# Exploiting Spatio-Temporal Correlation for Rate-Efficient Transmit Beamforming

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***Abstract*— With multiple antennas at the transmitter, either space-time block coding or beamforming can be used as the transmission scheme depending on the availability of channel state information at the transmitter. Although beamforming can provide superior performance over space-time block coding, it nevertheless suffers from feedback overhead. An efficient beamforming scheme in terms of feedback rate is proposed in this paper. The proposed scheme exploits the channel statistics by using space and time correlation in a joint manner in order to track the channel at the transmitter. Simulation results show improvements compared to a system with space-time block coding.**

## I. INTRODUCTION

The deployment of multiple transmit and receive antennas has been envisioned for several wireless communications standards. Two of the approaches that are being considered to exploit the potential of multiple transmit antennas with the purpose of maximizing the SNR at the receiver are space-time block coding (STBC) and beamforming. Space-time block coding is useful when channel state information (CSI) is not available at the transmitter. However, more can be done if the CSI is known to the transmitter, either completely or partially. In the case of complete CSI at the transmitter, beamforming can be used. Its drawback is the need for a feedback link to send back to the transmitter the estimated channel information. The feedback link results in overhead and reduces the achievable data rates in mobile environments. Different schemes have been proposed to combat and reduce the rate overhead caused by feedback. In [1], for example, a quantized version of the beamforming coefficients estimated at the receiver is sent back to the transmitter. The feedback rate is reduced by using an appropriate quantization scheme. In another approach [2], the channel covariance matrix is used at the transmitter to derive the beamforming coefficients from the eigenvector corresponding to the largest eigenvalue. Although this scheme requires a lower feedback rate (due to the slower variation of the covariance matrix), it nevertheless suffers from poor performance compared to the case when the channel realizations are used.

In this paper, we propose a scheme where the channel taps of a flat fading Rayleigh channel are efficiently tracked at the transmitter and then used directly to compute the beamforming coefficients. The channel correlations in both time and space are exploited by the transmitter for improved channel tracking and the receiver is required to feed back a decimated version of the received data. A reduced feedback rate is shown to be sufficient to track the channel and achieve performance close to that of an ideal beamformer.

The paper is organized as follows. The next section describes the model used for the channel. In Sec. III, a state-space model is derived that exploits the channel statistics in both time and space. Kalman and RLS channel tracking schemes are then presented in Sec. IV. Simulation results are presented in Sec. V. Conclusions are given in Sec. VI.

## II. CHANNEL MODELING

A multi-input single-output (MISO) system with $M$ transmit antennas is considered–see Fig. 1. A single-path channel is assumed between every transmit and receive antenna. Let $\mathbf{h}_n$ represent the collection of channel gains from the various transmit antennas to the receiver, say

$$\mathbf{h}_n = \text{col}\{h_1(n) \quad h_2(n) \quad \cdots \quad h_M(n)\}$$

where $h_i(n)$ is the complex channel gain from the $i$th antenna to the receiver at time $n$.

If the channel is known at the transmitter, then one could use beamforming to maximize the received SNR. The beamforming vector $\mathbf{w}_n$ at time $n$ could be chosen as [3]:

$$\mathbf{w}_n = \frac{\mathbf{h}_n^*}{\|\mathbf{h}_n\|} \tag{1}$$

where $\|.\|$ denotes the Euclidean norm of its argument.

One common way to obtain the Channel State Information (CSI) at the transmitter is to estimate the channel at the receiver and to feed this information back to the transmitter. This step involves some overhead and decreases the system throughput. An alternative solution would be to feed back some minimal information that enables the transmitter to perform both channel estimation and tracking. For example, for channels that could be modeled by a first order autoregressive (AR) model, say

