Joint Compensation of IQ Imbalance and Phase Noise in OFDM Systems

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***Abstract*— The joint effects of IQ imbalance and phase noise on OFDM systems are analyzed, and a compensation scheme is proposed to improve the system performance. The scheme consists of a joint channel estimation algorithm and a joint data symbol estimation algorithm. In the proposed channel estimation algorithm, the channel coefficients are jointly estimated with the IQ imbalance parameters and the phase noise components. Its performance is demonstrated to be close to the associated Cramer-Rao lower bound. In the proposed data symbol estimation algorithm, the joint compensation is decomposed into IQ imbalance compensation and phase noise compensation. It is shown both by theory and computer simulations that the proposed scheme can effectively improve the signal-to-noise ratio at the receiver. As a result, the sensitivity of OFDM receivers to the physical impairments can be significantly lowered, simplifying the RF and analog circuitry design in terms of implementation cost, power consumption, and silicon fabrication yield.**

I. Introduction

The OFDM modulated communication is susceptible to the impairments caused by the imperfectness in the radio-frequency (RF) signal down-conversion process. Its effects have been modeled as IQ imbalance and phase noise in the literature [1]. IQ imbalance is the mismatch in amplitude and phase between the I and Q branches in the receiver chain, while phase noise is the random unknown phase difference between the phase of the carrier signal and the phase of the local oscillator. The effects of IQ imbalance and phase noise on OFDM receivers have been investigated in previous works, such as [2], [3]. Some algorithms have also been developed for the compensation of IQ imbalance [4], [5] or the compensation of phase noise [6]–[8], separately. In [9], the joint effects of IQ imbalance and phase noise on OFDM systems were studied, but the analysis and proposed compensation scheme were based on the concatenation model of IQ imbalance and phase noise, where only the common error term of phase noise was considered.

In this paper, we pursue an explicit formulation for the joint effects of IQ imbalance and phase noise, and propose a joint compensation scheme with performance analysis. The scheme consists of a joint channel estimation algorithm and a joint data symbol estimation algorithm. In the channel estimation algorithm, block-type pilot symbols are transmitted periodically, and the channel coefficients are jointly estimated with the IQ imbalance parameters and phase noise components[1]. Instead of estimating the channel coefficients and phase noise in the frequency domain, we estimate them in the time domain by using interpolation techniques to reduce the number of unknowns. The joint estimation technique achieves a more accurate channel estimate than other conventional methods that either ignore the impairments or simply model them as additive Gaussian noise. The mean-squared errors of channel estimation are compared with their associated Cramer-Rao lower bounds, which shows that our scheme works well with performance close to the ideal case without the impairments. In the proposed data symbol estimation algorithm, it is shown that the joint compensation can be decomposed into the IQ imbalance compensation followed by the phase noise compensation. In the payload portion of OFDM packets, which contains both data tones and pilot tones, the data symbols and the phase noise components are jointly estimated at the receiver. The performance of the proposed algorithm is analyzed in terms of the improvements in the effective signal-to-noise ratio, and is compared with other compensation methods.

Throughout this paper, $(\cdot)^T$ denotes the matrix transpose, $(\cdot)^*$ represents the matrix conjugate transpose, and $\text{conj}\{\cdot\}$ takes the complex conjugate of its argument elementwisely. $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ return the real and imaginary parts of its argument, respectively. $\text{Tr}\{\cdot\}$ returns the trace of a matrix. $\mathbf{E}\{\cdot\}$ is the expected value with respect to the underlying probability measure. $\mathbf{I}_K$ is the identity matrix of size $K \times K$, and $\mathbf{I}_{\boldsymbol{\theta}}$ is the Fisher information matrix associated with the parameter vector $\boldsymbol{\theta}$.

II. System Model

At the OFDM transmitter, the information bits are first mapped into constellation symbols, and then converted into a block of N symbols $x[k]$, $k = 0, 1, \ldots, N-1$, by a serial-to-parallel converter. The N symbols are the frequency components to be transmitted using the N subcarriers of the OFDM modulator, and are converted to OFDM symbols by the unitary inverse Fast Fourier Transform (IFFT). After adding a cyclic prefix of length P, the resulting $N+P$ time-domain

[1]All standardized OFDM systems today provide such full pilot symbols at the beginning of every packet. Therefore, the proposed scheme does not require any modification to the packet structure and can be applied to existing standards.

