MULTI-RELAY STRATEGY FOR IMPERFECT CHANNEL INFORMATION IN SENSOR NETWORKS

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ABSTRACT

This paper describes a multi-relay strategy for wireless networks and examines the influence of imperfect channel information on system performance. A modified relay scheme is proposed to compensate for such imperfections.

1. INTRODUCTION

A fundamental task in a wireless sensor network is to broadcast some measured data from an origin sensor to a destination sensor. Since the sensors are typically small, power limited and low cost, they are only able to broadcast low-power signals. This means that the propagation loss from the origin to the destination sensor can attenuate the signals beyond detection. One way to deal with this problem is to pass the transmitted signal through one or more relay sensors [1].

We may categorize relay schemes into three general groups: amplify-forward, compress-forward and decode-forward. In the amplify-forward scheme, the relay nodes amplify the received signal and rebroadcast the amplified signals toward the destination node [2],[3],[4]. In the compress-forward method, the relay nodes compress the received signals by exploiting the statistical dependencies between the signals at the nodes [5],[6],[7]. In the decode-forward scheme, the relay nodes first decode the received signals and then forward the decoded signals toward the destination node [8],[9],[10]. In this paper we propose an amplify-forward scheme.

In traditional relay schemes, the relay nodes compensate for the phase of the incoming signal in order to result in coherent signal combination at the receiver. In such schemes, each node utilizes its maximum available power. In contrast, the scheme proposed in [11] and reviewed further ahead allows the relay nodes to adjust their power. Specifically, the scheme of [11] is based on a two-hop multi-sensor relay strategy that achieves path-loss saving, diversity gain and power efficiency. In the proposed scheme, the relay sensors do not need to share information about the received signal. However, as in conventional amplify-forward schemes, each relay node needs to know its local channels to the source and destination sensors. Due to channel estimation errors, the performance of the relay strategy may degrade. In this article, we examine the effect of channel uncertainties on system performance and propose a modified relay scheme to compensate for imperfect channel information. The article also examines the capacity and the power efficiency of the relay strategy.

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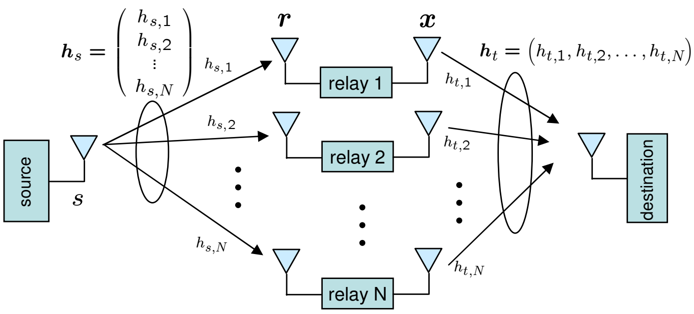

Figure 1: The relay network scheme.

2. RELAY NETWORK DESIGN

Consider a sensor network with N relay sensor nodes between a source sensor and a destination sensor. The relay nodes are labelled 1 through N – see Fig. 1. Let $\mathbf{h}_s$ denote the $N \times 1$ channel vector between the source sensor and the relay nodes, and let $\mathbf{h}_t$ denote the $1 \times N$ channel vector between the relay sensors and the destination sensor. A quasi-static fading condition is assumed for each channel gain so that the channel realizations stay fixed for the duration of a single frame. Let $h_{s,i}$ denote the ith element of $\mathbf{h}_s$, which stands for the channel coefficient from the source sensor to the ith relay node. Likewise, let $h_{t,i}$ denote the ith element of $\mathbf{h}_t$, which stands for the channel coefficient from the ith relay node to the destination sensor. We assume synchronous transmission and reception at relays nodes, so that the relay nodes relay their data at the same time instant.