$$\mathbf{h}_{n+1} = \mathbf{F}\mathbf{h}_n + \mathbf{G}\mathbf{u}_n \tag{2}$$

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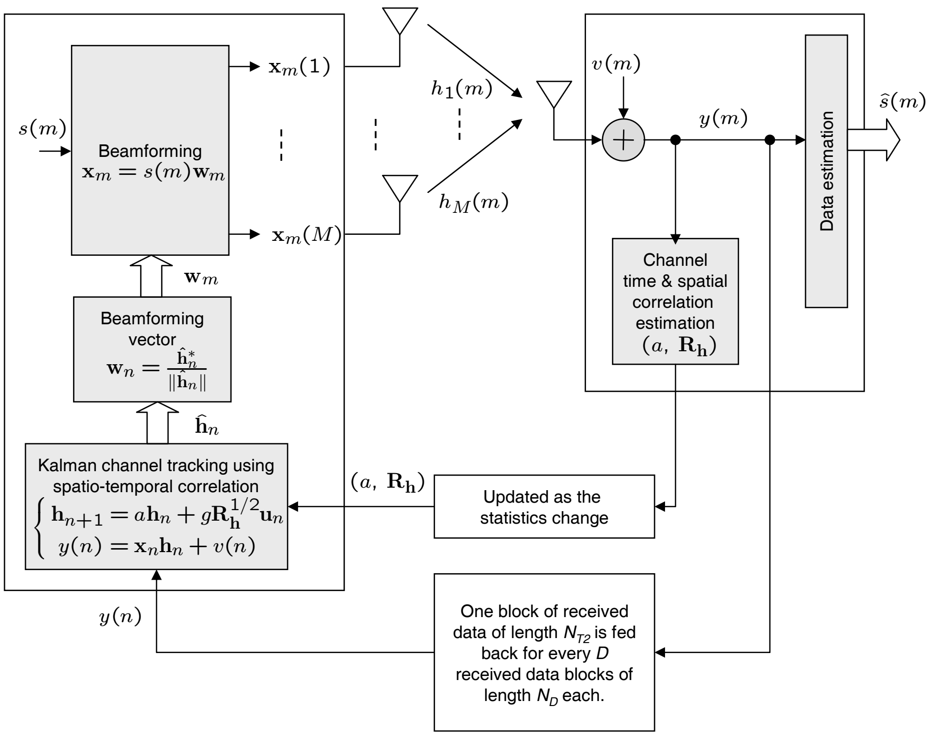


Fig. 1. The proposed beamforming architecture with feedback.

the channel estimation and tracking step could be performed at the transmitter by using a Kalman filter provided that the model parameters (i.e., the $\mathbf{F}$ and $\mathbf{G}$ matrices) are known by the transmitter. In the model (2), $\mathbf{u}_n$ is a zero-mean i.i.d random process with unit covariance matrix, $\mathbf{R}_\mathbf{u} = \mathsf{E}\mathbf{u}\mathbf{u}^* = \mathbf{I}$. In the case of Gaussian wide-sense stationary uncorrelated scattering fading channel gains (WSSUS), the matrices $\mathbf{F}$ and $\mathbf{G}$ in (2) can be taken as diagonal [8] (see below). However, in the case of spatially correlated channel gains, $\mathbf{F}$ and $\mathbf{G}$ would need to be chosen properly in order to reflect the inter-dependency among the elements of $\mathbf{h}_n$. This issue is addressed in Sec. III.

## III. State-Space Model

A channel vector $\mathbf{h}_n$ with a spatial covariance matrix $\mathbf{R}_\mathbf{h} = \mathsf{E}\mathbf{h}_n\mathbf{h}_n^*$ can be modeled as $\mathbf{h}_n = \mathbf{R}_\mathbf{h}^{1/2}\mathbf{h}_n^w$, where $\mathbf{h}_n^w$ is a vector whose elements are uncorrelated (i.e., $\mathbf{R}_\mathbf{h}^w = \mathbf{I}$). In other words, a spatially correlated channel vector can be regarded as the transformation of a spatially white channel vector $\mathbf{h}_n^w$. The dynamics of the vector $\mathbf{h}_n^w$ could be modeled as

$$\mathbf{h}_{n+1}^w = \mathbf{F}^w\mathbf{h}_n^w + \mathbf{G}^w\mathbf{u_n} \tag{3}$$

where $\mathbf{F}^w$ and $\mathbf{G}^w$ are diagonal matrices in the spatially uncorrelated case and found as follows [7]. Let $\mathbf{F}^w = a\mathbf{I}_{M\times M}$ and $\mathbf{G}^w = g\mathbf{I}_{M\times M}$. Then (3) becomes

