# HONEYBEE SWARMING BEHAVIOR USING DIFFUSION ADAPTATION 

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#### Abstract

Honeybees swarm when they move to a new site for their hive. During the process of swarming, their behavior can be analyzed by classifying them as informed scouts or uninformed bees, where the scouts have information about the destination while the uninformed bees follow the scouts. We model the network of bees as a network of mobile nodes with the nodes having asymmetric access to information about the location of the new hive. Diffusion adaptation is then used to model and explain the swarming behavior of bees.


Index Terms- Adaptive networks, diffusion adaptation, mobile networks, mobility, swarming.

## 1. INTRODUCTION

Many animal species move in groups, such as schools of fish, flocks of birds, and swarms of honeybees when they perform seasonal migrations, travel to food sources, or to new sites. For some species, all the individuals in the group process the information about the likelihood to travel in a certain direction. For other species, only some of the group members may have information about the destination, while the other uninformed members are guided by the informed individuals. For example, for honey bees, when they have made a decision about the new site and begin traveling, the location of the new nest site is known only to a small fraction of the swarm [1]. A curious feature in the home-site selection procedure for bees is that only $3 \%-5 \%$ of the bees [4] in the swarm has been to the new site and are called scout bees; these bees usually move at faster speeds than the other bees. So how can a small fraction of bees lead the rest of the swarm towards the new site? Several earlier studies [5]-[9] analyzed how bees reach their destination. One hypothesis is that a group of informed scouts, called streaker bees, have information about the destination and lead the group towards the target by flying back and forth in the swarm.

In this paper, and motivated by the observations in [5][9], we model the bee swarming behavior via a mobile adaptive network, with each node in the network corresponding to

[^0]a bee in the swarm. By using diffusion adaptation [12][13], each node makes their own estimation about the target location and shares information with their neighbors. The type of information shared among neighbors affects the efficiency of the swarm's behavior. We assume that nodes may share information either about the position of the destination or about its direction. Our work extends the study in [7], which assumed that the bees align their velocities to those of their neighbors via a consensus procedure. In our model, we use diffusion adaptation instead of consensus averaging in order to allow the bees to adjust their velocities by taking into account several additional factors such as: (a) the velocities of their neighbors as in [7]; (b) the velocities of the informed scouts; (c) the estimates of the target location by the neighbors; and (d) the ability to share different types of information in a localized manner. .

## 2. DIFFUSING SITE LOCATION

### 2.1. Information Processing by the Scouts

Consider $N_{s}$ informed scouts where scout $k$ is located at the vector location $x_{k, i}$ at time $i$. These scouts are assumed to have visited the new site at location $\omega^{o}$ before. However, during swarming, their measurements of the distance and direction of the new hive is not necessarily accurate. Thus, at every step $i$, they sense noisy measurements of the distance, $d_{k}(i)$, and direction, $u_{k, i}$, of the new hive, communicate with neighboring scouts, and share information to enhance the quality of their estimates of the actual distance and direction of the hive. Following this diffusion process, the scouts decide on how to set their velocity vectors and help guide the other uniformed bees.

To begin with, we assume that the measured distance, $d_{k}(i)$, is the true distance, denoted by $d_{k}^{o}(i)$, plus some additive noise with variance proportional to the true distance, i.e.,

$$
\begin{equation*}
\mathbf{d}_{k, i}=d_{k}^{o}(i)+\beta\left|\omega^{o}-x_{k, i}\right| \mathbf{n}_{k}(i) \tag{1}
\end{equation*}
$$

where $\mathbf{n}_{k}(i)$ is a normalized zero-mean Gaussian variable of unit variance, and $\beta$ is a positive parameter. The scaling factor $\left(w^{o}-x_{k, i}\right)$ is meant to scale the noise variance depending
on whether the scout is closer or further away from the destination; the noise variance would be expected to be larger in the latter case. In (1), we are using boldface letters to refer to variables that are stochastic in nature and normal font letters to denote deterministic quantities (such as actual measurements of the unknown location).

