A DIVIDE–AND–CONQUER ALGORITHM FOR CHANNEL ESTIMATION IN MULTI-USER SPACE-TIME CODED TRANSMISSIONS

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ABSTRACT

We develop channel estimation techniques for multiple–input multiple–output (MIMO) channels that employ space–time block–coded (STBC) transmissions. We derive the MMSE channel estimator for multi–user Alamouti STBC transmissions over both flat fading and frequency selective fading channels. The derivation exploits the structure of the space–time code to reduce the complexity of the algorithm.

1. INTRODUCTION

Wireless communications systems with multiple transmit and receive antennas provide large capacity gains, especially in rich scattering environment [1]. In order to achieve such high transmission rates with high performance and reliability, the receiver needs to have accurate information about the channels between the transmitter and the receiver. In multi–user multi–antenna scenarios, the channel estimation problem is more challenging due the high number of sub-channels.

Various space-time block codes (STBC) have been developed to provide high performance over both flat fading and frequency–selective fading channels (e.g., Alamouti–STBC [2], OFDM–STBC [3], and single–carrier frequency domain equalization STBC (SC–FDE STBC) [4]). They were shown to be useful for MIMO wireless systems with co–channel users since their structures could be exploited to suppress interference and simplify the receiver complexity [5]. Most of these receivers require explicit knowledge of the impulse response of all sub-channels at the receiver.

In this paper, we develop efficient channel estimation techniques for three multi–user STBC transmission schemes over both flat and frequency selective fading channels. We show how to exploit the STBC structure to reduce the computational complexity of the channel estimators. We also provide a unified approach to channel estimation for Alamouti-STBC, SC-FDE STBC, and OFDM-STBC transmissions.

2. PROBLEM FORMULATION

Consider a system consisting of M users, each equipped with two antennas. Each user transmits STBC data from its two antennas. The receiver is equipped with M receive antennas. The block diagram of the system is shown in Figure 1.

2.1 Flat Fading Channels

For each user, data are transmitted from its two antennas according to the Alamouti space–time block coding scheme of [2]. At times $k = 0, 2, 4, \cdots$, the data symbols of the i–th user, $x_{k,1}^{(i)}$ and $x_{k,2}^{(i)}$, are generated by an information source

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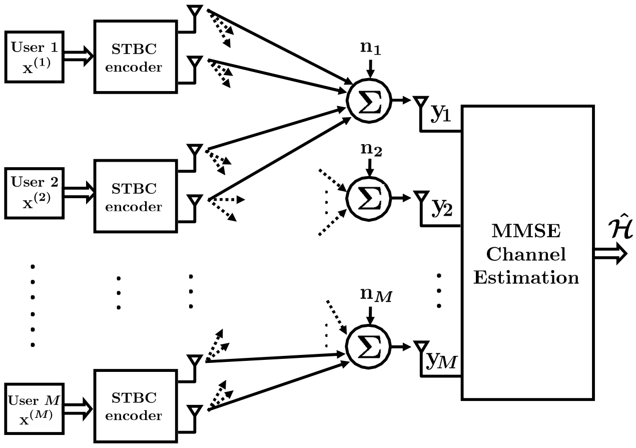

Figure 1: Block diagram of an M-user system.

according to the rule:

		Time	
		k	$k+1$
Antenna	1	$x_{k,1}^{(i)}$	$-x_{k,2}^{*(i)}$
	2	$x_{k,2}^{(i)}$	$x_{k,1}^{*(i)}$

(1)

The received signal at the l–th receive antenna at times k and $k+1$, denoted by $\mathbf{y}_{k,l}$, is given by

$$\mathbf{y}_{k,l}=\begin{pmatrix} y_{k,l} \\ y_{k+1,l}\end{pmatrix} = \sum_{i=1}^{M} \begin{pmatrix} x_{k,1}^{(i)} & x_{k,2}^{(i)} \\ -x_{k,2}^{*(i)} & x_{k,1}^{*(i)} \end{pmatrix} \begin{pmatrix} h_{1,l}^{(i)} \\ h_{2,l}^{(i)} \end{pmatrix} + \begin{pmatrix} n_{k,l} \\ n_{k+1,l} \end{pmatrix}$$

$$\triangleq \sum_{i=1}^{M} \mathbf{X}_k^{(i)} \mathbf{h}_l^{(i)} + \mathbf{n}_l \qquad (2)$$