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symbols are converted into a continuous-time signal $x(t)$ for transmission.

Fig. 1 shows the block diagram of an RF receiver with IQ imbalance α, θ and phase noise $\phi(t)$. Let T_s be the sampling period. It can be shown that the output symbols $y[k]$, $k = 0, 1, \ldots, N-1$, after OFDM demodulation are related to the data symbols $x[k]$, $k = 0, 1, \ldots, N-1$, by

$$y[k] = \mu \sum_{r=0}^{N-1} a[r] H[(k-r)_N] x[(k-r)_N] \quad (1)$$
$$+ \nu \sum_{r=0}^{N-1} a^*[r] H^*[(N-k-r)_N] x^*[(N-k-r)_N] + w[k],$$

where $(k)_N$ stands for $(k \bmod N)$, μ and ν account for the IQ imbalance and are related to α and θ by [5]

$$\mu = \cos(\theta/2) - j\alpha \sin(\theta/2), \;\; \nu = \alpha \cos(\theta/2) + j \sin(\theta/2),$$

$a[r]$, $r = 0, 1, \ldots, N-1$, are determined by the phase noise through

$$a[r] = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\phi(nT_s)} e^{-j\frac{2\pi rn}{N}}, \quad (2)$$

$H[k]$, $k = 0, 1, \ldots, N-1$, are the discrete-time Fourier transform of the baseband channel impulse response $h[n]$, $n = 0, 1, \ldots, L-1$, i.e.,

$$H[k] = \sum_{n=0}^{L-1} h[n] e^{-j\frac{2\pi kn}{N}}, \quad (3)$$

and $w[k]$ is the additive noise in the k^{th} subcarrier.

Using matrix notation, (1) can be represented by

$$\mathbf{y} = \mu \mathbf{A}\mathbf{H}\mathbf{x} + \nu \widetilde{\mathbf{A}}\widetilde{\mathbf{H}}\widetilde{\mathbf{x}} + \mathbf{w}, \quad (4)$$

where

$$\mathbf{y} = \begin{bmatrix} y[0] & y[1] & \ldots & y[N-1] \end{bmatrix}^T,$$
$$\mathbf{x} = \begin{bmatrix} x[0] & x[1] & \ldots & x[N-1] \end{bmatrix}^T,$$
$$\widetilde{\mathbf{x}} = \begin{bmatrix} x^*[0] & x^*[1] & \ldots & x^*[N-1] \end{bmatrix}^T,$$
$$\mathbf{A} = \begin{bmatrix} a[0] & a[N-1] & \ldots & a[1] \\ a[1] & a[0] & \ldots & a[2] \\ \vdots & \vdots & \ddots & \vdots \\ a[N-1] & a[N-2] & \ldots & a[0] \end{bmatrix},$$

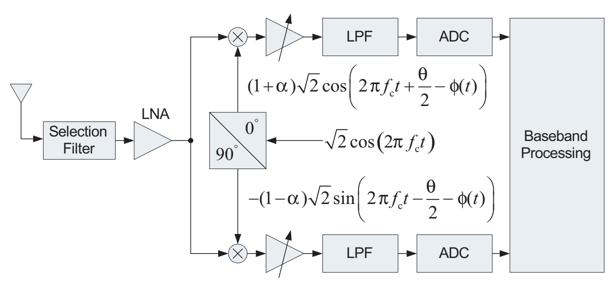


Fig. 1. An RF receiver with IQ imbalance α, θ and phase noise $\phi(t)$.

$$\widetilde{\mathbf{A}} = \begin{bmatrix} a^*[0] & a^*[N-1] & \ldots & a^*[1] \\ a^*[N-1] & a^*[N-2] & \ldots & a^*[0] \\ \vdots & \vdots & \ddots & \vdots \\ a^*[1] & a^*[0] & \ldots & a^*[2] \end{bmatrix},$$
$$\mathbf{H} = \text{diag}\{H[0],\; H[1],\; \ldots,\; H[N-1]\},$$
$$\widetilde{\mathbf{H}} = \text{diag}\{H^*[0],\; H^*[1],\; \ldots,\; H^*[N-1]\},$$
$$\mathbf{w} = \begin{bmatrix} w[0] & w[1] & \ldots & w[N-1] \end{bmatrix}^T.$$