Using the above formulation, the received vector at the relay sensors is given by

$$\boxed{\mathbf{r} = \mathbf{h}_s s + \mathbf{v}_s} \tag{1}$$

where

$$\mathbf{h}_s = [h_{s,1}, h_{s,2}, ..., h_{s,N}]^T$$

is a column vector and $\mathbf{v}_s$ is $N \times 1$ zero-mean complex noise with covariance matrix $\sigma_{v_s}^2 \mathbf{I}$. At the second phase of the relaying protocol, the relay sensors rebroadcast a transformed signal vector that is given by

$$\boxed{\mathbf{x} = \mathbf{F}\mathbf{r}} \tag{2}$$

where $\mathbf{F}$ is an $N \times N$ linear transformation matrix to be designed. The signal received at the destination sensor is

$$t = \mathbf{h}_t \mathbf{x} + v_t \tag{3}$$

where v_t is zero mean noise with variance $\sigma_{v_t}^2$. The uncorrupted received signal is $\mathbf{h}_t\mathbf{x}$, where

$$\mathbf{h}_t = [h_{t,1}, h_{t,2}, ..., h_{t,N}]$$

is a row vector.

3. MMSE RELAY STRATEGY

In [11] we selected $\mathbf{F}$ by solving

$$\hat{\mathbf{F}} = \arg\min_{\mathbf{F}} J(\mathbf{F}) \tag{4}$$

where

$$J(\mathbf{F}) \quad = \quad E|\eta s - \mathbf{h}_t\mathbf{x}|^2$$

and

$$\eta^2 = \text{SNR}_t \frac{\sigma_{v_t}^2}{\sigma_s^2}$$

where $E|s|^2 = \sigma_s^2$. This choice of η allows us to achieve a target signal-to-noise-ratio, SNR_t, at the destination node. Introducing $\mathbf{z} = \mathbf{h}_t\mathbf{F}$, and expanding (5) we get

$$J \quad = \quad \sigma_s^2\mathbf{z}\mathbf{h}_s\mathbf{h}_s^*\mathbf{z}^* + \sigma_{v_s}^2\mathbf{z}\mathbf{z}^* - \eta\sigma_s^2\mathbf{z}\mathbf{h}_s - \eta\sigma_s^2\mathbf{h}_s^*\mathbf{z}^* + \eta^2\sigma_s^2$$

Minimizing J over $\mathbf{z}$ gives

$$\hat{\mathbf{z}}^* = \eta\left(\frac{\sigma_s^2}{\sigma_{v_s}^2 + \sigma_s^2\|\mathbf{h}_s\|^2}\right)\mathbf{h}_s \tag{5}$$

and we are reduced to choosing a relay matrix $\mathbf{F}$ such that

$$\boxed{\hat{\mathbf{F}}^*\mathbf{h}_t^* = \eta\left(\frac{\sigma_s^2}{\sigma_{v_s}^2 + \sigma_s^2\|\mathbf{h}_s\|^2}\right)\mathbf{h}_s} \tag{6}$$

Expression (6) provides N independent equalities for N^2 unknown elements in $\hat{\mathbf{F}}$. The additional degrees of freedom can be exploited advantageously as follows. Since a wireless sensor network is fundamentally a distributed communications network, we assume that each node only has access to local channel information. Specifically, every node i will only have access to the channel gains $h_{s,i}$ and $h_{t,i}$ that connect it to the source and the destination. This structure motivates us to seek a diagonal $\hat{\mathbf{F}}$ that satisfies (6). Thus we may select diagonal entries $\{\hat{f}_i\}$ as [11]:

$$\boxed{\hat{f}_i = \eta\left(\frac{\sigma_s^2}{\sigma_s^2\|\mathbf{h}_s\|^2 + \sigma_{v_s}^2}\right)\cdot\frac{h_{s,i}^*h_{t,i}^*}{|h_{t,i}|^2}} \tag{7}$$

Now, in view of (3)-(5), we end up with

$$t \approx \eta s + v_t$$

and we can proceed to equalize t at the receiver in order to remove the effect of v_t and recover s. To do so, we choose a scalar α so as to minimize [11]:

$$\alpha = \arg\min_{\alpha} J(\alpha) \tag{8}$$

where now

$$J(\alpha) = E|s - \alpha t|^2 = E|s - \alpha\mathbf{h}_t\hat{\mathbf{F}}\mathbf{h}_s s - \alpha\mathbf{h}_t\hat{\mathbf{F}}\mathbf{v}_s - \alpha v_t|^2 \tag{9}$$