The informed scouts cooperate with each other to estimate the location $w^{o}$ by attempting to minimize the following cost function:

$$
\begin{equation*}
\min _{w} \sum_{k=1}^{N_{s}} E\left|\mathbf{d}_{k}(i)-\mathbf{u}_{k, i}\left(\omega-\mathbf{x}_{k, i}\right)\right|^{2} \tag{2}
\end{equation*}
$$

Using the Adapt-then-Combine (ATC) diffusion algorithm [12][13] to estimate $\omega^{o}$, we end up with the following threestep distributed procedure that can be used by each of the informed scouts:

Step I: Adaptation using local measurements:
$\phi_{k, i}=\psi_{k, i-1}+\mu_{k} u_{k, i}^{*}\left[d_{k}(i)-u_{k, i}\left(\psi_{k, i-1}-x_{k, i-1}\right)\right]$
Step II: Consultation with neighbors to combine their estimates:

$$
\begin{equation*}
\psi_{k, i}=\sum_{l \in \mathcal{N}_{k, s}^{(i)}} a_{k, l}^{s} \phi_{l, i} \tag{4}
\end{equation*}
$$

where $\psi_{k, i}$ is the estimate of $\omega^{o}$ by informed scout $k$ at time $i$ obtained after consultation (combination step), $\phi_{k, i}$ is the estimate obtained after local processing (adaptation), and $\mathcal{N}_{k, s}^{(i)}$ is the set of informed scouts in the neighborhood (within a distance less than $r_{s}$ ) of bee $k$ at time $i$. The coefficients $a_{k, l}^{s}$ are scaling factors that add up to one:

$$
\begin{equation*}
\sum_{l \in \mathcal{N}_{k, s}^{(i)}} a_{k, l}^{s}=1 \tag{5}
\end{equation*}
$$

Step III: Motion control. For this step, each informed scout checks the number of bees in its neighborhood. If the number exceeds a certain threshold (e.g., 5\% of the total number of bees), then the informed scout will move towards the direction of the nest site, which is estimated by the direction of the vector $\psi_{k, i}-x_{k, i-1}$. The velocity vector for the informed scout will be a weighted combination of the previous velocity vector and the current direction towards the target. We limit the maximum speed of the scout bee in this case to $\gamma v_{\max }$, where $\gamma$ is a number between 2 and 3 and $v_{\max }$ is the maximum speed for the uninformed bees. On the other hand, if the number of informed scouts is less than the threshold, then the informed bee will turn back and move towards the center of the bees in front of them until they reach the rear of the swarm. During this process, their velocity is set to $v_{\max }$, the same as the uninformed bees, so that the other bees do not recognize them as informed scouts and end up following them in the opposite direction to the new hive. The procedure


Fig. 1. Diffusion adaptation used to estimate the location of the new site and to control the velocity vectors of the informed scouts.
is summarized as follows:
$\underline{\text { If } \mathcal{N}_{k, s}^{(i)} \geq \text { threshold: }}$

$$
\begin{equation*}
v_{k, i}=(1-\lambda) v_{k, i-1}+\lambda \gamma v_{\max } \frac{\left(\psi_{k, i}-x_{k, i-1}\right)}{\left\|\psi_{k, i}-x_{k, i-1}\right\|} \tag{6}
\end{equation*}
$$

else if $\mathcal{N}_{k, s}^{(i)}<$ threshold:
The informed bee $k$ turns around and detects the bees that are in front of it and moves towards the center of this group. After each move, the informed bee detects the center again, and keeps doing this until it reaches the rear of the swarm. Bee $l$ is considered to be in front of informed bee $k$ when

$$
\begin{array}{r}
\left\|x_{l, i-1}-x_{k, i-1}\right\|<r_{r} \\
\left(x_{l, i-1}-x_{k, i-1}\right) v_{k, i-1}^{T}>0 \tag{8}
\end{array}
$$

where $r_{r}$ is the range of the perception area (the same value as the repulsion region for uninformed bees described further ahead). Then the velocity at time $i$ for informed bee $k$ is set as:

$$
\begin{equation*}
v_{k, i}=v_{\max } \frac{\sum_{l \in \mathcal{N}_{k, f}^{(i)}}\left(x_{l, i-1}-x_{k, i-1}\right)}{\left\|\sum_{l \in \mathcal{N}_{k, f}^{(i)}}\left(x_{l, i-1}-x_{k, i-1}\right)\right\|} \tag{9}
\end{equation*}
$$

where $\mathcal{N}_{k, f}^{(i)}$ is the set of bees in front of bee $k$ at time $i$. This velocity makes the bee move towards the center positions of the surrounding bees.

After determining the velocity vector, each informed scout updates its position accordingly:

$$
\begin{equation*}
x_{k, i}=x_{k, i-1}+v_{k, i} \Delta t \tag{10}
\end{equation*}
$$

The procedure is summarized in Fig.1.