$$\mathbf{h}_{n+1}^w = a\mathbf{h}_n^w + g\mathbf{u}_n \tag{4}$$

where the scalar $a$ denotes the first-order autocorrelation coefficient for each channel gain, namely,

$$a = \mathsf{E}h^w(n)h^{w*}(n-1) \tag{5}$$

In order to estimate $a$, we use the model from [4] where the variability of the wireless channel over time is modeled in terms of the autocorrelation function of a complex Gaussian process. It was shown in [5] that the theoretical power spectral density function associated with either the in-phase or quadrature portion of each channel element has the well-known U-shaped bandlimited form:

$$S(f) = \begin{cases} \frac{1}{\pi f_d\sqrt{1-(\frac{f}{f_d})^2}} & |f| \leq f_d \\ 0 & \text{elsewhere} \end{cases} \tag{6}$$

where $f_d$ is the maximum Doppler frequency. The corresponding normalized discrete-time autocorrelation sequence of each channel element is given by

$$a_k = \mathsf{E}h^w(n)h^{w*}(n-k) = J_0(2\pi f_d T|k|) \tag{7}$$

where $J_0(.)$ is the zeroth-order Bessel function of the first kind, and $T$ is the sampling period. Thus [6],

$$a = J_0(2\pi f_d T) \tag{8}$$

Moreover,

$$g = \sqrt{1-|a|^2} \tag{9}$$

Now let $\mathbf{R}_\mathbf{h}^{1/2}$ denote any square-root of $\mathbf{R}_\mathbf{h} > 0$. Using $\mathbf{h}_n = \mathbf{R}_\mathbf{h}^{1/2}\mathbf{h}_n^w$ we can formulate a state-space model for spatially correlated channels as follows. Substituting $\mathbf{h}_n^w = \mathbf{R_h}^{-1/2}\mathbf{h}_n$ into (3) gives

$$\mathbf{R_h}^{-1/2}\mathbf{h}_{n+1} = a\mathbf{R_h}^{-1/2}\mathbf{h}_n + g\mathbf{u}_n \tag{10}$$

or, equivalently,

$$\mathbf{h}_{n+1} = a\mathbf{h}_n + g\mathbf{R_h}^{1/2}\mathbf{u}_n \tag{11}$$

Thus, the dynamics of a spatially correlated channel vector could be modeled as

$$\mathbf{h}_{n+1} = \mathbf{F}\mathbf{h}_n + \mathbf{G}\mathbf{u}_n \tag{12}$$

where $\mathbf{F} = a\mathbf{I}_{M\times M}$ and $\mathbf{G} = g\mathbf{R_h}^{1/2}$, with $\mathbf{F}$ continuing to be a diagonal matrix[1].

## IV. Channel Estimation

Let $y(m)$ denote the received data at time $m$, i.e.,

$$y(m) = \mathbf{x}_m\mathbf{h}_m + v(m) \tag{13}$$

where $\mathbf{x}_m$ is the transmitted $1 \times M$ vector of data (known to the transmitter ) and $v(m)$ is the added noise at the receiver. Note that we are using two time indices, $m$ and $n$. It is assumed that data are transmitted at time instants $mT$, while the channel dynamics varies at a slower rate denoted by the time instants $nT$. Both $v(n)$ and $\mathbf{u}_n$ are assumed to be uncorrelated. Denoting the transmitted sample at time $m$ by $s(m)$, the transmitted vector $\mathbf{x}_m$ is given by $\mathbf{x}_m = s(m)\mathbf{w}_n$.

The channel covariance matrix $\mathbf{R_h} = \mathsf{E}\mathbf{h}_n\mathbf{h}_n^*$ can be initially estimated at the receiver by time averaging of the estimated channel elements and the result sent back to the transmitter. Subsequently, the receiver feeds back the received data $\{y(m)\}$ at a downsampled rate–see Figure 1. In this way, the transmitter ends up with the following state-space model:

$$\begin{aligned} \mathbf{h}_{n+1} &= a\mathbf{h}_n + g\mathbf{R_h}^{1/2}\mathbf{u}_n \\ y(n) &= \mathbf{x}_n\mathbf{h}_n + v(n) \end{aligned} \tag{14}$$

Note that the transmitted vector $\mathbf{x}_n$ is known at the transmitter and the receiver only feeds back $y(m)$ at a downsampled rate. This reduces the number of feedback bits significantly as opposed to the case when the entire channel vector is fed back.

