Tracking Behavior of Mobile Adaptive Networks

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***Abstract*—Adaptive networks consist of a collection of nodes with learning abilities that interact with each other locally in order to solve distributed processing and distributed inference problems in real-time. Various algorithms and performance analyses have been put forward for such networks, such as the adapt-then-combine (ATC) and combine-then-adapt (CTA) diffusion algorithms, the probabilistic diffusion algorithm, and diffusion with adaptive weights over the links. In this paper, we add mobility as another dimension and study the behavior of the network when the nodes move in pursuit/avoidance of a target. Mobility leads naturally to an adaptive topology with changing neighborhoods. Mobility also imposes physical constraints on the proximity among the nodes and on the velocity and location of the center of the network. We develop adaptation algorithms that exhibit self-organization properties and apply them to the modeling of collective behavior in biological systems, such as fish schooling. The results help provide an explanation for the agile adjustment of network patterns of fish schools in the presence of predators.**

***Index Terms*—Mobile adaptive networks, self-organization, diffusion adaptation, tracking, fish schooling, predator avoidance.**

I. Introduction

Self-organization is observed in many physical and biological systems [1][2]. In such systems, a global pattern of behavior emerges from the limited and localized interactions among the individual members of the system. One interesting organized behavior in animal groups is their collective motion, where animals move together in amazing synchrony such as fish schools swimming together [3], bees swarming towards a hive, or birds flying in V-formations [5].

In fish schools, the individual members tend to have similar speeds and to move almost in parallel while keeping a safe distance from their neighbors to avoid collisions. There are several biological hypotheses to explain how fish take advantage of schooling to avoid the presence of their predators. Fish forming schools confuse predators more easily and the individual member becomes harder to detect and track by predators [6]. This effect is called the dilution effect [7][8]. Another advantage of schooling is the many-eyes effect [9]. Fish within a school collaboratively detect predators such that the probability of detection increases appreciably. The school as a group can react and take actions earlier than what would be possible for a single fish.

In this paper, we apply the diffusion adaptation algorithms of [10]-[13] to explain how fish cooperatively pursue a food source while at the same time avoiding attack from a predator. It is observed in nature that fish schools spread out to escape from predators and regroup to continue with their schooling. We use the concept of adaptive networks, along with diffusion adaptation mechanisms, to explain how the behavior of each individual fish (or agent) contributes to this highly structured schooling behavior in its avoidance of mobile predators.

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II. Measurement Model

Let w° denote the location vector of a target that the network wishes to track (e.g., the location of a food source). As Fig. 1 shows, the distance between the target and node k at location $x_{k,i}$ at time i is given by the inner product

$$d_k^\circ(i) = u_{k,i}^\circ(w^\circ - x_{k,i}) \tag{1}$$

where $x_{k,i}$ denotes the location vector relative to some global coordinate system, and $u_{k,i}^\circ$ denotes the unit direction vector pointing to w° from $x_{k,i}$; this vector is defined in terms of the azimuth angle, $\theta_k(i)$, i.e.,

$$u_{k,i}^\circ = \begin{bmatrix}\cos\theta_k(i) & \sin\theta_k(i)\end{bmatrix} \tag{2}$$

The superscript $\circ$ in (1)-(2) is used to indicate true values. However, nodes observe noisy measurements of the direction $u_{k,i}^\circ$ and the distance $d_k^\circ(i)$ to the target, say,

$$u_{k,i} = u_{k,i}^\circ + n_{k,i}^u \tag{3}$$
$$d_k(i) = d_k^\circ(i) + n_k^d(i) \tag{4}$$

where $n_{k,i}^u$ and $n_k^d(i)$ denote additive noise terms of sizes M and 1, respectively. Rearranging the above equations, we obtain a linear regression model, namely,

$$\begin{aligned}\hat{d}_k(i) &\triangleq d_k(i) + u_{k,i}x_{k,i}\\ &= u_{k,i}w^\circ + n_k(i)\end{aligned} \tag{5}$$

where the scalar noise term $n_k(i)$ is given by

$$n_k(i) \triangleq -n_{k,i}^u(w^\circ - x_{k,i}) + n_k^d(i)$$

In addition, the measured location of the target by node k is denoted by $q_{k,i}$ and determined from $\{d_k(i), u_{k,i}, x_{k,i}\}$ as follows:

$$\begin{aligned}q_{k,i} &= x_{k,i} + d_k(i)u_{k,i}^T\\ &= w^\circ + \eta_{k,i}\end{aligned} \tag{6}$$

where the vector noise term is given by

$$\eta_{k,i} = n_k^d(i)u_{k,i}^{\circ T} + d_k^\circ(i)n_{k,i}^{uT} + n_k^d(i)n_{k,i}^{uT} \tag{7}$$