### 2.2. Information Processing by the Other Bees

The uninformed bees have not been to the destination before and they cannot sense the position of the destination. In order
to collect information to guide them in determining in what direction to move, they initially check whether there are informed bees in their neighborhood. If so, they combine these neighbors' estimates of the hive location; if not, they rely on their own previous estimates of the hive location. Specifically, the uninformed bees attempts to optimize the following cost function:

$$
\begin{equation*}
\min _{w} \sum_{k=1}^{N} E\left\|\sum_{l \in \mathcal{N}_{k, s}^{(i)}} c_{k, l} \psi_{l, i}-\omega\right\|^{2} \tag{11}
\end{equation*}
$$

where the coefficients $c_{k, l}$ are scaling factors that add up to one,

$$
\begin{equation*}
\sum_{l \in \mathcal{N}_{k, s}^{(i)}} c_{k, l}=1 \tag{12}
\end{equation*}
$$

By using diffusion adaptation again, we end up with the following procedure for the uninformed bees:

Step I: Adaptation using local adaptation using local data:

$$
\begin{equation*}
\phi_{k, i}=\psi_{k, i-1}+\mu_{k}\left(\sum_{l \in \mathcal{N}_{k, s}^{(i)}} c_{k, l} \psi_{l, i}-\psi_{k, i-1}\right) \tag{13}
\end{equation*}
$$

Step II: Consultation with neighbors:

$$
\begin{equation*}
\psi_{k, i}=\sum_{l \in \mathcal{N}_{k}^{(i)}} a_{k, l} \phi_{l, i} \tag{14}
\end{equation*}
$$

where $\mathcal{N}_{k}^{(i)}$ denotes the set of bees (both informed and uninformed) within the neighborhood of bee $k$ at time $i$.

Step III: Motion Control. After the diffusion step, the uninformed bees compute an initial value for their velocity vectors towards the estimated position in a manner similar to (6):

$$
\begin{equation*}
v_{k, m, i}=(1-\lambda) v_{k, i-1}+\lambda v_{\max } \frac{\left(\psi_{k, i}-x_{k, i-1}\right)}{\left\|\psi_{k, i}-x_{k, i-1}\right\|} \tag{15}
\end{equation*}
$$

This velocity vector need not be the velocity used by uninformed bee $k$ at time $i$. In order to keep the bees moving together as a group, the bees have to make sure that they will not go too far from, or get too close to, their neighbors [10]. This desirable feature can be implemented by defining attraction and repulsion regions around each bee. For attraction behavior, we introduce an intermediate velocity vector as the average of all vectors pointing from the current position of the bee to all of its neighbors [7]:

$$
\begin{equation*}
v_{k, a, i}=\frac{v_{\max }}{r_{a}} \frac{1}{\left|\mathcal{N}_{k, a}^{(i)}\right|} \sum_{l \in \mathcal{N}_{k, a}^{(i)}}\left(x_{l, i-1}-x_{k, i-1}\right) \tag{16}
\end{equation*}
$$



Fig. 2. Diffusion adaptation used to estimate the location of the new site and to control the velocity vectors of the uninformed bees.
where $\mathcal{N}_{k, a}^{(i)}$ is the set of bees within the attraction area of bee $k$ at time $i$. The factor $1 / r_{a}$ is used to bound the length of the vector to $[0,1]$, so that the value of $v_{k, a, i}$ does not exceed $v_{\text {max }}$.

For repulsion behavior, we introduce another intermediate velocity vector as the average of all vectors pointing from all neighbors within a given distance $r_{r}$ towards bee $k$ [7]:

$$
\begin{align*}
v_{k, r, i}= & \frac{v_{\max }}{r_{r}} \frac{1}{\left|\mathcal{N}_{k, r}^{(i)}\right|}  \tag{17}\\
& \sum_{l \in \mathcal{N}_{k, r}^{(i)}}\left(x_{k, i-1}-x_{l, i-1}\right)\left(\frac{r_{r}}{\left|x_{k, i-1}-x_{l, i-1}\right|}-1\right)
\end{align*}
$$

where $\mathcal{N}_{k, r}^{(i)}$ is set of bees within the repulsion area of bee $k$ at time $i$. The factor $1 / r_{r}$ is used to bound the length of the vector to $[0,1]$, so that the value of $v_{k, r, i}$ does not exceed $v_{\text {max }}$.

Taking these two intermediate velocities into consideration, the final bee velocity is set to:

$$
\begin{equation*}
v_{k, i}=\alpha_{m} v_{k, m, i}+\left(1-\alpha_{m}\right)\left(\rho_{a} v_{k, a, i}+\rho_{r} v_{k, r, i}\right) \tag{18}
\end{equation*}
$$

where $\rho_{a}, \rho_{r}$, and $\alpha_{m}$ are weighting scalars.
After determining the velocity, each uninformed bee updates its position according to:

$$
\begin{equation*}
x_{k, i}=x_{k, i-1}+v_{k, i} \Delta t \tag{19}
\end{equation*}
$$

Fig. 2 summarizes the behavior of the uninformed bees.