III. Channel Estimation

In the proposed channel estimation algorithm, block-type pilot symbols are transmitted, in which all subcarriers are used for the pilot symbols known to the receiver. For convenience of exposition, we assume that each time only *one* OFDM symbol is used as the block-type pilot symbol for channel estimation. Since the OFDM demodulation output $\mathbf{y}$ is related to the training symbol $\mathbf{x}$ through expression (4), the proposed algorithm is based on the following optimization problem:

$$\min_{\mu, \nu, \mathbf{A}, \mathbf{H}} \| \mathbf{y} - \mu \mathbf{A}\mathbf{H}\mathbf{x} - \nu \widetilde{\mathbf{A}}\widetilde{\mathbf{H}}\widetilde{\mathbf{x}} \|^2. \quad (5)$$

We notice that there are N unknowns in $\mathbf{H}$, N unknowns in $\mathbf{A}$, plus two additional unknowns μ and ν. Thus, the solution to this problem is not unique. To overcome this difficulty, we can reduce the number of unknowns by properly modeling the channel and the phase noise process with fewer parameters, as proposed in [8]. Since the length L of the discrete-time baseband channel impulse response is normally less than the OFDM symbol size N, we can relate $H[k]$, $k = 0, 1, \ldots, N-1$, to $h[n]$, $n = 0, 1, \ldots, L-1$, through

$$\mathbf{h} = \mathbf{F_h}\mathbf{h}',$$

where

$$\mathbf{h} = \begin{bmatrix} H[0] & H[1] & \ldots & H[N-1] \end{bmatrix}^T,$$
$$\mathbf{h}' = \begin{bmatrix} h[0] & h[1] & \ldots & h[L-1] \end{bmatrix}^T,$$

and $\mathbf{F_h}$ is the discrete Fourier transform matrix of appropriate size according to (3). Instead of estimating $\mathbf{h}$, we can estimate $\mathbf{h}'$. This reduces the number of unknown channel coefficients from N to L.

For the phase noise, instead of estimating $a[k]$, $k = 0, 1, \ldots, N-1$, we can estimate the phase noise components in the time domain, i.e., $e^{j\phi(nT_s)}$, $n = 0, 1, \ldots, N-1$. In order to reduce the number of unknowns, we can estimate $e^{j\phi(m(N-1)T_s/(M-1))}$ for $m = 0, 1, \ldots, M-1$ ($M < N$), and then obtain the approximation of $e^{j\phi(nT_s)}$, $n = 0, 1, \ldots, N-1$, by interpolation. Let

$$\mathbf{c} = \begin{bmatrix} e^{j\phi(0)} & e^{j\phi(T_s)} & \ldots & e^{j\phi((N-1)T_s)} \end{bmatrix}^T,$$
$$\mathbf{c}' = \begin{bmatrix} e^{j\phi(0)} & e^{j\phi\left(\frac{(N-1)T_s}{M-1}\right)} & \ldots & e^{j\phi((N-1)T_s)} \end{bmatrix}^T.$$

Then,

$$\mathbf{c} \approx \mathbf{P}\mathbf{c}'$$

where $\mathbf{P}$ is an interpolation matrix[2]. Using (2), we have

$$\mathbf{a} = \frac{1}{N}\mathbf{F_a c} \approx \frac{1}{N}\mathbf{F_a P c}', \tag{6}$$

where

$$\mathbf{a} = \begin{bmatrix} a[0] & a[1] & \dots & a[N-1] \end{bmatrix}^T,$$

and $\mathbf{F_a}$ is the discrete Fourier transform matrix. Instead of estimating $\mathbf{a}$, we can estimate $\mathbf{c}'$, which reduces the number of unknowns from N to M.