where $\mathbf{y}_{k,l}$ is the 2×1 received k-th block at the l-th receive antenna, $\mathbf{n}_{k,l}$ is the 2×1 noise vector at the l-th receive antenna from the k-th block, and $\mathbf{X}_k^{(i)}$ is the 2×2 Alamouti matrix of the i-th user and the k-th block. The structure of $\mathbf{X}_k^{(i)}$ will appear frequently throughout the paper, and we shall use the terminology *Alamouti–like matrix* to refer to it. The coefficients of the flat fading channels, $h_{1,l}^{(i)}$ and $h_{2,l}^{(i)}$, are modelled as iid complex Gaussian random variables with variance equal to 0.5 per dimension, i.e., the covariance matrix is $\mathbf{R}_h = \mathbf{I}$. The noise is modelled as AWGN with zero mean and covariance matrix $\mathbf{R}_n = \sigma_n^2 \mathbf{I}$, and the transmitted symbols have variance σ_x^2. Assuming the channel to be constant for $2M$ symbols, we collect M blocks of data at the receiver. The received signal from all antennas is written in matrix form as

$$\mathcal{Y} = \mathcal{X}\mathcal{H} + \mathcal{N} \qquad (3)$$

where

$$\mathcal{Y} = \underbrace{\begin{pmatrix} \mathbf{y}_{1,1} & \dots & \mathbf{y}_{1,M} \\ \vdots & \ddots & \vdots \\ \mathbf{y}_{M,1} & \dots & \mathbf{y}_{M,M} \end{pmatrix}}_{2M\times M}, \quad \mathcal{X} = \underbrace{\begin{pmatrix} \mathbf{X}_1^{(1)} & \dots & \mathbf{X}_1^{(M)} \\ \vdots & \ddots & \vdots \\ \mathbf{X}_M^{(1)} & \dots & \mathbf{X}_M^{(M)} \end{pmatrix}}_{2M\times 2M}$$

$$\mathcal{H} = \underbrace{\begin{pmatrix} \mathbf{h}_1^{(1)} & \dots & \mathbf{h}_1^{(M)} \\ \vdots & \ddots & \vdots \\ \mathbf{h}_M^{(1)} & \dots & \mathbf{h}_M^{(M)} \end{pmatrix}}_{2M\times M}, \quad \mathcal{N} = \underbrace{\begin{pmatrix} \mathbf{n}_{1,1} & \dots & \mathbf{n}_{1,M} \\ \vdots & \ddots & \vdots \\ \mathbf{n}_{M,1} & \dots & \mathbf{n}_{M,M} \end{pmatrix}}_{2M\times M}$$

Then the MMSE channel estimator is given by [6]

$$\hat{\mathcal{H}} = \mathcal{X}^* \left[\mathcal{X}\mathcal{X}^* + \sigma_n^2 \mathbf{I}_{2M}\right]^{-1} \mathcal{Y} \qquad (4)$$

By examining the structure of the matrix $\mathcal{X}\mathcal{X}^* + \sigma_n^2 \mathbf{I}_{2M}$, we find that it is Hermitian and it has a block structure with 2×2 Alamouti-like subblocks . The matrix inversion needed to compute $\hat{\mathcal{H}}$ could be done efficiently by exploiting the properties of Alamouti-like matrices as shown further ahead in Section 3.

2.2 Frequency-Selective Fading Channels

For frequency-selective fading channels, the Alamouti scheme is implemented on a block level to achieve multi-path diversity. Different schemes have been proposed for the block level implementation. In this section we describe two schemes, namely, single–carrier frequency domain equalization STBC (SC-FDE STBC) and orthogonal frequency division multiplexing STBC (OFDM-STBC). For each scheme, we show how to exploit the code structure for MIMO channel estimation of multiple–user transmissions.