### 2.3. Simulation Results

We set the simulation parameters as in Table 1 to ensure that the density of bees in the simulation is similar to the density of bees in the real world. The units of $r_{a}$ and $r_{r}$ are m (meter), and around 15 steps in the simulation would be equivalent to one second in the real world. The Fig. 3 indicates that


Fig. 3. Distribution of the swarm of bees at the start of the simulation (top) and at the end (bottom).


Fig. 4. MSE performance with different numbers of scout bees.
the swarm can reach the destination even if the percentage of informed scouts is small ( $5 \%$ ). The simulations seem to indicate that the swarm is able to reach the destination even when there is only one informed bee in the swarm. Fig. 4 shows the convergence speed and mean square error between the estimated position and the true position for different percentages of informed bees.

Table 1. Simulation parameters.
$N$ : the number of bees in the swarm.
$N_{s}$ : the number of informed bees in the swarm.
$\sigma_{v}^{2}$ : variance of noise added to the velocity vector of the bees to simulate inaccuracies in the calculations by the bees.

| $N$ | $N_{s}$ | $\rho_{a}$ | $\rho_{r}$ | $\sigma_{v}^{2}$ | $r_{r}$ | $r_{a}$ | $v_{\max }$ | $\mu$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 100 | 5 | 0.5 | 0.5 | 0.5 | 0.8 | 1.5 | 0.1 | 0.2 |

It is seen from Fig. 4 that as the percentage of informed bees increases, the location estimates will converge towards the true location more quickly. Fig 4 also shows that when the percentage of informed bees increases from $5 \%$ to $10 \%$, the difference in convergence time does not change as much as when the percentage of scouts increases from $1 \%$ to $5 \%$. On


Fig. 5. MSE performance with different numbers of bees (5\% scout bees).
the other hand, given the same proportion of informed bees, when the total number of bees increases, the estimation of the swarm will converge more quickly (Fig. 5). From these results, we see that the larger the swarm is, the smaller the number of scouts that is needed. This result is consistent with the observation in [6] that the larger the swarm is, the less leaders the swarm needs. Moreover, Fig. 4 has a staircase shape. This is because when the informed bees go back to the rear of the swarm, they fly with low speeds so that the uninformed bees do not recognize them. The MSE performance is expected to improve only when the informed bees are moving towards the desired target location.

## 3. DIFFUSING SITE DIRECTION

In this section, we assume that the information that is shared among the bees is no longer the estimates of the destination location, but rather the direction vector towards the destination.

### 3.1. Information Processing by the Scouts

For the informed bees, they estimate the position of the new site, set their own velocity vectors, and then communicate the direction information to the surrounding scout bees. Just as in the previous model, when the scout bees fly back to the rear of the swarm, they use low speeds, and will not be counted as scouts by the surrounding bees:

Step I: Adaptation by using local data:

$$
\begin{equation*}
\psi_{k, i}=\psi_{k, i-1}+\mu_{k} u_{k, i}^{*}\left[d_{k}(i)-u_{k, i}\left(\psi_{k, i-1}-x_{k, i-1}\right)\right] \tag{20}
\end{equation*}
$$

Step II: Motion control.
$\underline{\text { If } \mathcal{N}_{k, s}^{(i)} \geq \text { threshold: }}$

$$
\begin{equation*}
u_{k, i}=(1-\lambda) v_{k, i-1}+\lambda \gamma v_{\max } \frac{\left(\psi_{k, i}-x_{k, i-1}\right)}{\left\|\psi_{k, i}-x_{k, i-1}\right\|} \tag{21}
\end{equation*}
$$

$\underline{\text { else if } \mathcal{N}_{k, s}^{(i)}<\text { threshold: }}$

$$
\begin{equation*}
v_{k, i}=v_{\max } \frac{\sum_{l \in \mathcal{N}_{k, f}^{(i)}}\left(x_{l, i-1}-x_{k, i-1}\right)}{\left\|\sum_{l \in \mathcal{N}_{k, f}^{(i)}}\left(x_{l, i-1}-x_{k, i-1}\right)\right\|} \tag{22}
\end{equation*}
$$

where the position of bee $l$ should satisfy:

$$
\begin{array}{r}
\left\|x_{l, i-1}-x_{k, i-1}\right\|<r_{r} \\
\left(x_{l, i-1}-x_{k, i-1}\right) v_{k, i-1}^{T}>0 \tag{24}
\end{array}
$$

