbf{A} = \begin{bmatrix} \alpha[0] & \alpha[N-1] & \ldots & \alpha[1] \\ \alpha[1] & \alpha[0] & \ldots & \alpha[2] \\ \vdots & \vdots & \ddots & \vdots \\ \alpha[N-1] & \alpha[N-2] & \ldots & \alpha[0] \end{bmatrix}, \qquad (4)$$

$$\mathbf{H} = \begin{bmatrix} H[0] & 0 & \ldots & 0 \\ 0 & H[1] & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & H[N-1] \end{bmatrix}.$$

3. MODELING OF PHASE NOISE

There are mainly two types of oscillators used in practice, depending on whether or not they are used in a phase-locked loop (PLL). We briefly describe the phase noise model for each one of them.

3.1 Free-running Oscillator

The frequency deviation $\mu(t)$ of an oscillator is modeled as a zero-mean white Gaussian random process with single-sided power spectral density (PSD) $N_0 = \nu/\pi$, where ν is the oscillator linewidth. The phase noise $\phi_{Free}(t)$ generated by a free-running oscillator is modeled by integrating $\mu(t)$, i.e.,

$$\phi_{Free}(t) = 2\pi \int_0^t \mu(\lambda) d\lambda,$$

which turns out to be a Wiener process. The single-sided PSD of $\phi_{Free}(t)$ is

$$S_{\phi,Free}(f) = \frac{\nu}{\pi f^2}.$$

3.2 Oscillator with PLLs

If the free-running oscillator is used in the 1^{st}-order PLL, whose closed loop transfer function is modeled as

$$H(s) = \frac{\omega_L}{s + \omega_L},$$

then the single-sided PSD of the phase noise $\phi_{PLL}(t)$ is given by

$$S_{\phi,PLL}(f) = \frac{\nu}{\pi\left(f^2 + f_L^2\right)},$$

where $f_L = \omega_L/(2\pi)$ is a measure of loop bandwidth.

4. THE PROPOSED ALGORITHM

If both $\mathbf{A}$ and $\mathbf{H}$ in (3) were known, then the data vector $\mathbf{x}$ could be recovered, e.g., by solving [8]:

$$\widehat{\mathbf{x}} = \arg \min_{\mathbf{x}} \| \mathbf{y} - \mathbf{A}\mathbf{H}\mathbf{x} \|^2 .$$

However, in practice, neither the channel matrix $\mathbf{H}$ nor the phase noise matrix $\mathbf{A}$ are known to the receiver. In this section, we propose a solution to deal with the situation when both $\mathbf{A}$ and $\mathbf{H}$ are unknown at the receiver. The proposed algorithm consists of two stages: one is the channel estimation stage, and the other is the data transmission stage. In the channel estimation stage, we use block-type pilot symbols to jointly estimate $\mathbf{H}$ and $\mathbf{A}$. In the data transmission stage, comb-type symbols are transmitted such that $\mathbf{x}$ can be jointly estimated with $\mathbf{A}$ by using the $\mathbf{H}$ estimated in the channel estimation stage. The motivation for this algorithm is based on the fact that wireless channels are usually slowly time-varying compared to phase noise. Since the phase noise components may change significantly from one OFDM symbol to another, it is harmful to use the previous estimate of phase noise to help detect the data symbols in the subsequently received OFDM symbols. However, we can use the channel estimate for a few subsequent OFDM symbols due to the slowly time-varying nature of wireless channels. This motivates our approach to compensate for phase noise by using the joint channel estimation (with phase noise) first and then followed by the joint data symbol estimation (with phase noise). The algorithm is illustrated by Fig. 2.

4.1 Joint Channel and Phase Noise Estimation

In the block-type pilot symbols, all subcarriers are used to transmit pilot symbols. For convenience of exposition, we assume that each time only one OFDM symbol is used as the block-type pilot symbol for channel estimation. Since there are only N pilot tones in each block-type pilot symbol, it is underdetermined to directly estimate the $2N$ unknowns, i.e., $\alpha[k]$ and $H[k]$, $k = 0,1,\ldots,N-1$. To overcome this difficulty, we can reduce the number of unknowns by properly modeling the channel and the phase noise process with fewer parameters as follows. Since the length L of the discrete-time baseband channel impulse response is usually less than the OFDM symbol size N, we can relate $H[k]$, $k = 0,1,\ldots,N-1$, to $h[n]$, $n = 0,1,\ldots,L-1$, through