Moreover, we realize that in (5) there exists an ambiguity of a scaling factor among the estimates of μ, $\mathbf{A}$ and $\mathbf{H}$. To resolve the ambiguities, we add the following two constraints to the original problem:

$$\mu = 1 \text{ and } a[0] = 1.$$

Consequently, knowing $\mathbf{x}$ and $\mathbf{y}$, we can estimate $\mathbf{H}$ by solving

$$\min_{\nu,\mathbf{c}',\mathbf{h}'} \| \mathbf{y} - \mathbf{AHx} - \nu\widetilde{\mathbf{A}}\widetilde{\mathbf{H}}\widetilde{\mathbf{x}} \|^2 \text{ subject to } a[0] = 1.$$

The optimization problem is nonlinear and nonconvex. A sub-optimal solution can be found by using numerical methods.

A. Cramer-Rao Lower Bound (CRLB)

To evaluate the proposed algorithm, we compare its performance with the Cramer-Rao lower bound (CRLB) that gives a lower bound on the covariance matrix of any unbiased estimator of unknown parameters. In the following derivation, it is assumed that 1) all pilot symbols $x[k]$ have the same power and let $\sigma_p^2 = \mathbf{E}\{|x[k]|^2\}$; 2) the pilot symbols, the phase noise, the channel coefficients and the additive noise are independent of each other; 3) the channel coefficients $H[k]$ are independently identically distributed and circularly symmetric Gaussian with mean zero and variance $\sigma_H^2 = \mathbf{E}\{|H[k]|^2\}$; 4) the additive noise $\mathbf{w}$ is circularly symmetric Gaussian with covariance matrix $\sigma_w^2\mathbf{I}_N$.

Scenario 1: No Impairment

In this scenario, we consider two cases: one tries to estimate $\mathbf{h}$ and the other tries to estimate $\mathbf{h}'$. If $\mathbf{h}$ is directly estimated, the CRLB for estimating $H[k]$ is

$$\mathbf{E}\{|\widehat{H}[k] - H[k]|^2\} \geq \frac{\sigma_w^2}{\sigma_p^2}.$$

If $\mathbf{h}'$ is estimated instead, the CRLB for estimating $H[k]$ is

$$\mathbf{E}\{|\widehat{H}[k] - H[k]|^2\} \geq \frac{L\sigma_w^2}{N\sigma_p^2}.$$

Scenario 2: Without any Compensation when Both IQ Imbalance and Phase Noise are Present

If $\mathbf{h}$ is estimated, the system model is given by

$$\mathbf{y} = \mu a[0]\mathbf{Hx} + \mu(\mathbf{A} - a[0]\mathbf{I}_N)\mathbf{Hx} + \nu\widetilde{\mathbf{A}}\widetilde{\mathbf{H}}\widetilde{\mathbf{x}} + \mathbf{w}.$$

Since there exists a scalar ambiguity among the estimates of μ, $\mathbf{A}$ and $\mathbf{H}$, we treat $\mu a[0]\mathbf{H}$ as the "true" channel response to be estimated. The term $\mu(\mathbf{A} - a[0]\mathbf{I}_N)\mathbf{Hx} + \nu\widetilde{\mathbf{A}}\widetilde{\mathbf{H}}\widetilde{\mathbf{x}} + \mathbf{w}$ can be *approximately* regarded as additive Gaussian noise with covariance matrix $\{\sigma_w^2 + [(1-\sigma_{a,0}^2)|\mu|^2 + |\nu|^2]\sigma_H^2\sigma_p^2\} \times \mathbf{I}_N$, where $\sigma_{a,0}^2 = \mathbf{E}\{|a[0]|^2\}$. The CRLB can then be computed as

$$\begin{aligned}&\mathbf{E}\{|\mu a[0]\widehat{H}[k] - \mu a[0]H[k]|^2\} \\ &\qquad \geq \frac{\sigma_w^2}{\sigma_p^2} + [(1-\sigma_{a,0}^2)|\mu|^2 + |\nu|^2]\sigma_H^2.\end{aligned}$$

Similarly, if $\mathbf{h}'$ is estimated, then

$$\begin{aligned}&\mathbf{E}\{|\mu a[0]\widehat{H}[k] - \mu a[0]H[k]|^2\} \\ &\qquad \geq \frac{L\sigma_w^2}{N\sigma_p^2} + \frac{L}{N}[(1-\sigma_{a,0}^2)|\mu|^2 + |\nu|^2]\sigma_H^2.\end{aligned}$$