[1]The approach presented in this paper for a MISO channel can be generalized to a MIMO channel. Let $\mathbf{H}$ be an $N \times M$ matrix representing the channel. The spatially correlated channel matrix can be modelled as

$$\mathbf{H} = \mathbf{R}_r^{1/2}\mathbf{H}^w\mathbf{R}_t^{1/2}$$

where all the entries of $\mathbf{H}^w$ have unit variance and for some transmit and receive covariance matrices $\{\mathbf{R}_t, \mathbf{R}_r\}$. Then

$$\underbrace{\text{vec}(\mathbf{H})}_{\mathbf{h}} = \left(\mathbf{R}_t^{\mathrm{T}/2} \otimes \mathbf{R}_r^{1/2}\right)\underbrace{\text{vec}(\mathbf{H}^w)}_{\mathbf{h}^w}$$

and results can be extended to the MIMO case by substituting $\mathbf{R_h}^{1/2}$ with $\left(\mathbf{R}_t^{\mathrm{T}/2} \otimes \mathbf{R}_r^{1/2}\right)$.

The Kalman equations [3] can now be used by the transmitter to track the channel as follows:

$$\begin{aligned} e(n) &= y(n) - \mathbf{x}_n\hat{\mathbf{h}}_n, \quad \hat{\mathbf{h}}_0 = 0 \\ \hat{\mathbf{h}}_{n+1} &= \hat{\mathbf{h}}_n + k_{p,n}e(n) \\ k_{p,n} &= aP_n\mathbf{x}_n^*r_e^{-1}(n) \\ r_e(n) &= \sigma_v^2 + \mathbf{x}_nP_n\mathbf{x}_n^* \\ P_{n+1} &= |a|^2P_n + |g|^2\mathbf{R_h} - k_{p,n}r_ek_{p,n}^*, \quad P_0 = \mathbf{R_h} \end{aligned}$$

Alternatively, the RLS algorithm could be used to estimate and track the channel. RLS is independent of the underlying state-space model and it does not require knowledge of the state-space parameters $\mathbf{F}$ and $\mathbf{G}$. Based on the measurements $y(n)$, the RLS update equations are given by [3]:

$$\begin{aligned} \mathbf{P}_n &= \lambda^{-1}\left(\mathbf{P}_{n-1} - \frac{\lambda^{-1}\mathbf{P}_{n-1}\mathbf{x}_n^*\mathbf{x}_n\mathbf{P}_{n-1}}{1 + \lambda^{-1}\mathbf{x}_n\mathbf{P}_{n-1}\mathbf{x}_n^*}\right), \quad \mathbf{P}_{-1} = \delta\mathbf{I} \\ \hat{\mathbf{h}}_n &= \hat{\mathbf{h}}_{n-1} + \mathbf{P}_n\mathbf{x}_n^*\left[(y(n) - \mathbf{x}_n\hat{\mathbf{h}}_{n-1}\right], \quad \hat{\mathbf{h}}_{-1} = 0 \end{aligned} \tag{15}$$

where $0 \ll \lambda \leq 1$. The RLS algorithm is used for comparison purposes in order to show how much improvement in performance can be obtained when the channel model is estimated and exploited. This improvement is reflected in the BER curves shown in the following section.

## V. Simulation Results

### A. Simulation Set-Up

A typical MISO system is simulated to evaluate the performance of the proposed transmission scheme in comparison to a beamformer with perfect channel information, as well as a system with orthogonal space-time block coding. We consider a 4-transmit 1-receive configuration. QPSK constellation is used at the transmitter. A single tap channel between every transmit and receive antenna is simulated.