2.2.1 SC-FDE STBC

For each user, data are transmitted from its two antennas in blocks of length N according to the following space–time coding scheme. Denote the n–th symbol of the k–th transmitted block from antenna j of user i by $\mathbf{x}_{k,j}^{(i)}(n)$. At times $k = 0, 2, 4, \cdots$, the blocks $\mathbf{x}_{k,1}^{(i)}(n)$ and $\mathbf{x}_{k,2}^{(i)}(n)$ ($0 \le n \le N-1$) are generated by an information source according to the rule [4, 5]:

		Time k	$k+1$
Block	1	$\mathbf{x}_{k,1}^{(i)}(n)$	$-\mathbf{x}_{k,2}^{*(i)}\left((-n)_N\right)$
	2	$\mathbf{x}_{k,2}^{(i)}(n)$	$\mathbf{x}_{k,1}^{*(i)}\left((-n)_N\right)$

(5)

where each data vector $\mathbf{x}_{k,j}^{(i)}$ has a covariance matrix equal to $\sigma_x^2 \mathbf{I}_N$, and where $(\cdot)^*$ and $(\cdot)_N$ denote complex conjugation and modulo–N operations, respectively. In addition, a cyclic prefix (CP) of length ν is added to each transmitted block to eliminate inter–block interference (IBI) and to make all channel matrices *circulant*. Here, ν denotes the longest channel memory between the transmit antennas and the receive antennas. With two transmit antennas per user and M receive antennas, and assuming all channels are fixed over two consecutive blocks, the received blocks k and $k+1$ at the l–th antenna, in the presence of additive white noise, are described by

$$\begin{aligned} \mathbf{y}_{k,l} &= \sum_{i=1}^{M} \left(\mathbf{H}_{1,l}^{(i)} \mathbf{x}_{k,1}^{(i)} + \mathbf{H}_{2,l}^{(i)} \mathbf{x}_{k,2}^{(i)}\right) + \mathbf{n}_{k,l} \\ \mathbf{y}_{k+1,l} &= \sum_{i=1}^{M} \left(\mathbf{H}_{1,l}^{(i)} \mathbf{x}_{k+1,1}^{(i)} + \mathbf{H}_{2,l}^{(i)} \mathbf{x}_{k+1,2}^{(i)}\right) + \mathbf{n}_{k+1,l} \end{aligned} \qquad (6)$$

where $\mathbf{n}_{k,l}$ and $\mathbf{n}_{k+1,l}$ are the noise vectors for the received blocks k and $k+1$, respectively, at the l-th receive antenna with covariance matrix $\sigma_n^2 \mathbf{I}_N$, and $\mathbf{H}_{1,l}^{(i)}$ and $\mathbf{H}_{2,l}^{(i)}$ are the *circulant* channel matrices from the first and second transmit antennas of the i–th user, respectively, to the l–th receive antenna [5]. Applying the DFT matrix $\mathbf{Q}$ to $\mathbf{y}_{k,l}$ and $\mathbf{y}_{k+1,l}$ in (6), we get a relation in terms of frequency–transformed variables:

$$\begin{aligned} \mathbf{Y}_{k,l} &= \sum_{i=1}^{M} \left(\mathbf{\Lambda}_{1,l}^{(i)} \mathbf{X}_{k,1}^{(i)} + \mathbf{\Lambda}_{2,l}^{(i)} \mathbf{X}_{k,2}^{(i)}\right) + \mathbf{N}_{k,l} \\ \mathbf{Y}_{k+1,l} &= \sum_{i=1}^{M} \left(\mathbf{\Lambda}_{1,l}^{(i)} \mathbf{X}_{k+1,1}^{(i)} + \mathbf{\Lambda}_{2,l}^{(i)} \mathbf{X}_{k+1,2}^{(i)}\right) + \mathbf{N}_{k+1,l} \end{aligned} \qquad (7)$$

where $\mathbf{Y} = \mathbf{Q}\mathbf{y}$, $\mathbf{X} = \mathbf{Q}\mathbf{x}$, $\mathbf{N} = \mathbf{Q}\mathbf{n}$, and $\mathbf{\Lambda}_{1,l}^{(i)}$ and $\mathbf{\Lambda}_{2,l}^{(i)}$ are diagonal matrices given by $\mathbf{\Lambda}_{1,l}^{(i)} = \mathbf{Q}\mathbf{H}_{1,l}^{(i)}\mathbf{Q}^*$ and $\mathbf{\Lambda}_{2,l}^{(i)} = \mathbf{Q}\mathbf{H}_{1,l}^{(i)}\mathbf{Q}^*$, respectively. Using the encoding rule (5) and properties of the DFT [7], we have that

$$\mathbf{X}_{k+1,1}^{(i)}(m) = -\mathbf{X}_{k,2}^{*(i)}(m), \quad \mathbf{X}_{k+1,2}^{(i)}(m) = \mathbf{X}_{k,1}^{*(i)}(m) \qquad (8)$$