$$\mathbf{h} = \mathbf{F}_h \mathbf{h}', \tag{5}$$

where

$$\mathbf{h} = [\; H[0] \quad H[1] \quad \ldots \quad H[N-1] \;]^T,$$
$$\mathbf{h}' = [\; h[0] \quad h[1] \quad \ldots \quad h[L-1] \;]^T,$$
$$\mathbf{F}_h = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & e^{-j\frac{2\pi}{N}} & \ldots & e^{-j\frac{2\pi(L-1)}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\frac{2\pi(N-1)}{N}} & \ldots & e^{-j\frac{2\pi(N-1)(L-1)}{N}} \end{bmatrix}.$$

Instead of estimating $\mathbf{h}$, we can estimate $\mathbf{h}'$. This reduces the number of unknowns from N to L.

For the phase noise, instead of estimating $\alpha[k]$, $k = 0,1,\ldots,N-1$, we can estimate the phase noise components in the time domain, i.e., $e^{j\phi(nT_s)}$, $n = 0,1,\ldots,N-1$. In order to reduce the number of unknowns, we can estimate $e^{j\phi(m(N-1)T_s/(M-1))}$ for $m = 0,1,\ldots,M-1$ ($M < N$), and then obtain the approximation of $e^{j\phi(nT_s)}$, $n = 0,1,\ldots,N-1$, by interpolation. Let

$$\mathbf{c} = [\; e^{j\phi(0)} \quad e^{j\phi(T_s)} \quad \ldots \quad e^{j\phi((N-1)T_s)} \;]^T,$$
$$\mathbf{c}' = \left[\; e^{j\phi(0)} \quad e^{j\phi\left(\frac{(N-1)T_s}{M-1}\right)} \quad \ldots \quad e^{j\phi((N-1)T_s)} \;\right]^T.$$

Then,

$$\mathbf{c} \approx \mathbf{P}\mathbf{c}',$$

where $\mathbf{P}$ is an interpolation matrix to be determined. Using (2), we have

$$\mathbf{a} = \frac{1}{N}\mathbf{F}_a \mathbf{c} \approx \frac{1}{N}\mathbf{F}_a \mathbf{P}\mathbf{c}', \tag{6}$$

where

$$\mathbf{a} = [\; \alpha[0] \quad \alpha[1] \quad \ldots \quad \alpha[N-1] \;]^T,$$
$$\mathbf{F}_a = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & e^{-j\frac{2\pi}{N}} & \ldots & e^{-j\frac{2\pi(N-1)}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\frac{2\pi(N-1)}{N}} & \ldots & e^{-j\frac{2\pi(N-1)^2}{N}} \end{bmatrix}.$$

Consequently, knowing $\mathbf{x}$ during training, we estimate $\mathbf{A}$ and $\mathbf{H}$ by solving

$$\min_{\mathbf{c}',\mathbf{h}'} \; \| \mathbf{y} - \mathbf{A}\mathbf{H}\mathbf{x} \|^2, \tag{7}$$

where $\mathbf{A}$ is related to $\mathbf{c}'$ by (6) and $\mathbf{H}$ is related to $\mathbf{h}'$ by (5). The estimates of $\mathbf{A}$ and $\mathbf{H}$ from (7) will have an ambiguity of a scaling factor, which can be resolved by constraining $\alpha[0]$ to be 1. The problem is overdetermined if $L + M < N$.

The optimization problem given by (7) is nonlinear and nonconvex. Suppose that $\mathbf{A}$ is known. The optimal $\mathbf{h}'$ is then given by

$$\mathbf{h}'_o = \left(\mathbf{F}_h^* \mathbf{X}^* \mathbf{A}^* \mathbf{A}\mathbf{X}\mathbf{F}_h\right)^{-1} \mathbf{F}_h^* \mathbf{X}^* \mathbf{A}^* \mathbf{y},$$

where $\mathbf{X} = \mathrm{diag}\{\mathbf{x}\}$. By substituting $\mathbf{H} = \mathrm{diag}\{\mathbf{F}_h \mathbf{h}'_o\}$ into (7), the optimal $\mathbf{c}'$ is given by

$$\mathbf{c}'_o = \arg\min_{\mathbf{c}'} \|\mathbf{y} - \mathbf{A}\cdot \mathrm{diag}\left\{\mathbf{F}_h \mathbf{h}'_o\right\}\cdot \mathbf{x}\|^2.$$