A spatially white channel is generated through a first order AR model:

$$\mathbf{h}_n^w = a\mathbf{h}_{n-1}^w + g\mathbf{u}_{n-1}$$

with parameters $a$ and $g$ calculated using the channel Doppler frequency as

$$a = J_0(2\pi f_dT), \quad g = \sqrt{1 - |a|^2}$$

with $f_d = 60$Hz, $T = 1\mu sec$. The spatially correlated channel used in the simulations is generated as

$$\mathbf{h}_n = \mathbf{R_h}^{1/2}\mathbf{h}_n^w$$

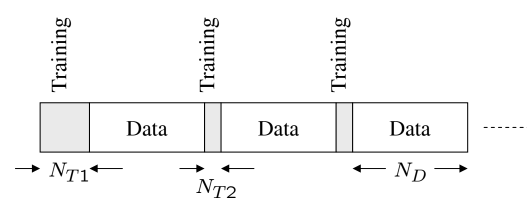


Fig. 2. The training pattern used in the simulations.

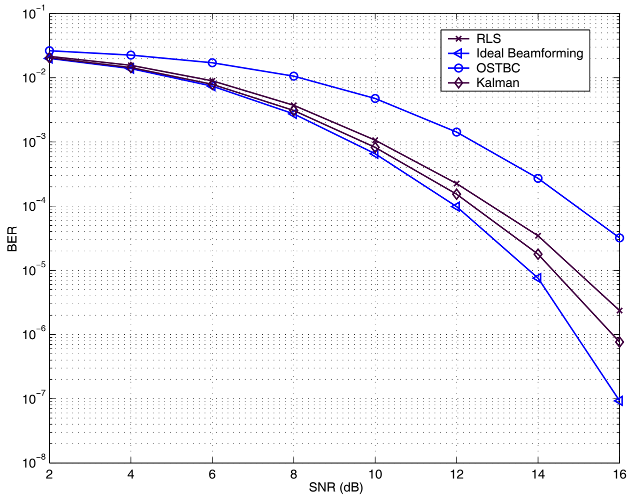


Fig. 3. BER versus SNR for a $4 \times 1$ system with $f_d = 60$Hz, $\rho = 0.8$, $N_D$=512, $N_{T1} = 6$, and $N_{T2} = 1$.

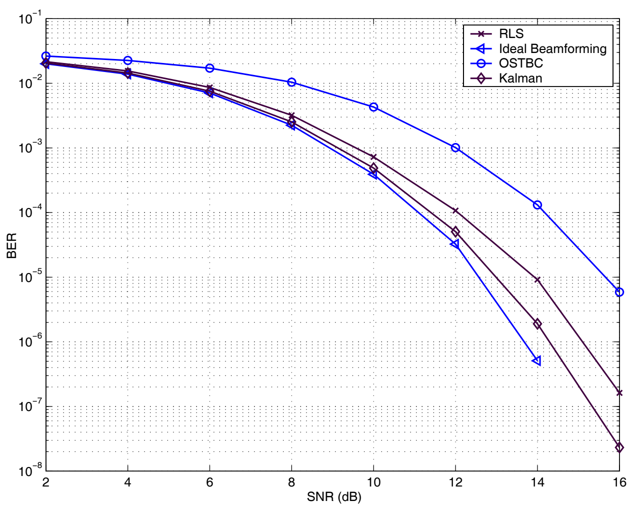


Fig. 4. BER versus SNR for a $4 \times 1$ system with $f_d = 60$Hz, $\rho = 0.4$, $N_D$=512, $N_{T1} = 6$, and $N_{T2} = 1$.

where $\mathbf{R_h}$ is the channel covariance matrix. We performed simulations for a covariance matrix of form

$$\mathbf{R_h} = \begin{bmatrix} 1 & \rho & \rho^4 & 0 \\ \rho & 1 & \rho & \rho^4 \\ \rho^4 & \rho & 1 & \rho \\ 0 & \rho^4 & \rho & 1 \end{bmatrix}$$

where the spatial correlation of the channel is controlled through the parameter $\rho < 1$. In the simulations, we use an estimated version of $\mathbf{R_h}$ at the transmitter. This estimate is obtained at the receiver using time-averaging and is fed back to the transmitter. The packet structure used in the simulations is shown in Figure 2.