for $m = 0, 1, \cdots, N-1$ and $k = 0, 2, 4, \cdots$. Combining (7) and (8), we arrive at the linear relation (9) shown at the top of the next page where $\bar{(\cdot)}$ denotes complex conjugation of the entries of the vector. Equation (9) tells us how the entries of the transformed vectors at the l–th receive antenna, $\{\mathbf{Y}_{k,l}, \mathbf{Y}_{k+1,l}\}$, are related to the entries of the transformed transmitted blocks $\{\mathbf{X}_{k,1}^{(i)}, \mathbf{X}_{k,2}^{(i)}\}$ from the two antennas of the i–th user. We conjugate the entries of $\mathbf{Y}_{k+1,l}$ and reorder the entries of (9) to obtain the relation shown in (10). Let

$$\mathcal{Y}_{k,l}(m) = \begin{pmatrix} \mathbf{Y}_{k,l}(m) \\ \mathbf{Y}_{k+1,l}(m) \end{pmatrix}$$

Then the m–th entry of received vectors from the l–th receive antenna can be written as

$$\begin{aligned} \mathcal{Y}_{k,l}(m) &= \sum_{i=1}^{M} \begin{pmatrix} \mathbf{X}_{k,1}^{(i)}(m) & \mathbf{X}_{k,2}^{(i)}(m) \\ -\mathbf{X}_{k,2}^{*(i)}(m) & \mathbf{X}_{k,1}^{*(i)}(m) \end{pmatrix} \begin{pmatrix} \mathbf{\Lambda}_{1,l}^{(i)}(m) \\ \mathbf{\Lambda}_{2,l}^{(i)}(m) \end{pmatrix} \\ &\quad + \begin{pmatrix} \mathbf{N}_{k,l}(m) \\ \mathbf{N}_{k+1,l}(m) \end{pmatrix} \\ &\triangleq \sum_{i=1}^{M} \mathcal{X}_k^{(i)}(m) \mathbf{\Lambda}_l^{(i)}(m) + \mathcal{N}_{k,l}(m) \end{aligned} \qquad (11)$$

By inspecting the structure of (11), we find that it has the same form as (2). This allows the estimation of $\mathbf{\Lambda}_{1,l}^{(i)}(m)$ and $\mathbf{\Lambda}_{1,l}^{(i)}(m)$ in a fashion similar to the flat fading case discussed earlier. The only difference is that we now assume the channel to be fixed over M blocks of data. Thus collect M blocks of data at the receiver. The m–th entry of the received vectors from all antennas is written in matrix form as:

$$\mathcal{Y}(m) = \mathcal{X}(m)\mathbf{\Lambda}(m) + \mathcal{N}(m) \qquad (12)$$

where

$$\mathcal{Y}(m) = \underbrace{\begin{pmatrix} \mathcal{Y}_{1,1}(m) & \dots & \mathcal{Y}_{1,M}(m) \\ \vdots & \ddots & \vdots \\ \mathcal{Y}_{M,1}(m) & \dots & \mathcal{Y}_{M,M}(m) \end{pmatrix}}_{2M\times M}, \mathcal{X}(m) = \underbrace{\begin{pmatrix} \mathcal{X}_1^{(1)}(m) & \dots & \mathcal{X}_1^{(M)}(m) \\ \vdots & \ddots & \vdots \\ \mathcal{X}_M^{(1)}(m) & \dots & \mathcal{X}_M^{(M)}(m) \end{pmatrix}}_{2M\times 2M}$$

$$\mathbf{\Lambda}(m) = \underbrace{\begin{pmatrix} \mathbf{\Lambda}_1^{(1)}(m) & \dots & \mathbf{\Lambda}_1^{(M)}(m) \\ \vdots & \ddots & \vdots \\ \mathbf{\Lambda}_M^{(1)}(m) & \dots & \mathbf{\Lambda}_M^{(M)}(m) \end{pmatrix}}_{2M\times M}, \mathcal{N}(m) = \underbrace{\begin{pmatrix} \mathcal{N}_{1,1}(m) & \dots & \mathcal{N}_{1,M}(m) \\ \vdots & \ddots & \vdots \\ \mathcal{N}_{M,1}(m) & \dots & \mathcal{N}_{M,M}(m) \end{pmatrix}}_{2M\times M}$$