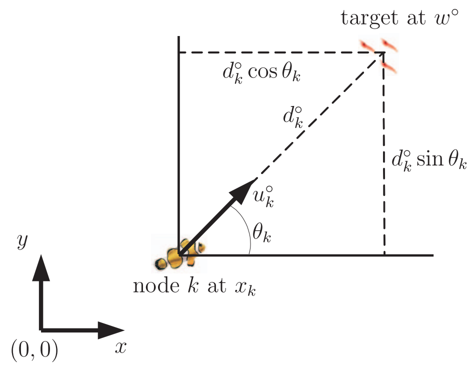

Fig. 1. Distance and direction of the target w° from node k at location x_k. The unit direction vector u_k° points towards w°.

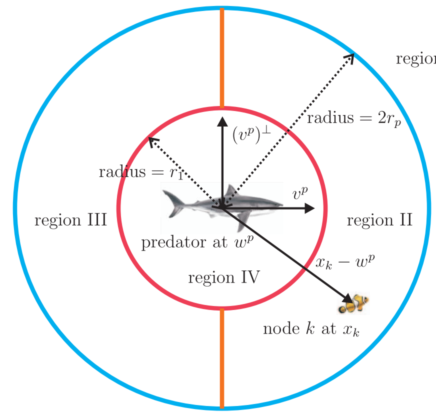

Fig. 2. Four regions around the predator.

We assume that $\eta_{k,i}$ is a zero mean white random process with covariance matrix $C_{k,i}$.

In the application we are studying in this paper, the nodes of the network wish to track two separate targets: the location of the food source and the location of a predator. The modeling equations described above apply to either target. To distinguish between them, we shall use superscripts f and p for food and predator, respectively (thus, instead of w°, we shall write w^f for the location of the food source and w^p for the location of the predator). Moreover, the superscript n will represent values related to the adaptive network.

III. Motion Control Mechanism

In a mobile network, every node k will update its location vector over time according to the rule:

$$x_{k,i+1} = x_{k,i} + \triangle t \cdot v_{k,i+1} \tag{8}$$

where $\triangle t$ represents the time step and $v_{k,i+1}$ is the velocity vector of the node. Several factors influence the velocity vector of node k such as (a) the desire to move away from a predator at location w^p, (b) the desire to move towards a food source at w^f, (c) the desire to move in coordination with the other nodes, and (d) the desire to avoid collisions. We assume the predator is moving as well, so that its actual location should be denoted by w_i^p. The nodes in the network are therefore interested in estimating the fixed quantity w^f and in tracking the time-variant quantity w_i^p. Since the nodes do not have access to the actual locations of the food and the predator, we will use $w_{k,i}^f$ and $w_{k,i}^p$ to denote the local estimates at node k at time i. We now explain the mechanism by which $v_{k,i+1}$ can be set by node k.

A. Pursuing Food and Avoiding Predators

To begin with, the nodes in the network would like to get to the food source and avoid the predator. The action of each node depends on the location of the predator. Figure 3 shows four regions around the predator. There are two concentric circles with their centers origin at the predator and with radii r_p and $2r_p$. The four regions represent the areas outside the circle of radius $2r_p$, inside the circle of radius r_p, and in front and behind the predator within the disc $r_p < r < 2r_p$. If the predator is far away (i.e., if the distance from node k to the predator is larger than $2r_p$, meaning $d_k^p(i) > 2r_p$), the fish stays in region I and focuses on exploring the food location. In this case, the velocity vector is set along the direction of the food, i.e.,

$$v_{k,i+1}^a = \frac{w_{k,i}^f - x_{k,i}}{\|w_{k,i}^f - x_{k,i}\|} \quad \text{(region I)} \tag{9}$$