In implementation, we use the following two-step iterative algorithm to find a sub-optimal solution to (7):

Start with an initial guess $\widehat{\mathbf{c}}'_0$. For example,

$$\widehat{\mathbf{c}}'_0 = [\; 1 \quad 1 \quad \ldots \quad 1 \;]^T.$$

Then, execute Step 1.) and Step 2.) iteratively for $i = 1,2,\ldots$, until there is no significant improvement in the objective function $\|\mathbf{y} - \widehat{\mathbf{A}}_i \widehat{\mathbf{H}}_i \mathbf{x}\|^2$:

Step 1.) Let $\widehat{\mathbf{a}}_{i-1} = \frac{1}{N}\mathbf{F}_a \mathbf{P}\widehat{\mathbf{c}}'_{i-1}$, and find the associated optimal $\widehat{\mathbf{h}}'_{i-1}$ by solving the following least-squares problem:

$$\widehat{\mathbf{h}}'_{i-1} = \arg\min_{\mathbf{h}'} \| \mathbf{y} - \widehat{\mathbf{A}}_{i-1}\mathbf{H}\mathbf{x} \|^2,$$

where $\mathbf{x}$, $\mathbf{y}$ are known and $\widehat{\mathbf{A}}_{i-1}$ is determined by $\widehat{\mathbf{a}}_{i-1}$ through (4).

Step 2.) Let $\widehat{\mathbf{h}}_{i-1} = \mathbf{F}_h \widehat{\mathbf{h}}'_{i-1}$, and find the associated optimal $\widehat{\mathbf{c}}'_i$ by solving the following least-squares problem:

$$\widehat{\mathbf{c}}'_i = \arg\min_{\mathbf{c}'} \| \mathbf{y} - \mathbf{A}\widehat{\mathbf{H}}_{i-1}\mathbf{x} \|^2,$$

where $\widehat{\mathbf{H}}_{i-1} = \mathrm{diag}\left\{\widehat{\mathbf{h}}_{i-1}\right\}$.

The obtained $\widehat{\mathbf{H}}$ will be used in the data transmission stage when comb-type symbols are transmitted. In the next subsection, we assume that $\mathbf{H}$ is known to be $\widehat{\mathbf{H}}$.

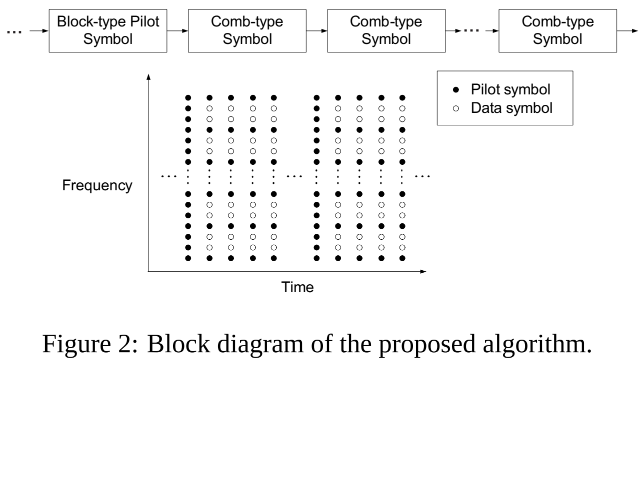


Figure 2: Block diagram of the proposed algorithm.

4.2 Joint Data Symbol and Phase Noise Estimation

In each comb-type OFDM symbol, assume that Q carriers are used for pilot tones and $N-Q$ carriers for data symbols. Similarly, instead of estimating $\alpha[k]$, $k=0,1,\ldots,N-1$, we estimate the phase noise components in the time domain, i.e., $e^{j\phi(m(N-1)T_s/(M-1))}$ for $m=0,1,\ldots,M-1$, and then obtain $e^{j\phi(nT_s)}$, $n=0,1,\ldots,N-1$, by interpolation. The estimation problem can be formulated as