Scenario 3: With Perfect Knowledge of μ and ν but No Compensation for Phase Noise

In this case, if $\mathbf{h}$ is estimated,

$$\begin{aligned}&\mathbf{E}\{|\mu a[0]\widehat{H}[k] - \mu a[0]H[k]|^2\} \\ &\qquad \geq \frac{|\mu|^2\sigma_w^2}{(|\mu|^2 + |\nu|^2)\sigma_p^2} + (1-\sigma_{a,0}^2)|\mu|^2\sigma_H^2.\end{aligned}$$

If $\mathbf{h}'$ is estimated, then

$$\begin{aligned}&\mathbf{E}\{|\mu a[0]\widehat{H}[k] - \mu a[0]H[k]|^2\} \\ &\qquad \geq \frac{L|\mu|^2\sigma_w^2}{N(|\mu|^2 + |\nu|^2)\sigma_p^2} + \frac{L}{N}(1-\sigma_{a,0}^2)|\mu|^2\sigma_H^2.\end{aligned}$$

Scenario 4: With the Proposed Joint Estimation when Both IQ Imbalance and Phase Noise are Present

In this case, the CRLB for estimating $\mathbf{H}$ is computed based on the following model:

$$\begin{aligned}\mathbf{y} &= \mu\mathbf{A}_{\text{appro}}\mathbf{Hx} + \nu\widetilde{\mathbf{A}}_{\text{appro}}\widetilde{\mathbf{H}}\widetilde{\mathbf{x}} + \mu(\mathbf{A} - \mathbf{A}_{\text{appro}})\mathbf{Hx} \\ &\quad + \nu(\widetilde{\mathbf{A}} - \widetilde{\mathbf{A}}_{\text{appro}})\widetilde{\mathbf{H}}\widetilde{\mathbf{x}} + \mathbf{w},\end{aligned}$$

where $\mathbf{h} = \mathbf{F_h h}'$, and $\mathbf{A}_{\text{appro}}$ is determined by the vector $\mathbf{a}_{\text{appro}} = \frac{1}{N}\mathbf{F_a P c}'$ according to the construction of $\mathbf{A}$. Note that $\mathbf{A} - \mathbf{A}_{\text{appro}}$ represents the modeling error existing in the approximation given by (6). The parameter vector to be estimated is

$$\begin{aligned}\boldsymbol{\theta} = \big[\, &\text{Re}\{\nu\} \;\; \text{Im}\{\nu\} \;\; \text{Re}\{\mathbf{c}''^T\} \;\; \text{Im}\{\mathbf{c}''^T\} \\ &\text{Re}\{\mathbf{h}'^T\} \;\; \text{Im}\{\mathbf{h}'^T\} \,\big]^T,\end{aligned}$$

where the vector $\mathbf{c}'' = \begin{bmatrix} c'[1] & c'[2] & \dots & c'[M-1] \end{bmatrix}^T$ contains all elements of $\mathbf{c}'$ except its first element $c'[0]$. Hence, the desired signal component is $\mu\mathbf{A}_{\text{appro}}\mathbf{Hx} + \nu\widetilde{\mathbf{A}}_{\text{appro}}\widetilde{\mathbf{H}}\widetilde{\mathbf{x}}$, and the noise term is

$$\mathbf{w}' = \mu(\mathbf{A} - \mathbf{A}_{\text{appro}})\mathbf{Hx} + \nu(\widetilde{\mathbf{A}} - \widetilde{\mathbf{A}}_{\text{appro}})\widetilde{\mathbf{H}}\widetilde{\mathbf{x}} + \mathbf{w}.$$

The covariance matrix of $\mathbf{w}'$ is *approximately* equal to $\sigma_{w'}^2\mathbf{I}_N$, where

$$\begin{aligned}\sigma_{w'}^2 &= \sigma_w^2 + (|\mu|^2 + |\nu|^2) \times \mathbf{E}\{\|\mathbf{a} - \mathbf{a}_{\text{appro}}\|^2\} \times \sigma_H^2\sigma_p^2 \\ &= \sigma_w^2 + (|\mu|^2 + |\nu|^2)(1 - \text{Tr}\{\mathbf{Q}(\mathbf{Q}^*\mathbf{Q})^{-1}\mathbf{Q}^*\mathbf{R_a}\})\sigma_H^2\sigma_p^2.\end{aligned}$$

[2] $\mathbf{P}$ can be constructed from linear interpolation.