$$\begin{pmatrix}\mathbf{Y}_{k,l}\\ \bar{\mathbf{Y}}_{k+1,l}\end{pmatrix}=\begin{pmatrix}\mathbf{Y}_{k,l}(0)\\ \vdots\\ \mathbf{Y}_{k,l}(N-1)\\ \hline \bar{\mathbf{Y}}_{k+1,l}(0)\\ \vdots\\ \bar{\mathbf{Y}}_{k+1,l}(N-1)\end{pmatrix}=\sum_{i=1}^{M}\left(\begin{array}{ccc|ccc}\mathbf{\Lambda}^{(i)}_{1,l}(0) & & & \mathbf{\Lambda}^{(i)}_{2,0}(1) & & \\ & \ddots & & & \ddots & \\ & & \mathbf{\Lambda}^{(i)}_{1,l}(N-1) & & & \mathbf{\Lambda}^{(i)}_{2,l}(N-1)\\ \hline \mathbf{\Lambda}^{*(i)}_{2,l}(0) & & & -\mathbf{\Lambda}^{*(i)}_{1,l}(0) & & \\ & \ddots & & & \ddots & \\ & & \mathbf{\Lambda}^{*(i)}_{2,l}(N-1) & & & -\mathbf{\Lambda}^{*(i)}_{1,l}(N-1)\end{array}\right)\begin{pmatrix}\mathbf{X}^{(i)}_{k,1}(0)\\ \vdots\\ \mathbf{X}^{(i)}_{k,1}(N-1)\\ \hline \mathbf{X}^{(i)}_{k,2}(0)\\ \vdots\\ \mathbf{X}^{(i)}_{k,2}(N-1)\end{pmatrix}+\begin{pmatrix}\mathbf{N}_{k,l}(0)\\ \vdots\\ \mathbf{N}_{k,l}(N-1)\\ \hline \bar{\mathbf{N}}_{k+1,l}(0)\\ \vdots\\ \bar{\mathbf{N}}_{k+1,l}(N-1)\end{pmatrix} \tag{9}$$

$$\mathcal{Y}_{k,l}\triangleq\begin{pmatrix}\mathbf{Y}_{k,l}(0)\\ \mathbf{Y}_{k+1,l}(0)\\ \hline \vdots\\ \hline \mathbf{Y}_{k,l}(N-1)\\ \mathbf{Y}_{k+1,l}(N-1)\end{pmatrix}=\sum_{i=1}^{M}\begin{pmatrix}\mathbf{X}^{(i)}_{k,1}(0) & \mathbf{X}^{(i)}_{k,2}(0) & & & \\ -\mathbf{X}^{*(i)}_{k,2}(0) & \mathbf{X}^{*(i)}_{k,1}(0) & & & \\ & & \ddots & & \\ & & & \mathbf{X}^{(i)}_{k,1}(N-1) & \mathbf{X}^{(i)}_{k,2}(N-1)\\ & & & -\mathbf{X}^{*(i)}_{k,2}(N-1) & \mathbf{X}^{*(i)}_{k,1}(N-1)\end{pmatrix}\begin{pmatrix}\mathbf{\Lambda}^{(i)}_{1,l}(0)\\ \mathbf{\Lambda}^{(i)}_{2,l}(0)\\ \hline \vdots\\ \hline \mathbf{\Lambda}^{(i)}_{1,l}(N-1)\\ \mathbf{\Lambda}^{(i)}_{2,l}(N-1)\end{pmatrix}+\begin{pmatrix}\mathbf{N}_{k,l}(0)\\ \mathbf{N}_{k+1,l}(0)\\ \hline \vdots\\ \hline \mathbf{N}_{k,l}(N-1)\\ \mathbf{N}_{k+1,l}(N-1)\end{pmatrix} \tag{10}$$

The MMSE channel estimator is now given by [6]

$$\hat{\mathbf{\Lambda}}(m)=\mathcal{X}^*(m)\left[\mathcal{X}(m)\mathcal{X}^*(m)+\sigma_n^2\mathbf{I}_{2M}\right]^{-1}\mathcal{Y}(m) \tag{13}$$

Again, $\mathcal{X}(m)\mathcal{X}^*(m)+\sigma_n^2\mathbf{I}_{2M}$ is Hermitian with 2×2 Alamouti-like subblocks similar to that of a flat–fading implementation described earlier. Hence, the matrix inversion complexity can be significantly reduced by exploiting its structure as shown in Section 3.