$$\min_{\mathbf{c}',\mathbf{x}_{data}} \|\mathbf{y}-\mathbf{AHx}\|^2,$$

which can be rewritten as

$$\min_{\mathbf{c}',\mathbf{x}_{data}} \left\|\mathbf{y}-\mathbf{A}_{pilot}\mathbf{H}_{pilot}\mathbf{x}_{pilot}-\mathbf{A}_{data}\mathbf{H}_{data}\mathbf{x}_{data}\right\|^2, \quad (8)$$

where $\mathbf{x}_{pilot}$ is the sub-vector of $\mathbf{x}$ that consists of all the pilot symbols and $\mathbf{A}_{pilot}$, $\mathbf{H}_{pilot}$ are its associated sub-matrices from $\mathbf{A}$ and $\mathbf{H}$, and $\mathbf{x}_{data}$ is the sub-vector of $\mathbf{x}$ that consists of all the data symbols and $\mathbf{A}_{data}$, $\mathbf{H}_{data}$ are its associated sub-matrices from $\mathbf{A}$ and $\mathbf{H}$. Since there are $N-Q$ unknown data symbols in $\mathbf{x}_{data}$ and M unknowns in $\mathbf{c}'$, then the problem is overdetermined if $Q>M$.

Since the optimization problem given by (8) is similar to the joint channel estimation problem we just solved, we can follow the same procedure to solve it. If $\mathbf{A}$ is known, the optimal $\mathbf{x}_{data}$ is given by

$$\mathbf{x}_{data,o}=\mathbf{H}_{data}^{-1}\left(\mathbf{A}_{data}^{*}\mathbf{A}_{data}\right)^{-1}\mathbf{A}_{data}^{*}\left(\mathbf{y}-\mathbf{A}_{pilot}\mathbf{H}_{pilot}\mathbf{x}_{pilot}\right).$$

Then the optimal $\mathbf{c}'$ is given by

$$\mathbf{c}'_o=\arg\min_{\mathbf{c}'}\left\|\mathbf{y}-\mathbf{A}_{pilot}\mathbf{H}_{pilot}\mathbf{x}_{pilot}-\mathbf{A}_{data}\mathbf{H}_{data}\mathbf{x}_{data,o}\right\|^2.$$

A sub-optimal solution can be found by the following iterative procedure:

Use the method given in [4] to estimate the CPE coefficient $\alpha[0]$, and let

$$\widehat{\mathbf{c}}'_0=[\ \widehat{\alpha}[0]\quad \widehat{\alpha}[0]\quad \ldots\quad \widehat{\alpha}[0]\ ]^T.$$

Then, recursively execute Step 1.) and Step 2.) for $i=1,2,\ldots$, until there is no significant improvement in the objective function:

Step 1.) Let $\widehat{\mathbf{a}}_{i-1}=\frac{1}{N}\mathbf{F}_a\mathbf{P}\widehat{\mathbf{c}}'_{i-1}$, and find the associated optimal $\widehat{\mathbf{x}}_{data,i-1}$ by solving the following least-squares problem:

$$\widehat{\mathbf{x}}_{data,i-1}=\arg\min_{\mathbf{x}_{data}}\ \|\ \mathbf{y}-\widehat{\mathbf{A}}_{pilot,i-1}\mathbf{H}_{pilot}\mathbf{x}_{pilot}-\widehat{\mathbf{A}}_{data,i-1}\mathbf{H}_{data}\mathbf{x}_{data}\ \|^2,$$

where $\widehat{\mathbf{A}}_{pilot,i-1}$ and $\widehat{\mathbf{A}}_{data,i-1}$ are determined by $\widehat{\mathbf{a}}_{i-1}$ according to (4).

Step 2.) Find the optimal $\widehat{\mathbf{c}}'_i$ by solving the following least-squares problem:

$$\widehat{\mathbf{c}}'_i=\arg\min_{\mathbf{c}'}\left\|\mathbf{y}-\mathbf{A}_{pilot}\mathbf{H}_{pilot}\mathbf{x}_{pilot}-\mathbf{A}_{data}\mathbf{H}_{data}\widehat{\mathbf{x}}_{data,i-1}\right\|^2.$$

We first estimate the CPE coefficient $\alpha[0]$ in order to correct the scaling ambiguity in $\widehat{\mathbf{H}}$. Finally, the obtained $\widehat{\mathbf{x}}_{data}$ is mapped into bits.