On the other hand, if the predator is close, (i.e., if $d_k^p(i) < r_p$ so that the node is in region IV), then the node focuses on escaping the attack by the predator by moving away from it. In this case, the velocity vector is chosen as

$$v_{k,i+1}^a = \left(\frac{r_p}{\|x_{k,i} - w_{k,i}^p\|} - 1\right)(x_{k,i} - w_{k,i}^p) \quad \text{(region IV)} \tag{10}$$

In (10), the speed of the node depends on the distance to the predator. The node will move faster if the predator is closer to it. The final situation we need to consider is when the predator is located at a distance between r_p and $2r_p$ from the node. There are two possible regions in this case. If the predator is moving towards the node (i.e., if the node lies in region II), then the node should stop foraging and follow its neighbors (see section III.C). The velocity vector in this case becomes

$$v_{k,i+1}^a = 0 \quad \text{(region II)} \tag{11}$$

Likewise, if the predator is moving away, the node (i.e., if the node lies in region III), then the node should move in the opposite direction of the predator maintain a safe distance from the predator. The velocity vector would be set as:

$$v_{k,i+1}^a = -\frac{v_{k,i}^p}{\|v_{k,i}^p\|} \quad \text{(region III)} \tag{12}$$

where $v_{k,i}^p$ is the estimate by node k of the velocity of the predator. Note that the two regions II and III can be distinguished by the value of the inner product $(x_{k,i} - w_{k,i}^p)^T v_{k,i}^p$. The predator moves towards node k if the value is greater than zero, and vice versa.

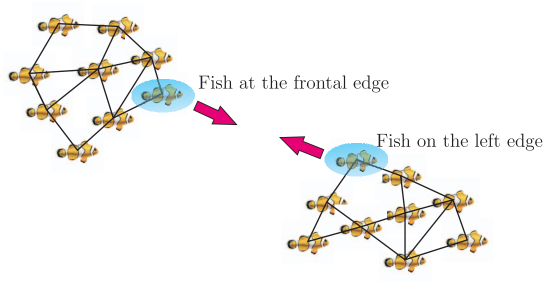

Fig. 3. Two fragmental groups. Connections among the fish are indicated by lines. One fish at the frontal edge (left group) and one fish on the left edge (right group) are highlighted. They will move along the arrow directions to cause regrouping.

B. Reunion

Following an attack by a predator, a network becomes fragmented with some nodes lying at the outer edges of the new smaller groups that resulted from the fragmentation. To reunite, nodes on the outer boundaries have to estimate the location of the other groups and move towards them. To do so, a node first needs to detect whether it is on the edge of the fragmented groups. We consider three kinds of edges - frontal edge, left edge and right edge. The node computes the number of its neighbors in each direction according to the coordinate of node l with respect to node k:

$$x_{l,i}^{(k)} = W(v_{k,i})^T(x_{l,i} - x_{k,i}) \tag{13}$$

where

$$W(v) = \begin{bmatrix} v_1/\|v\| & -v_2/\|v\| \\ v_2/\|v\| & v_1/\|v\| \end{bmatrix} \tag{14}$$

is an orthonormal matrix for a local coordinate system centered at a node that is moving with velocity vector $v = (v_1, v_2)$. If the first coordinate of $x_{l,i}^{(k)}$ is greater than zero, node l lies in front of node k. Similarly, if the second coordinate of $x_{l,i}^{(k)}$ is greater than zero, node l lies to the left of node k; otherwise, it lies to the right side of node k. We say node k belongs to the frontal edge of the fragmented groups if the number of neighbors in the front is less than one. Likewise for the left and right edges. Nodes on the edge then search for other groups. For example, nodes in the front edge will find the nearest frontal node outside its neighborhood and move towards that node. That is, node k would perform the following operation:

$$\hat{l} = \arg\min_{l}\{\|x_{l,i}^{(k)}\| | l \in \mathcal{N}_k^F \setminus \mathcal{N}_k\} \tag{15}$$

$$v_{k,i+1}^b = \begin{cases} 0, & \text{if } \hat{l} = \phi \\ \frac{x_{\hat{l},i} - x_{k,i}}{\|x_{\hat{l},i} - x_{k,i}\|}, & \text{otherwise} \end{cases} \tag{16}$$

where $\mathcal{N}_k^F$ is the set of indexes of nodes that lie in the front of node k and ϕ denotes the empty set. Nodes in the left and right edges conduct the same procedure.