The optimal α is given by

$$\hat{\alpha} = \frac{\sigma_s^2\mathbf{h}_s^*\hat{\mathbf{F}}^*\mathbf{h}_t^*}{\sigma_s^2|\mathbf{h}_t\hat{\mathbf{F}}\mathbf{h}_s|^2 + \sigma_{v_s}^2\|\mathbf{h}_t\hat{\mathbf{F}}\|^2 + \sigma_{v_t}^2} \tag{10}$$

Let $\text{SNR} = \sigma_s^2/\sigma_{v_s}^2$. Assuming $\text{SNR}.\|\mathbf{h}_s\|^2 \gg 1$ which is a reasonable assumption for N large, gives

$$\boxed{\hat{\alpha} \approx \frac{\sigma_s^2}{\eta^2\sigma_s^2 + \sigma_{v_t}^2}} \tag{11}$$

This expression indicates that when the number of relay sensors is large, the destination node does not need the power of the broadcast channel, $\|\mathbf{h}_s\|^2$, in order to perform equalization. For further details in this relay strategy, and on variations with power constrains, the reader may refer to [11].

4. CAPACITY OF THE RELAY NETWORK

Due to the two-phase protocol scheme, the relay sensors are busy with receiving data during the first phase and with relaying data during the second phase. Thus the source sensor is able to transmit only at half of the time. As a result we shall scale the capacity of the AWGN channel by two. The received signal at the destination node is given by

$$t \quad = \quad \mathbf{h}_t\hat{\mathbf{F}}\mathbf{r} + v_t = \underbrace{\mathbf{h}_t\hat{\mathbf{F}}\mathbf{h}_s}_{h}s + \underbrace{\mathbf{h}_t\hat{\mathbf{F}}\mathbf{v}_s + v_t}_{v} \tag{12}$$

and the capacity of the resulting channel (assuming $\mathbf{v}$ is Gaussian) is [4, 14]:

$$C = \frac{1}{2}E\left[\log_2\left(1 + \frac{\sigma_s^2|h|^2}{\sigma_v^2}\right)\right] \text{ (bits/Hz/sec)} \tag{13}$$

where $\frac{1}{2}$ is due to transmitting only at half of the times. Moreover,

$$\begin{aligned} \sigma_v^2 \quad &= \quad E|\mathbf{h}_t\hat{\mathbf{F}}\mathbf{v}_s + v_t|^2 \qquad (14) \\ &= \quad \sigma_{v_t}^2 + \frac{\eta^2\sigma_s^4\|\mathbf{h}_s\|^2}{(\sigma_{v_s}^2 + \sigma_s^2\|\mathbf{h}_s\|^2)^2}\sigma_{v_s}^2 \end{aligned}$$

and

$$|h|^2 \quad = \quad \frac{\eta^2\sigma_s^4\|\mathbf{h}_s\|^4}{(\sigma_{v_s}^2 + \sigma_s^2\|\mathbf{h}_s\|^2)^2} \tag{15}$$

Substituting into (13) gives

$$C = \tag{16}$$

$$\frac{1}{2}E\left[\log_2\left(1 + \frac{\eta^2\sigma_s^6\|\mathbf{h}_s\|^4}{\sigma_{v_t}^2(\sigma_{v_s}^2 + \sigma_s^2\|\mathbf{h}_s\|^2)^2 + \eta^2\sigma_s^4\sigma_{v_s}^2\|\mathbf{h}_s\|^2}\right)\right]$$

Assuming $\sigma_{v_s}^2 + \sigma_s^2\|\mathbf{h}_s\|^2 \approx \sigma_s^2\|\mathbf{h}_s\|^2$, the above expression is approximated by

$$C \quad \approx \quad \frac{1}{2}E\left[\log_2\left(1 + \frac{\eta^2\sigma_s^2}{\sigma_{v_t}^2 + \frac{\eta^2\sigma_{v_s}^2}{\|\mathbf{h}_s\|^2}}\right)\right] \tag{17}$$