Before the transmission starts, a burst of $N_{T1}$ (e.g., $N_{T1} = 6$) training data is transmitted for initial estimation of the channel and of $\mathbf{R_h}$. Payloads of length $N_D$ are then transmitted

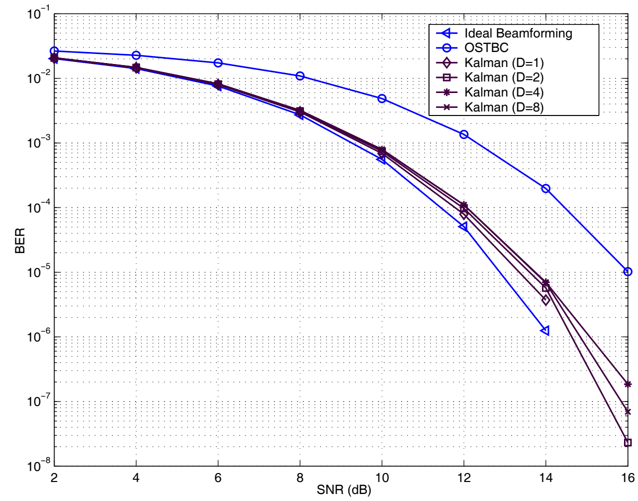


Fig. 5. BER versus SNR for a $4 \times 1$ system with $f_d = 60$Hz, $\rho = 0.8$, $N_D$=512, $N_{T1} = 8$, and $N_{T2} = 1$. See Figure 1 for an explanation of the parameter $D$.

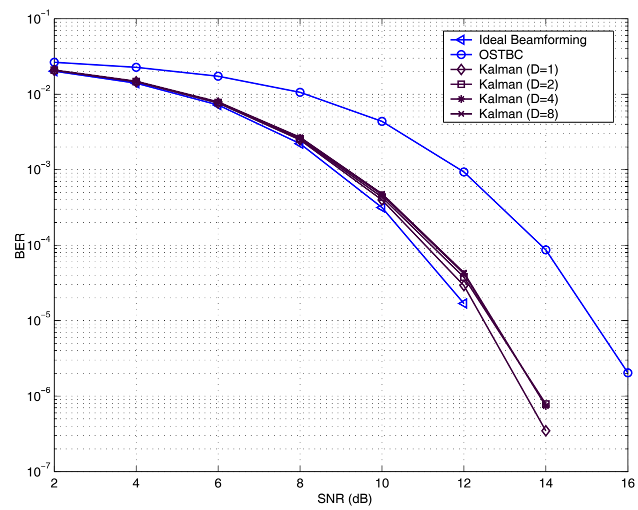


Fig. 6. BER versus SNR for a $4 \times 1$ system with $f_d = 60$Hz, $\rho = 0.4$, $N_D$=512, $N_{T1} = 8$, and $N_{T2} = 1$. See Figure 1 for an explanation of the parameter $D$.

(e.g., $N_D = 512$). To enable channel tracking, training data is transmitted between the payloads. To reduce the training overhead, we choose $N_{T2} < N_{T1}$, e.g., $N_{T2} = 1$ in which case only one measurement $y(m)$ is fed back every $N_D$ data points. More generally, one block of received data $y(m)$ corresponding to a training block of length $N_{T2}$ could be sent back to the transmitter for every $D$ received data blocks of length $N_D$ each. In the simulations, we show how we can increase the payload length $N_D$ and decrease the training lengths $N_{T1}$ and $N_{T2}$ in order to reduce the feedback overhead.

The simulation results demonstrate the effects of the parameters $f_d$, $\rho$, and the feedback rate, on the proposed Kalman and standard RLS channel estimation schemes. In all simulations we compare the suggested channel tracking system to both ideal beamforming and Orthogonal Space Time Coding (OSTBC). In all the simulations, we have used the $3/4$-rate orthogonal space-time block code proposed for a $4 \times 1$ MIMO system [9]. Due to the rate difference between the beamforming and OSTBC scheme (full versus $3/4$), the total transmit power for a frame of data is normalized for both schemes. Figures 5 and 6 depict the results for different feedback rates $D = 1, 2, 4, 8$–see Figure 1. The results show a significant improvement over the OSTBC scheme and a performance that is close to ideal beamforming for different simulation parameters.

## VI. Concluding Remarks

An efficient beamforming scheme in terms of feedback rate is proposed in this paper. The proposed scheme exploits the channel statistics using space and time correlation in a joint manner to track the channel at the transmitter. The proposed channel tracking scheme is based on a state-space model derived for channel evolution in the spatially correlated case. Simulation results show improvements compared to a system with space-time block coding.

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