Here, $\mathbf{Q} = \frac{1}{N}\mathbf{F_a}\mathbf{P}$, $\mathbf{R_a} = \mathbf{E}\{\mathbf{a}\mathbf{a}^*\}$, and $\mathbf{E}\{\|\mathbf{a} - \mathbf{a}_{\text{appro}}\|^2\} = 1 - \text{Tr}\{\mathbf{Q}(\mathbf{Q}^*\mathbf{Q})^{-1}\mathbf{Q}^*\mathbf{R_a}\}$ is given by the minimum mean-squared error of estimating $\mathbf{a}$ by $\mathbf{a}_{\text{appro}} = \mathbf{Q}\mathbf{c}'$ with noting that $\|\mathbf{a}\|^2 = 1$. Consequently, $\mathbf{I}_{\boldsymbol{\theta}}$ and the associated CRLB for $\mathbf{h}'$ can be computed. By using the relation $\mathbf{h} = \mathbf{F_h}\mathbf{h}'$, we have

$$\mathbf{E}\{|\mu a[0]\widehat{H}[k] - \mu a[0]H[k]|^2\} = \mathbf{E}\{\|\mu a[0]\widehat{\mathbf{h}'} - \mu a[0]\mathbf{h}'\|^2\}.$$

The lower bound for $H[k]$ can then be derived from the lower bound for $\mathbf{h}'$.

IV. Data Symbol Estimation

Assume that the receiver has acquired the channel response $\mathbf{H}$ and the IQ imbalance parameters μ and ν. Given the system model (4), we are now interested in how to estimate the transmitted vector $\mathbf{x}$. It is noticed that expression (4) can be represented as

$$\mathbf{y} = \mu\mathbf{z} + \nu\widetilde{\mathbf{z}} + \mathbf{w},$$

where $\mathbf{z} = \mathbf{A}\mathbf{H}\mathbf{x}$ is denoted by

$$\mathbf{z} = \left[\, z[0] \quad z[1] \quad \ldots \quad z[N-1] \,\right],$$

and $\widetilde{\mathbf{z}}$ is defined accordingly as

$$\widetilde{\mathbf{z}} = \left[\, z^*[0] \quad z^*[N-1] \quad \ldots \quad z^*[1] \,\right].$$

Then the problem can be decomposed into two separate compensation problems: the IQ imbalance compensation and the phase noise compensation, as illustrated in Fig. 2. First, $\mathbf{z}$ is estimated from $\mathbf{y}$ by using any IQ imbalance compensation method; then, $\mathbf{x}$ is estimated from $\widehat{\mathbf{z}}$ by using any phase noise compensation method. In this paper, we apply the post-FFT IQ and phase noise compensation techniques proposed in [5] and [8].

A. Performance Analysis

We can analyze the effects of IQ imbalance and phase noise on OFDM systems in terms of the signal-to-noise ratio degradation. The expressions of the effective signal-to-noise ratio at the receiver are derived by assuming that 1) the data symbols $x[k]$ are independent and identically distributed with mean zero and variance $\sigma_x^2 = \mathbf{E}\{|x[k]|^2\}$; 2) the data symbols, the phase noise, the channel coefficients and the additive noise are independent of each other. 3) the channel coefficients $H[k]$ are independently identically distributed and circularly symmetric Gaussian with mean zero and variance $\sigma_H^2 = \mathbf{E}\{|H[k]|^2\}$. The expressions of the effective signal-to-noise ratio for different scenarios are listed below:

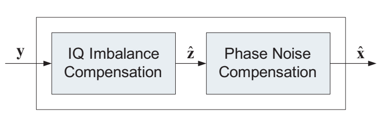


Fig. 2. Block diagram of the data symbol estimation algorithm. It can be decomposed into the IQ imbalance compensation block and the phase noise compensation block.