This result shows that as the number of relay sensors grows, the capacity of the relay network converges to the capacity of a SISO channel between the source sensor and the destination sensor with the channel link achieving the target SNR, i.e.,

$$\lim_{N\to\infty} C \approx \frac{1}{2}\log_2\left(1+\frac{\eta^2\sigma_s^2}{\sigma_{v_t}^2}\right) = \frac{1}{2}\log_2\left(1+\text{SNR}_t\right) \quad (18)$$

On the other hand, when the target SNR is large, the dominant noise term will be the relay noise, $\mathbf{v}_s$, and the asymptotic capacity will be

$$\lim_{\eta\to\infty} C \approx \frac{1}{2}\log_2\left(1+\frac{\sigma_s^2\|\mathbf{h}_s\|^2}{\sigma_{v_s}^2}\right) = O(\log(N)) \quad (19)$$

which is similar to the capacity scaling law for amplify-forward relay scheme of [3] and [4]. In order to compare the capacity results for the case of small target SNR with the capacity that would result from the conventional scheme, let us define the power efficiency of a relay network as the ratio between the capacity and the average power spent at the relay nodes from [11]. Using (18) the power efficiency for large number of relay nodes can be written as

$$P_{\text{eff}} = \frac{C}{P_{\text{Total}}} \approx \frac{\frac{1}{2}\log_2\left(1+\frac{\eta^2\sigma_s^2}{\sigma_{v_t}^2}\right)}{\frac{\gamma\eta^2}{N\sigma_{h_s}^4}\left(\sigma_s^2\sigma_{h_s}^2+\sigma_{v_s}^2\right)} \approx \frac{O(1)}{O\left(\frac{1}{N}\right)} = O(N)$$

where the approximation for P_{Total} is from [11]. In comparison, for the conventional relay scheme, the capacity scales by $O(\log(N))$ [4, 3] and the power scales as $O(N)$, so that the power efficiency scales as $O(\log(N)/N)$, which decreases as N increases.

5. CHANNEL UNCERTAINTIES

Each relay sensor needs to know its local channels to the source and destination sensor in order to form the relay factor $\hat{f}_i$ given by (7). Due to channel estimation errors, the estimated channels at the relay nodes will not be accurate. In order to compensate for the expected degradation in performance, we modify the design of the relay matrix $\hat{\mathbf{F}}$. Let $\hat{\mathbf{h}}_s$ and $\hat{\mathbf{h}}_t$ denote the available estimates of $\mathbf{h}_s$ and $\mathbf{h}_t$, respectively, at the relay sensor nodes, i.e.,

$$\hat{\mathbf{h}}_s = \mathbf{h}_s - \Delta_s, \quad \hat{\mathbf{h}}_t = \mathbf{h}_t - \Delta_t$$

where the elements of the disturbances Δ_s and Δ_t will be assumed to be complex i.i.d. zero-mean random variables with variances $\sigma_{\Delta_s}^2$ and $\sigma_{\Delta_t}^2$, respectively. The received signal at the destination sensor will now be given by

$$\begin{aligned} t &= \mathbf{h}_t\mathbf{F}\mathbf{r}+v_t \quad (20)\\ &= (\hat{\mathbf{h}}_t+\Delta_t)\mathbf{F}(\hat{\mathbf{h}}_s+\Delta_s)s+(\hat{\mathbf{h}}_t+\Delta_t)\mathbf{F}\mathbf{v}_s+v_t \end{aligned}$$

Using the same approach we used for the case of perfect channel information, we again seek $\mathbf{F}$ in order to minimize

$$\begin{aligned} J &= E|\eta s-\mathbf{h}_t\mathbf{x}|^2 \quad (21)\\ &= E|\hat{\mathbf{h}}_t\mathbf{F}\hat{\mathbf{h}}_s s+\hat{\mathbf{h}}_t\mathbf{F}\Delta_s s+\Delta_t\mathbf{F}\hat{\mathbf{h}}_s s\\ &\quad +\Delta_t\mathbf{F}\Delta_s s+\hat{\mathbf{h}}_t\mathbf{F}\mathbf{v}_s+\Delta_t\mathbf{F}\mathbf{v}_s-\eta s|^2 \end{aligned}$$