C. Coherent motion

The nodes do not only want to approach to the food source and avoid the predator, they also want to move in harmony to confuse the predator and would like to avoid collisions by maintaining a safe distance r from their neighbors. This can be achieved if the node updates its velocity vector as follows [4]:

$$v_{k,i+1}^c = v_{k,i}^g + \gamma\delta_{k,i} \tag{17}$$

where γ is a nonnegative scalar and

$$\delta_{k,i} = \frac{1}{|\mathcal{N}_k| - 1}\sum_{l\in\mathcal{N}_k\setminus\{k\}}\left(1 - \frac{r}{\|x_{l,i} - x_{k,i}\|}\right)(x_{l,i} - x_{k,i}) \tag{18}$$

The term $\delta_{k,i}$ suggests that nodes should adjust their velocity direction to be consistent with the average of displacement vectors, $\{x_{l,i} - x_{k,i}\}$, in the neighborhood while maintaining a distance r from their neighbors. Expression (17) also incorporates the term $v_{k,i}^g$, which refers to a local estimate for the velocity of the center of gravity of the network, v^g, which is defined as

$$v^g \triangleq \frac{1}{N}\sum_{l=1}^{N} v_l \tag{19}$$

Based on the preceding criteria, we assume that nodes adjust their velocity vectors as follows:

$$\boxed{\begin{aligned} v_{k,i+1} = \lambda \cdot I_{k,i}(\alpha v_{k,i+1}^a + \beta v_{k,i+1}^b) \\ + (1 - \lambda \cdot I_{k,i})v_{k,i}^g + \gamma\delta_{k,i} \end{aligned}} \tag{20}$$

where $\{\lambda, \alpha, \beta, \gamma\}$ are non-negative weighting factors and $I_{k,i}$ is an indicator function, whose value is equal to 0 if both $v_{k,i+1}^a$ and $v_{k,i+1}^b$ are equal to zero (i.e., node k is in region II and does not need to reunite); otherwise it is equal to 1. In addition, we bound the maximum speed of nodes by v_{max}. That is, the magnitude of $v_{k,i+1}$ will be scaled to v_{max} if it is larger than v_{max}.

Moving forward, we assume that every node in the network is adjusting its velocity vector according to (20). We now develop diffusion mechanisms that allow the nodes to obtain the local estimates $w_{k,i}^p$, $v_{k,i}^p$, $w_{k,i}^f$, and $v_{k,i}^g$ in a distributed manner and in real-time.

IV. Estimation of Global Variables

A. Estimating w^p, v^p, and w^f

The algorithms to estimate w^p and w^f are the same. Here, we only show how to estimate w^p. At every time instant i, every node k has access to the local measurements $\{\hat{d}_k^p(i), u_{k,i}^p\}$ in (5). The nodes would like to estimate in a distributed manner the global parameter w^p that minimizes the following cost function:

$$J^p(w^p) = \sum_{k=1}^{N} E|\hat{\boldsymbol{d}}_k^p(i) - \boldsymbol{u}_{k,i}^p w^p|^2 \tag{21}$$

Applying the Adapt-then-Combine (ATC) diffusion algorithm [11], we have

$$\begin{cases} \psi_{k,i}^p = w_{k,i-1}^p + \mu_k^p u_{k,i}^{pT}[\hat{d}_k^p(i) - u_{k,i}^p w_{k,i-1}^p] & \text{(22a)} \\ w_{k,i}^p = \sum_{l\in\mathcal{N}_k} a_{l,k}^p \psi_{l,i}^p & \text{(22b)} \end{cases}$$

where the coefficients $\{a^p_{l,k}\}$ satisfy

$$\sum_{l=1}^{N} a^p_{l,k} = 1 \quad a^p_{l,k} = 0 \text{ if } l \notin \mathcal{N}_k \tag{23}$$

Since our model (5) is geometry-bearing (see (1)), we can exploit this fact to simplify (22a) under reasonable approximations:

$$w^p_{k,i-1} - x_{k,i} \approx \|w^p_{k,i-1} - x_{k,i}\| u^{pT}_{k,i} \tag{24}$$