1) *No Impairment:*

$$\text{SNR}_0 = \frac{\mathbf{E}\{|H[k]x[k]|^2\}}{\mathbf{E}\{|w[k]|^2\}} = \frac{\sigma_H^2\sigma_x^2}{\sigma_w^2};$$

2) *No Compensation for Phase Noise and IQ Imbalance:*

$$\text{SNR}_{\text{no}} = \frac{\text{SNR}_0}{1 + (1 - 2\text{Re}\{\mu a[0]\} + |\mu|^2 + |\nu|^2)\,\text{SNR}_0};$$

3) *IQ and Common Phase Error (CPE) Compensation, i.e., μ, ν and $a[0]$ are Known:*

$$\text{SNR}_{\text{IQ+CPE}} = \frac{(|\mu|^2 + |\nu|^2)\sigma_{a,0}^2\text{SNR}_0}{1 + (1 - \sigma_{a,0}^2)(|\mu|^2 + |\nu|^2)\text{SNR}_0};$$

4) *Proposed Joint Compensation Scheme:*

$$\text{SNR}_{\text{prop}} = \frac{\left(|\mu|^2 + |\nu|^2\right) \times \text{Tr}\{\mathbf{Q}(\mathbf{Q}^*\mathbf{Q})^{-1}\mathbf{Q}^*\mathbf{R_a}\} \times \text{SNR}_0}{1 + \left(|\mu|^2 + |\nu|^2\right) \times \left(1 - \text{Tr}\{\mathbf{Q}(\mathbf{Q}^*\mathbf{Q})^{-1}\mathbf{Q}^*\mathbf{R_a}\}\right) \times \text{SNR}_0}.$$

V. Computer Simulations

In the simulations, the system bandwidth is 20 MHz, i.e., $T_s = 0.05\ \mu s$, and the constellation used for symbol mapping is 64-QAM. The OFDM symbol size is $N = 64$ and the prefix length is $P = 20$. The channel length is 6, and each tap is independently Rayleigh distributed with the power profile specified by 3 dB decay per tap. The average power of the channel response is normalized to 1, i.e., $\sigma_H^2 = 1$. We simulate an OFDM receiver with the IQ imbalance specified by $\alpha = 0.1$ and $\theta = 10°$. The spectrum of simulated phase noise is shown in Fig. 3.

We first examine the performance of different channel estimation algorithms for different scenarios. In the simulations, only one block-type pilot symbol is used for each time of channel estimation. The assumed channel length in the time domain is $L = 10$ and the length of the phase noise vector to be estimated is $M = 8$. Fig. 4(a) plots the mean-squared errors (MSE) of different channel estimation algorithms vs. the normalized signal-to-noise ratio at the receiver, i.e., $\text{SNR} = \sigma_p^2/\sigma_w^2$. In Fig. 4(b), the CRLB is plotted by using the expressions derived in Section III-A. By comparing Fig. 4(a)

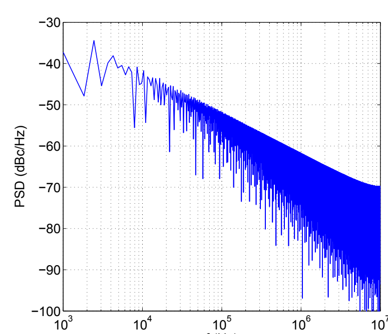
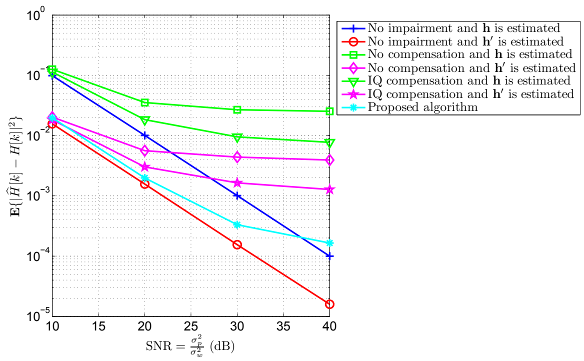
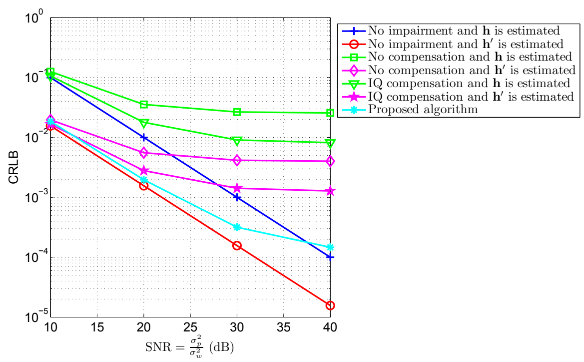