Since each relay sensor only has access to its received signal, we again limit $\mathbf{F}$ to a diagonal matrix. Let the $1\times N$ vector $\mathbf{f}=\text{diag}(\mathbf{F})$ denote the diagonal elements of $\mathbf{F}$. Then ignoring terms $\Delta_t\mathbf{F}\Delta_s s$ and $\Delta_t\mathbf{F}\mathbf{v}_s$ with second order disturbance/noise factors, we write

$$\begin{aligned} J &\approx E|\mathbf{f}\text{diag}(\hat{\mathbf{h}}_t)\hat{\mathbf{h}}_s s+\mathbf{f}\text{diag}(\hat{\mathbf{h}}_t)\Delta_s s\\ &\quad +\mathbf{f}\text{diag}(\Delta_t)\hat{\mathbf{h}}_s s+\mathbf{f}\text{diag}(\hat{\mathbf{h}}_t)\mathbf{v}_s-\eta s|^2 \end{aligned}$$

and the optimum $\mathbf{f}$ that minimizes the right-hand side expression is given by (22). It can be verified that (22) will collapse to (7) in the case of no channel uncertainty by replacing the Δ_s and Δ_t by zero.

Moreover, in order to remove the effect of the receiving noise v_t, we use the same approach we used before to equalize t. Namely, we choose a scalar α so as to minimize

$$\alpha = \arg\min_{\alpha} J(\alpha)$$

where

$$\begin{aligned} J(\alpha) &= E|s-\alpha t|^2 \quad (23)\\ &= E|s-\alpha(\hat{\mathbf{h}}_t+\Delta_t)\hat{\mathbf{F}}(\hat{\mathbf{h}}_s+\Delta_s)s-\alpha(\hat{\mathbf{h}}_t+\Delta_t)\hat{\mathbf{F}}\mathbf{v}_s-\alpha v_t|^2 \end{aligned}$$

The optimal α is given by

$$\hat{\alpha} = \frac{\sigma_s^2\hat{\mathbf{h}}_s^*\hat{\mathbf{F}}^*\hat{\mathbf{h}}_t^*}{\sigma_s^2|\hat{\mathbf{h}}_t\hat{\mathbf{F}}\hat{\mathbf{h}}_s|^2+(\sigma_s^2\sigma_{\Delta_s}^2+\sigma_{v_s}^2)\|\hat{\mathbf{h}}_t\hat{\mathbf{F}}\|^2+\sigma_s^2\sigma_{\Delta_t}^2\|\hat{\mathbf{F}}\hat{\mathbf{h}}_s\|^2+\sigma_{v_t}^2} \quad (24)$$

It can again be verified that (24) will collapse to (10) in the case of no channel uncertainty.

6. SIMULATION RESULTS

The performance of the proposed scheme is investigated for a relay network with one source and one destination. We assume that all relay sensors are essentially at the same distance from the source and destination sensors. Using this assumption, the channels from the source sensor to the relay sensors have the same second moment statistics as the channels from the relay sensors to the destination sensor, i.e., $E(\mathbf{h}_s\mathbf{h}_s^*) = E(\mathbf{h}_t^*\mathbf{h}_t)$. Moreover, we use zero-mean unit variance complex Gaussian channel models for $\mathbf{h}_s$ and $\mathbf{h}_t$, and the transmitted signal from the source sensor is assumed to be QPSK with unit power. Fig. 2 shows the BER performance of the scheme (7) when the destination sensor has less noise variance than the relay sensors, i.e., $10\log\left(\sigma_{v_t}^2/\sigma_{v_s}^2\right) = -8$dB. Fig. 3 illustrates the performance when there is uncertainty in the channel estimation of the relay sensor nodes.