4.3 Selection of the Interpolation Matrix P

4.3.1 *PSD of the phase noise is known*

If the PSD of the phase noise is known, the optimal interpolation matrix $\mathbf{P}_o$ can be obtained by minimizing the mean square error of interpolating $\mathbf{c}$ from $\mathbf{c}'$, i.e.,

$$\mathbf{P}_o=\arg\min_{\mathbf{P}}\mathbf{E}\left\|\ \mathbf{c}-\mathbf{P}\ \mathbf{c}'\right\|^2,$$

from which the optimal $\mathbf{P}_o$ is given by $\mathbf{P}_o=\mathbf{R}_{\mathbf{cc}'}\mathbf{R}_{\mathbf{c}'}^{-1}$, where $\mathbf{R}_{\mathbf{cc}'}=\mathbf{E}\{\mathbf{cc}'^*\}$ and $\mathbf{R}_{\mathbf{c}'}=\mathbf{E}\{\mathbf{c}'\mathbf{c}'^*\}$.

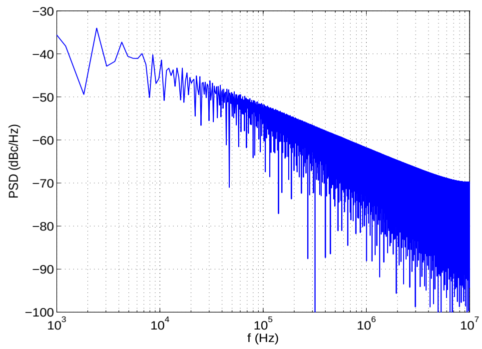
4.3.2 *PSD of the phase noise is unknown*

In this case, an interpolation matrix $\mathbf{P}_L\in\mathbb{R}^{N\times M}$ is constructed from linear interpolation, and its element at the n^{th} row and m^{th} column is given by (9) (see the equation at the top of the next page) for $n=1,2,\ldots,N$ and $m=1,2,\ldots,M$. It can be shown that for free-running oscillators, the optimal interpolator $\mathbf{P}_o$ is approximately equal to the linear interpolator $\mathbf{P}_L$ if $\nu T_s \ll 1$.

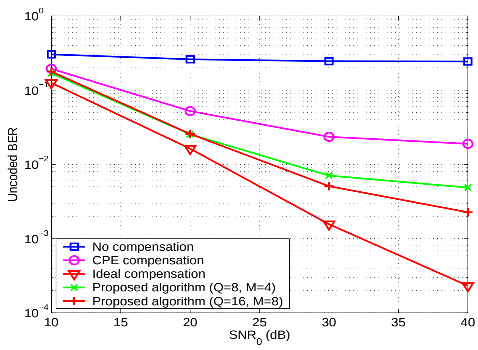
5. COMPUTER SIMULATIONS

The proposed scheme is simulated in comparison with the ideal OFDM receiver with perfect phase noise compensation, the CPE correction scheme [4], and the ICI compensation schemes proposed in [5–7]. The system bandwidth is 20 MHz, i.e., $T_s=0.05\ \mu s$, and the constellation used for symbol mapping is 16-QAM. The OFDM symbol size is $N=64$ and the prefix length is $P=20$. The channel length is 6, and each tap is independently Rayleigh distributed with the power profile specified by 3 dB decay per tap. The assumed channel length in the time domain for channel estimation is $L=12$, the length of the phase noise vector to be estimated is $M=4$ or 8, and the number of pilot tones in the comb-type symbols is $Q=8$ or 16.

Fig. 3 shows the simulated system performance when the phase noise is generated by a free-running oscillator. The phase noise spectrum for $\nu=5$ kHz is plotted in Fig. 3(a). Fig. 3(b) compares different schemes in terms of the uncoded bit error rate (BER) for the phase noise spectrum with $\nu=5$ kHz. It is also demonstrated in Fig. 4 that the proposed scheme can significantly improve the system performance for the phase-loop locked oscillators.



(a) PSD of phase noise with $\nu=5$ kHz.