2.2.2 OFDM-STBC

OFDM-STBC is similar to SC-FDE STBC except for transmitting the IDFT of the data blocks rather than the data blocks themselves. At times $k=0,2,4,\cdots$, N-symbol data blocks $\mathbf{x}^{(i)}_{k,1}$ and $\mathbf{x}^{(i)}_{k,2}$ are generated by an information source. The data blocks are then transmitted from the antennas of the i–th user according to the following rule:

		Block k	$k+1$
Antenna	1	$\mathbf{Q}^*\mathbf{x}^{(i)}_{k,1}$	$-\mathbf{Q}^*\mathbf{x}^{*(i)}_{k,2}$
	2	$\mathbf{Q}^*\mathbf{x}^{(i)}_{k,2}$	$\mathbf{Q}^*\mathbf{x}^{*(i)}_{k,1}$

(14)

where $\mathbf{Q}^*$ is the IDFT matrix of size $N\times N$. A cyclic prefix (CP) is also added to each transmitted block to eliminate inter–block interference (IBI) and to make all channel matrices *circulant*. The received blocks k and $k+1$ at the l–th antenna, in the presence of additive white noise, are described by

$$\mathbf{y}_{k,l}=\sum_{i=1}^{M}\left(\mathbf{H}^{(i)}_{1,l}\mathbf{Q}^*\mathbf{x}^{(i)}_{k,1}+\mathbf{H}^{(i)}_{2,l}\mathbf{Q}^*\mathbf{x}^{(i)}_{k,2}\right)+\mathbf{n}_{k,l}$$
$$\mathbf{y}_{k+1,l}=\sum_{i=1}^{M}\left(-\mathbf{H}^{(i)}_{1,l}\mathbf{Q}^*\mathbf{x}^{*(i)}_{k,2}+\mathbf{H}^{(i)}_{2,l}\mathbf{Q}^*\mathbf{x}^{(i)}_{k,1}\right)+\mathbf{n}_{k+1,l} \tag{15}$$

Applying the DFT matrix $\mathbf{Q}$ to $\mathbf{y}_{k,l}$ and $\mathbf{y}_{k+1,l}$ in (6), we get a relation in terms of frequency–transformed variables:

$$\mathbf{Y}_{k,l}\;=\;\sum_{i=1}^{M}\left(\mathbf{\Lambda}^{(i)}_{1,l}\mathbf{x}^{(i)}_{k,1}+\mathbf{\Lambda}^{(i)}_{2,l}\mathbf{x}^{(i)}_{k,2}\right)+\mathbf{N}_{k,l}$$
$$\mathbf{Y}_{k+1,l}\;=\;\sum_{i=1}^{M}\left(\mathbf{\Lambda}^{(i)}_{1,l}\mathbf{x}^{(i)}_{k+1,1}+\mathbf{\Lambda}^{(i)}_{2,l}\mathbf{x}^{(i)}_{k+1,2}\right)+\mathbf{N}_{k+1,l} \tag{16}$$

By examining the structure of (16), we find that it is similar to (7). The only difference is that the DFTs of the data blocks have been replaced by the data blocks themselves. We then conclude that all expressions derived in Equations (9)– (13) are also valid for the OFDM-STBC case except for changing $\mathcal{X}^{(i)}_k(m)$ to

$$\mathcal{X}^{(i)}_k(m)=\begin{pmatrix}\mathbf{x}^{(i)}_{k,1}(m) & \mathbf{x}^{(i)}_{k,2}(m)\\ -\mathbf{x}^{*(i)}_{k,2}(m) & \mathbf{x}^{*(i)}_{k,1}(m)\end{pmatrix} \tag{17}$$

3. EXPLOITING STBC STRUCTURE

In this section, we show how to exploit the STBC structure to reduce the computational complexity of the MMSE channel estimator. By inspecting the structure of $\mathcal{X}\mathcal{X}^*+\sigma_n^2\mathbf{I}_{2M}$ in (4) and $\mathcal{X}(m)\mathcal{X}^*(m)+\sigma_n^2\mathbf{I}_{2M}$ in (13), we see that they are Hermitian and they have a block matrix structure with Alamouti–like subblocks. Furthermore, the diagonal subblocks are multiples of the identity matrix $\mathbf{I}_2$. In order to compute the inverse of a matrix $\mathcal{A}$ that has such structure, we proceed by partitioning the matrix as follows:

$$\mathcal{A}=\left(\begin{array}{c|c}\mathbf{A}_1 & \mathbf{B}_1\\ \hline \mathbf{B}_1^* & \mathbf{D}_1\end{array}\right) \tag{18}$$

with $\mathbf{A}_1$, $\mathbf{B}_1$, $\mathbf{D}_1$ denoting the 2×2 upper–left, $2\times 2(M-1)$ upper–right, and $2(M-1)\times 2(M-1)$ lower–right matrices, respectively. We then partition the right–lower matrix D_1 in a similar manner, i.e.,

$$\mathbf{D}_1=\left(\begin{array}{c|c}\mathbf{A}_2 & \mathbf{B}_2\\ \hline \mathbf{B}_2^* & \mathbf{D}_2\end{array}\right) \tag{19}$$

with $\mathbf{A}_2$, $\mathbf{B}_2$, $\mathbf{D}_2$ denoting the 2×2 upper–left, $2\times 2(M-2)$ upper–right, and $2(M-2)\times 2(M-2)$ lower–right matrices, respectively. We repeat the partitioning of the lower–right matrix until we get the following partitioned matrix

$$\mathcal{A}=\left(\begin{array}{c|c}\mathbf{A}_1 & \mathbf{B}_1\\ \hline \mathbf{B}_1^* & \left(\begin{array}{c|c}\mathbf{A}_2 & \mathbf{B}_2\\ \hline \mathbf{B}_2^* & \begin{array}{cc}\ddots & \\ & \left(\begin{array}{c|c}\mathbf{A}_{M-1} & \mathbf{B}_{M-1}\\ \hline \mathbf{B}_{M-1}^* & \mathbf{A}_M\end{array}\right)\end{array}\end{array}\right)\end{array}\right) \tag{20}$$

with $\mathbf{A}_i=\alpha_i\mathbf{I}_2$, $i=1,\cdots,M$ with α_i being the product of the i–th row of $\mathcal{X}$, respectively $\mathcal{X}(m)$, and its transpose plus σ_n^2. Next, we find the inverse of $\mathcal{A}$ using the block matrix inversion of (18):

$$\mathcal{A}^{-1}=\begin{pmatrix}\mathbf{\Sigma}_{D_1}^{-1} & -\mathbf{\Sigma}_{D_1}^{-1}\mathbf{B}_1\mathbf{D}_1^{-1}\\ -\mathbf{D}_1^{-1}\mathbf{B}_1^*\mathbf{\Sigma}_{D_1}^{-1} & \mathbf{D}_1^{-1}+\mathbf{D}_1^{-1}\mathbf{B}_1^*\mathbf{\Sigma}_{D_1}^{-1}\mathbf{B}_1\mathbf{D}_1^{-1}\end{pmatrix} \tag{21}$$

Table 1: Exploiting STBC structure for matrix inversion.

Let $\mathbf{D}_{M-1} = \mathbf{A}_M$, then $\mathbf{D}_{M-1}^{-1} = \alpha_M^{-1}\mathbf{I}_2$. From (20), we have

$$\mathbf{D}_{M-2} = \left(\begin{array}{c|c} \mathbf{A}_{M-1} & \mathbf{B}_{M-1} \\ \hline \mathbf{B}_{M-1}^* & \mathbf{D}_{M-1} \end{array}\right)$$

Starting from $i = M-1$,

1. Compute $\mathbf{\Sigma}_{D_i}$, the 2×2 Schur complement of D_i as follows
$$\mathbf{\Sigma}_{D_i} = \mathbf{A}_i - \mathbf{B}_i\mathbf{D}_i^{-1}\mathbf{B}_i^* = \gamma_i\mathbf{I}_2$$
2. Compute $\mathbf{\Phi}_i$ defined as
$$\mathbf{\Phi}_i = \mathbf{\Sigma}_{D_i}^{-1}\mathbf{B}_i\mathbf{D}_i^{-1} = \gamma_i^{-1}\mathbf{B}_i\mathbf{D}_i^{-1}$$
3. construct $\mathbf{D}_{i-1}^{-1}$ as follows
$$\mathbf{D}_{i-1}^{-1} = \begin{pmatrix} \gamma_i^{-1}\mathbf{I}_2 & -\mathbf{\Phi}_i \\ -\mathbf{\Phi}_i^* & \mathbf{D}_i^{-1} + \gamma_i\mathbf{\Phi}_i^*\mathbf{\Phi}_i \end{pmatrix}$$
4. Let $i = i-1$ and repeat steps 1-3 until $i = 1$ to get $\mathcal{A}^{-1}$.