We then can rewrite (22) as

$$\boxed{\begin{aligned} \psi^p_{k,i} &= (1-\mu^p_k) w^p_{k,i-1} + \mu^p_k q^p_{k,i} \\ w^p_{k,i} &= \sum_{l \in \mathcal{N}_k} a^p_{l,k} \psi^p_{l,i} \end{aligned}} \tag{25}$$

The update is simply a convex combination of the current measurements $\{q^p_{k,i}\}$ and the previous estimate of the target location $w^p_{k,i-1}$. The estimation for w^f can be implemented in the same way by replacing the superscript p by f. In addition, we estimate the velocity of the predator as follows:

$$\boxed{v^p_{k,i} = \frac{1}{\triangle t}(w^p_{k,i} - w^p_{k,i-1})} \tag{26}$$

B. Estimating v^g

The velocity of the center of gravity, v^g, should be also estimated in a distributed manner. By definition, v^g is the average velocity of all nodes in the network as in (19). Consider the global cost function for estimating v^g:

$$J^v(v^g) = \sum_{k=1}^{N} \|v_{k,i} - v^g\|^2 \tag{27}$$

Using the same diffusion structure (22), and the arguments in [11], we can arrive at the following diffusion algorithm for computing $v^g_{k,i}$:

$$\boxed{\begin{aligned} \varphi_{k,i} &= (1-\mu^v_k) v^g_{k,i-1} + \mu^v_k v_{k,i} \\ v^g_{k,i} &= \sum_{l \in \mathcal{N}_k} a^v_{l,k} \varphi_{l,i} \end{aligned}} \tag{28}$$

where $\{a^v_{l,k}\}$ is a set of non-negative real coefficients satisfying (23) and the superscript v is used to indicate that these coefficients are for the estimation problem involving v^g.

V. Behavior of the Predator

The predator tracks the location of one node at a time. We assume that the predator keeps tracking the nearest node. At each time instant i, the predator measures the distance, $d^n(i)$, and direction, u^n_i, of the nearest node. These observations are then used to compute the measured location of the node

$$q^n_i = w^p_i + d^n(i) u^{nT}_i \tag{29}$$

The predator then updates the location of the node according to

$$w^n_i = (1-\nu) w^n_{i-1} + \nu q^n_i \tag{30}$$

After estimating the location, the predator moves towards the node. That is, the velocity and location vectors of the predator are updated as:

$$v^p_{i+1} = c \cdot v_{max} \frac{w^n_i - w^p_i}{\|w^n_i - w^p_i\|} \tag{31}$$

$$w^p_{i+1} = w^p_i + \triangle t \cdot v^p_{i+1} \tag{32}$$

where c is a positive scalar to control the speed of the predator.

VI. Simulation Results

In this section, we simulate the motion of mobile networks with 50 nodes and illustrate the effectiveness of the algorithms. We first specify the neighbors of a node. Let R represent the maximum distance within which two nodes can communicate successfully. All nodes within a radius R of one node are candidate neighbors. However, to reduce computational and communication overhead, the number of neighbors will be constrained, say to N_B. In this paper, a node chooses its neighbors from the nearest ones.

The simulation parameters are set as follows. The unit length is the body length of a node (e.g., body length of a fish). All step sizes are set to 0.5. The combination coefficients are set to $a^p_{l,k} = a^f_{l,k} = a^v_{l,k} = 1/|\mathcal{N}_{k,i}|$ if $l \in \mathcal{N}_{k,i}$. For velocity control, the coefficients are $\lambda = 0.5$, $\alpha = 1$, $\beta = 2$ and $\gamma = 1$. Moreover, the time duration is $\triangle t = 0.5$ sec. In addition, we set $R = 5$, $N_B = 6$, the optimal distance between two neighbors is $r = 3$, and the the distance r_p is equal to 10.

We illustrate the maneuver of a mobile network in $\mathbb{R}^2$ over time in Fig. 4. The symbol, "■", denotes the target of interest. In addition, "•" and "–" indicate the locations and moving directions of the nodes, respectively. The ones with bigger sizes represent the predator. We observe that the nodes in the network move harmoniously and approach the food source. When the predator tries to attack them, the nodes spread out and regroup after the attack. The simulation results regenerate the behavior of fish schools in nature.