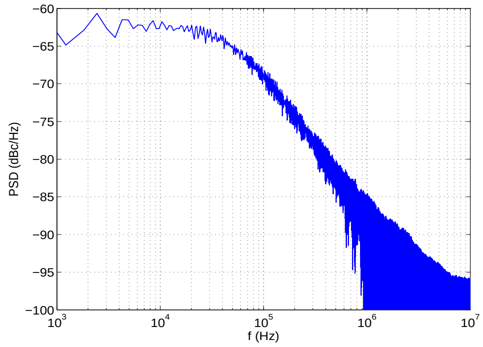


(b) Uncoded BER with estimated channel information.

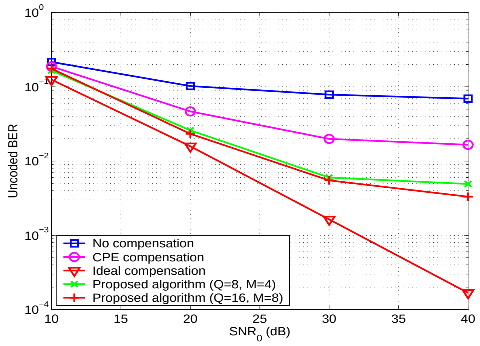
Figure 3: Simulation results when the phase noise is generated by a free-running oscillator.

In Fig. 5, the proposed phase noise compensation scheme

$$\mathbf{P}_L(n,m) = \begin{cases} m - \frac{(n-1)(M-1)}{N-1}, & \text{if } \frac{(m-1)(N-1)}{M-1} \leq n-1 < \frac{m(N-1)}{M-1}, \\ \frac{(n-1)(M-1)}{N-1} - (m-2), & \text{if } \frac{(m-2)(N-1)}{M-1} \leq n-1 < \frac{(m-1)(N-1)}{M-1}, \\ 0, & \text{otherwise}, \end{cases} \quad (9)$$



(a) PSD of phase noise with $\nu = 5$ kHz and $f_L = 50$ kHz.



(b) Uncoded BER with estimated channel information.

Figure 4: Simulation results when the phase noise is generated by a 1^{st}-order PLL oscillator.

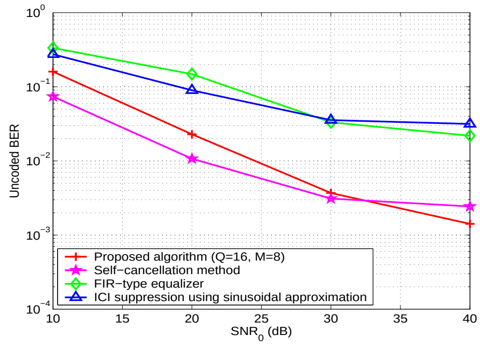


Figure 5: Comparison of different ICI compensation methods. The phase noise is generated by a 1^{st}-order PLL oscillator with $\nu = 5$ kHz and $f_L = 50$ kHz.

is compared with the self-cancellation scheme proposed in [5], the frequency-domain FIR-type equalizer proposed in [6], and the ICI suppression method using sinusoidal approximation proposed in [7]. In this simulation, we assume that the receivers have perfect channel information and the phase noise is generated by a 1^{st}-order phase-loop locked oscillator with $\nu = 5$ kHz and $f_L = 50$ kHz. It is shown in the figure that the FIR-type equalizer and the ICI suppression method using sinusoidal approximation do not perform well for this type of fast-changing phase noise because they can only compensate for the ICI from adjacent subcarriers. The self-cancellation scheme works as well as the proposed method since it uses two subcarriers to transmit one data symbol, which brings the benefit of diversity gain as in MIMO communications. However, the self-cancellation method reduces the spectral efficiency by one half. In the simulation, each OFDM symbol can transmit $N - Q = 48$ data symbols in the proposed scheme but only $N/2 = 32$ symbols in the self-cancellation scheme, which demonstrates that the proposed method can achieve 50% more spectral efficiency than the self-cancellation method.

6. CONCLUSIONS

In this paper, a phase noise compensation scheme is proposed for OFDM-based wireless communications. The proposed scheme consists of two phases. One phase is the joint channel and phase noise estimation, and the other phase is the joint data symbol and phase noise estimation. The simulations show that the proposed scheme can effectively improve the system performance in terms of the bit error rate. Since oscillators with ultra-low phase noise usually have the disadvantage of high implementation cost and high power consumption, the improvement will significantly reduce the cost and power consumption from the perspective of hardware designers.

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