Fig. 3. Power spectral density (PSD) of the phase noise with $\xi = 2.5$ kHz. The PSD is measured in dB with respect to the carrier power, namely, dBc.



(a) Mean-squared error obtained by computer simulations.



(b) CRLB computed by using the formulas derived in Subsection III-A.

Fig. 4. Plots of the MSE and CRLB for channel estimation when $\alpha = 0.1$, $\theta = 10°$ and $\xi = 2.5$ kHz. Seven cases are simulated: i) There is no impairment and $\mathbf{h}$ is estimated. ii) There is no impairment and $\mathbf{h}'$ is estimated. iii) Both IQ imbalance and phase noise are present but the receiver assumes no impairment when estimating $\mathbf{h}$. iv) Both IQ imbalance and phase noise are present but the receiver assumes no impairment when estimating $\mathbf{h}'$. v) Perfect knowledge about IQ imbalance is available at the receiver, but there is no compensation for phase noise when estimating $\mathbf{h}$. vi) Perfect knowledge about IQ imbalance is available at the receiver, but there is no compensation for phase noise when estimating $\mathbf{h}'$. vii) Both IQ imbalance and phase noise are present and the proposed channel estimation algorithm is applied.

and Fig. 4(b), it can be seen that the CRLB gives a good measure about the accuracy of different algorithms.

The proposed data symbol estimation algorithm is simulated in comparison with the ideal OFDM receiver with no impairment and the IQ+CPE (common phase error) correction scheme proposed in [9]. During the payload portion of OFDM packets, 16 out of the 64 subcarriers are used for pilot tones, i.e., $Q = 16$. Fig. 5 shows the uncoded BER performance when the receiver only has the estimated channel information. Compared to the IQ+CPE scheme, the proposed method achieves lower BERs, because it not only corrects the common phase rotation of the received constellation but also suppresses part of the inter-carrier interference caused by phase noise. In other words, the proposed algorithm can reduce the sensitivity of OFDM receivers to the analog impairments effectively.

VI. Conclusions

In this paper, the joint effects of IQ imbalance and phase noise on OFDM systems are studied. A compensation scheme

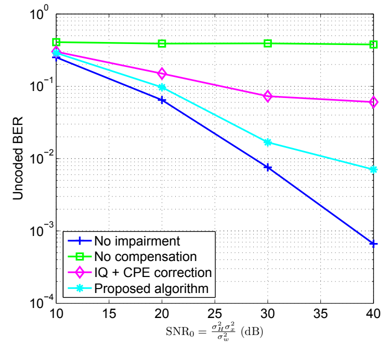


Fig. 5. Plots of uncoded BER vs. SNR_0 when the receiver only has the estimated channel information. Four scenarios are simulated: i) There is no impairment. ii) Both IQ imbalance and phase noise are present, but no compensation is applied. iii) Both IQ imbalance and phase noise are present, and the IQ+CPE correction scheme proposed in [9] is applied. iv) Both IQ imbalance and phase noise are present, and the proposed data symbol estimation algorithm is applied.

is proposed that consists of two stages. One stage is the joint channel estimation, and the other is the joint data symbol estimation. The proposed channel estimation algorithm performs close to the derived Cramer-Rao lower bound in the presence of the impairments. Also, the analysis and simulations show that the compensation scheme can effectively improve the system performance and reduce the sensitivity of OFDM receivers to the analog impairments. Since receivers with less analog impairments usually have the disadvantage of high implementation cost, our technique enables the use of low-cost receivers for OFDM communications.

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