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$$\hat{\mathbf{f}} = \eta \hat{\mathbf{h}}_s^* \mathrm{diag}(\hat{\mathbf{h}}_t^*)^{-1} \left[\sigma_{\Delta_t}^2 \mathrm{diag}(\hat{\mathbf{h}}_t)^{-1} \hat{\mathbf{h}}_s \hat{\mathbf{h}}_s^* \mathrm{diag}(\hat{\mathbf{h}}_t)^{-*} + \hat{\mathbf{h}}_s \hat{\mathbf{h}}_s^* + \left(\sigma_{\Delta_s}^2 + \frac{1}{\mathrm{SNR}} \right) \mathbf{I} \right]^{-1} \tag{22}$$

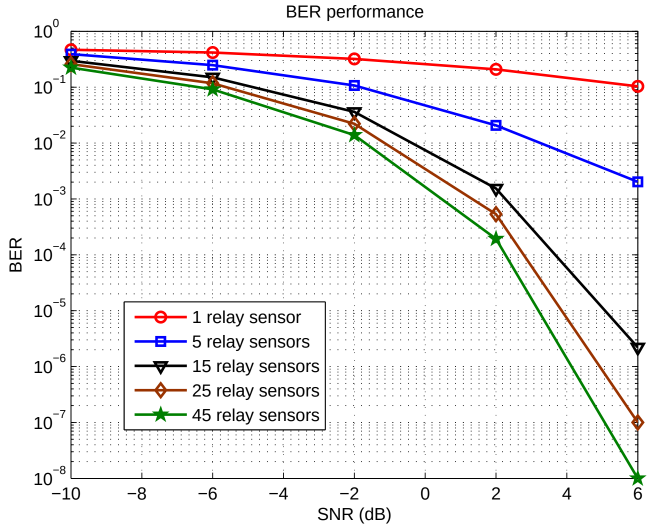

Figure 2: The BER performance of the proposed scheme(7) when the relay sensors are assumed to have more noise power than the destination sensor. The relay sensors are placed such that they have the same distance from the source and destination sensors.

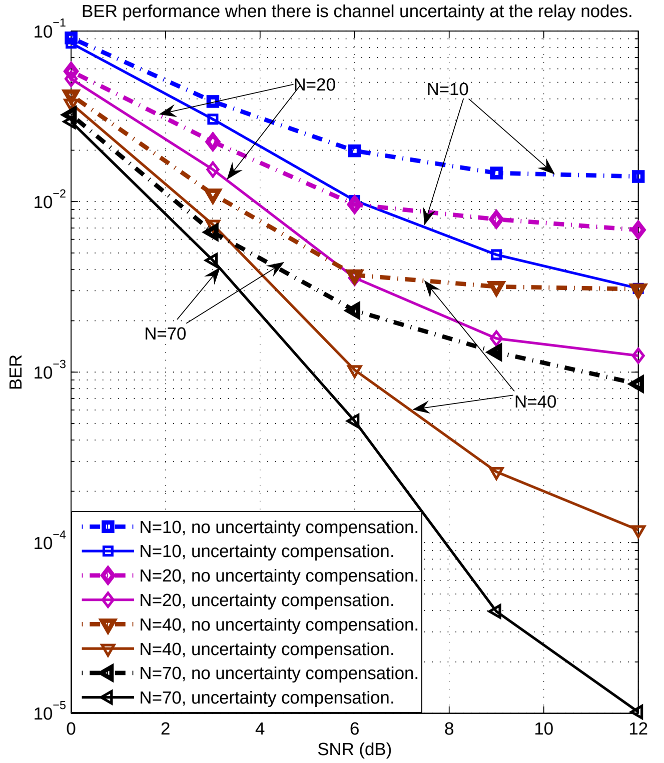

Figure 3: The BER comparison when the channel uncertainty compensation scheme in (22) is used vs. the relay scheme in (7) assuming $10\log\sigma_{\Delta_s}^2/\sigma_s^2 = 10\log\sigma_{\Delta_t}^2/\sigma_s^2 = -10$ dB uncertainty at the relay sensor nodes.

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