Step III: Consultation with neighbors:
$\underline{\text { If } \mathcal{N}_{k, s}^{(i)} \geq \text { threshold: }}$

$$
\begin{equation*}
v_{k, i}=\sum_{l \in \mathcal{N}_{k, s}^{(i)}} a_{k, l}^{s} u_{l, i} \tag{25}
\end{equation*}
$$

$\underline{\text { else if } \mathcal{N}_{k, s}^{(i)}<\text { threshold: }}$

$$
\begin{equation*}
v_{k, i}=u_{k, i} \tag{26}
\end{equation*}
$$

Step IV: Update location

$$
\begin{equation*}
x_{k, i}=x_{k, i-1}+v_{k, i} \Delta t \tag{27}
\end{equation*}
$$

Compared with the discussion in Section II, the difference now is that the previous diffusion Eq. (4) is performed on the estimates of the position of the new site, while diffusion in (25) is performed on the vectors related to the direction of the hive.

### 3.2. Information Processing by the Other Bees

In the previous model, uninformed bees shared information about the position of the new site, and each bee determined its velocity vector through diffusion. In contrast, in the current model, the uninformed bees can only get information of velocity from the neighboring bees, and the diffusion process is performed over velocities:

## Step I: Adaptation.

$$
\begin{equation*}
u_{k, i}=v_{k, i-1}+\mu_{k}\left(\sum_{l \in \mathcal{N}_{k, s}^{(i)}} c_{k, l} v_{l, i}-v_{k, i-1}\right) \tag{28}
\end{equation*}
$$

where $\mathcal{N}_{k, u}^{(i)}$ is the set of uninformed bees in the neighborhood of bee $k$ at time $i$.


Fig. 6. Distribution of the swarm of bees at the start of the simulation (top) and at the end (bottom).

## Step II: Consultation.

$$
\begin{align*}
v_{k, m, i} & =\sum_{l \in \mathcal{N}_{k, u}^{(i)}} a_{k, l}^{u} u_{l, i}+\sum_{l \in \mathcal{N}_{k, s}^{(i)}} a_{k, l}^{s} v_{l, i}  \tag{29}\\
v_{k, i} & =\alpha_{m} v_{k, m, i}+\left(1-\alpha_{m}\right)\left(\rho_{a} v_{k, a, i}+\rho_{r} v_{k, r, i}\right) \tag{30}
\end{align*}
$$

## Step III: Motion control.

$$
\begin{equation*}
x_{k, i}=x_{k, i-1}+v_{k, i} \Delta t \tag{31}
\end{equation*}
$$

In the simulations, we use the same parameters from Table 1. Again, the bees are able to reach the destination as in the previous model (see Fig. 6).

### 3.3. Diffusion of Information

The model used in [7] for velocity control assumes that uninformed bees average the velocities of the surrounding bees. In our model, the uninformed bees first check the velocities of the informed bees, set their own velocities, and then communicate with other bees as shown by equations (29)-(30). By doing so, uninformed bees pay more attention to the informed bees. In order to illustrate this difference, we set up two simulations, one for the method of [7] and one for our method. The same parameters are used in both simulations. For the first 250 steps, the destination is set at $[20,20,20]$ and after that, the destination is changed to $[0,0,0]$. One factor that we measure is the difference between the direction towards the target and the estimated direction for each uninformed bee. We use mean-square error (MSE) to assess this factor. Averaging over 50 experiments, Figs. 7 and 8 show how the MSE and the distance to the destination vary with time for the method of [7] and for the proposed diffusion adaptation models. The results suggest that it takes longer for the uninformed bees in model [7] to re-orient themselves to the new destination. Roughly, it takes about 350 steps for the bees to gather sufficient information about the new direction using the model in [7], while diffusion adaptation seems to need only about 10 steps.


Fig. 7. Method of [7]: MSE(left) vs. distance to the destination(right).


Fig. 8. Our method: MSE(left) vs. distance to the destination(right).

## 4. CONCLUDING REMARKS

In this paper, we used diffusion adaptation to model the swarming behavior of bees moving towards a new hive site. It was assumed that the scout bees estimate either the position or direction of the destination at each step, communicate with the surrounding scout bees through a diffusion process, and decide where to go. They also broadcast their estimation results to their neighbors. The uninformed bees set their own velocities and communicate with neighbors. Through diffusion adaptation, it was seen that even if only a few nodes have access to relatively accurate estimation, this information is sufficient for the network to self-adjust.

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