with $\mathbf{\Sigma}_{D_1} = \mathbf{A}_1 - \mathbf{B}_1\mathbf{D}_1^{-1}\mathbf{B}_1^*$ is the 2×2 Schur complement of $\mathbf{D}_1$. We still need to evaluate $\mathbf{D}_1^{-1}$ which could be found by applying the block matrix inversion formula again to (19). Since $\mathbf{D}_1^{-1}$ depends on $\mathbf{D}_2^{-1}$, we need to proceed in a similar manner to find the matrix inverse of $\mathbf{D}_2$. Evaluating $\mathbf{D}_2^{-1}$ requires the knowledge of $\mathbf{D}_3^{-1}$ which can also be computed by another block matrix inversion. The following properties of Alamouti–like matrices further reduce the complexity:

- The sum, difference, or product of two Alamouti–like matrices is an Alamouti-like matrix.
- The inverse of an Alamouti-like matrix is Alamouti-like.
- The inverse of a Hermitian block matrix with Alamouti-like subblocks is another Hermitian block matrix with Alamouti-like subblocks.

By using these properties, we can show that

- $\mathbf{\Sigma}_{D_i}$, $i = 1, \cdots, M-1$, is equal to $\gamma_i\mathbf{I}_2$. This means that no matrix inversion is needed to compute the inverse of $\mathcal{A}$.
- Since $\mathcal{A}^{-1}$ and $\mathbf{D}_i^{-1}$, $i = 1, \cdots, M-1$, are Hermitian, then the lower–left submatrix is the complex conjugate of upper–right submatrix. This also reduces the complexity since only half of the entries of the matrix need to be computed.

We summarize the algorithm for matrix inversion in Table 1. By exploiting the structure of the space–time block code, the computational complexity of the solution is reduced by approximately 70% over an algorith that would invert the matrices in (4) or (13) directly.

4. SIMULATION RESULTS

We simulate three different scenarios; the flat fading case, the frequency selective case with SC-FDE STBC, and the frequency selective case with OFDM-STBC. The system has M users, each equipped with two transmit antennas. The number of receive antennas is equal to the number of users. The channels from each transmit antenna to each receive antenna are assumed to be independent. The data bits of each user are mapped into an 8-PSK signal constellation. The processed symbols are transmitted at a symbol rate equal to 271 KSymbols/sec. The signal to noise ratios of all users at the receiver are assumed to be equal. For the flat fading scenario, we assume single tap independent channels between transmit and receive antennas and symbol level Alamouti STBC [2]. For the frequency selective one, a Typical Urban (TU) channel model with overall channel impulse response memory ν equals to 3 is considered for all channels. The transmitted symbols are grouped into blocks of 32 symbols. A cyclic prefix is added to each block by copying the first ν symbols after the last symbol of the same block.

Figure 2 shows the overall system MSE associated with the different channel estimation techniques presented in this paper for $M = 2$.

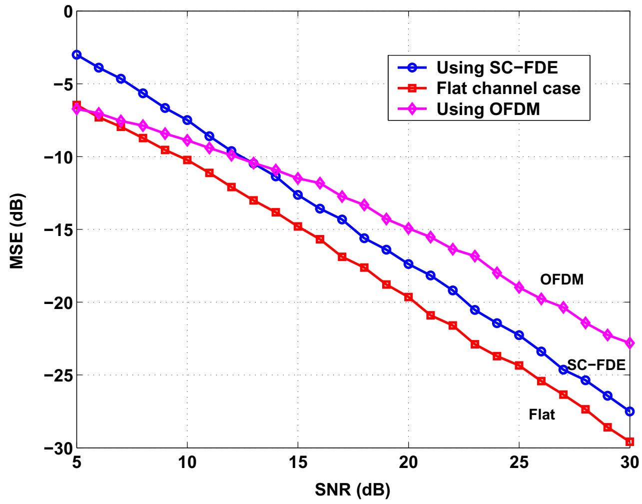

Figure 2: MSE performance of channel estimator.

5. CONCLUSIONS

In this paper, we described an efficient channel estimation technique for STBC transmissions over flat and frequency selective fading channels. In particular, we showed that the channel estimation for SC-FDE STBC and OFDM-STBC collapses to the problem of estimating N parallel flat fading channels where N is the block size.

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