VII. Concluding Remarks

In this paper, we proposed a diffusion adaptation model to simulate the behavior of fish schools in the presence of a predator. With the aid of diffusion adaptation, the algorithm is implemented in a distributed manner and in real time. Each node only communicates with its immediate neighbors. Simulation results regenerate rather well the maneuver behavior of fish in nature.

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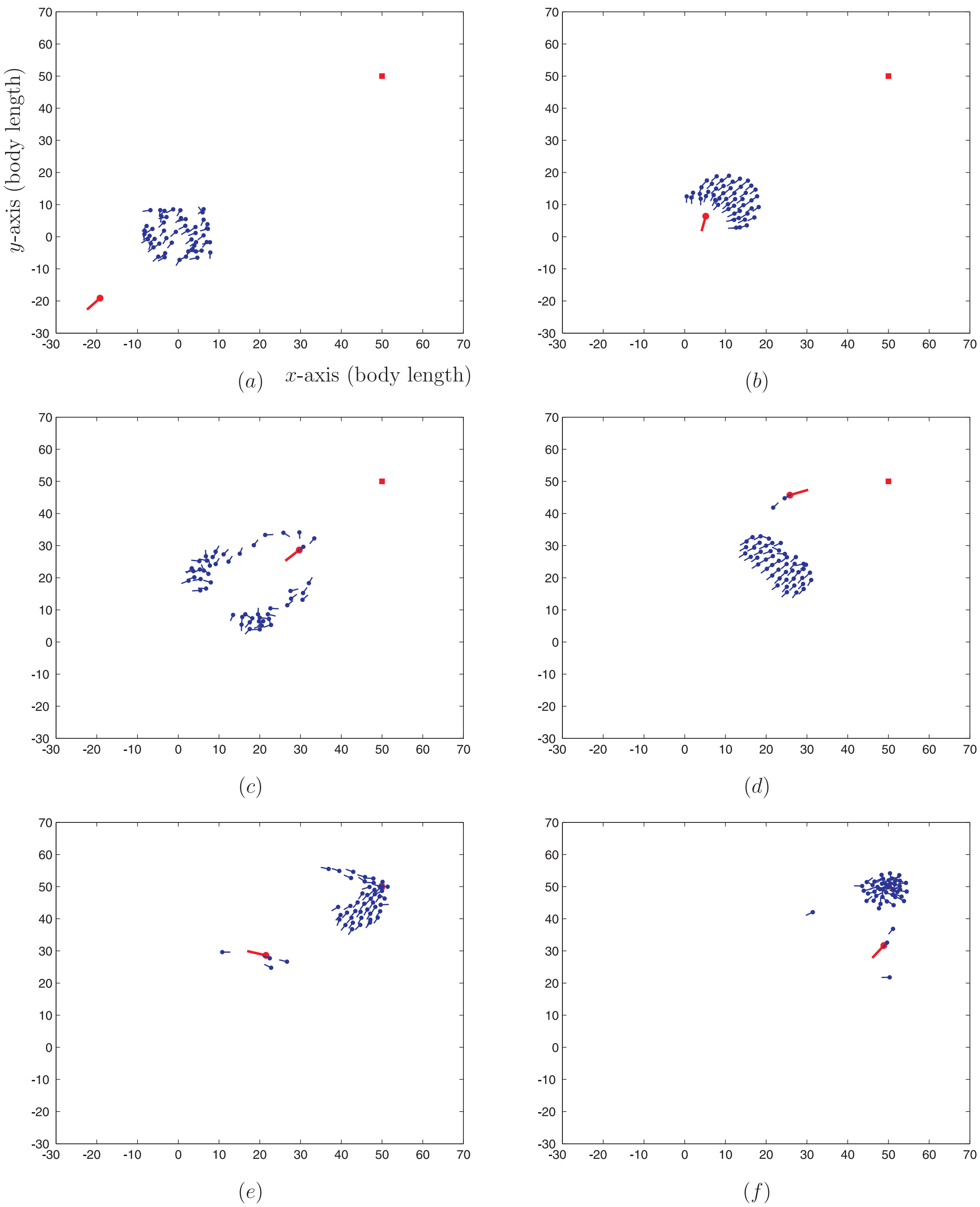

Fig. 4. Maneuvers of mobile networks in $\mathbb{R}^2$ over time: (a) 0 sec, (b) 15 sec, (c) 30 sec, (d) 45 sec, (e) 105 sec, and (e) 120 sec.

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