A New Adaptive Estimation Algorithm for Wireless Location Finding Systems*

Nabil R. Yousef and Ali H. Sayed

Adaptive and Nonlinear Systems Laboratory
Electrical Engineering Department
University of California
Los Angeles, CA 90095

Abstract

Wireless location finding is receiving increasing attention in the field of wireless communications. This is is due to a recent order issued by the Federal Communications Commission (FCC) that mandates all wireless service providers to locate an emergency 911 caller within a high accuracy, by the year 2001. In this paper, we derive a new (sub-optimal) maximum likelihood estimation algorithm for the time and amplitude of arrival of a known transmitted sequence over a single path fading channel. The new algorithm is then applied to the case of CDMA wireless location finding. The paper discusses both simulation and field trial results, all of which demonstrate significant estimation accuracy improvement over known algorithms.

1 INTRODUCTION

Wireless location finding has recently emerged as a very essential public safety feature of future cellular systems. This has been emphasized by a recent federal order issued by the federal communications commission (FCC), which mandates all wireless service providers to provide public safety answering points with information to locate an emergency 911 caller with an accuracy of 125 meters for 67% of the cases [1]. It is also expected that the FCC will tighten its requirements in the near future. This has boosted research in the field of wireless location finding, which has many other potential applications in areas such as location sensitive billing, fraud protection, mobile yellow pages, and fleet management (see, e.g., [2]-[6].

Location finding mainly requires accurate estimates of the time and amplitude of arrival of the mobile station signal when received at various base stations. Obtaining such estimates is usually difficult due to the low signal to noise ratios and to fast channel fading conditions encountered in wireless propagation environments [5].

Although several signal parameters estimation algorithms already exist in the literature (see, e.g., [7, 8, 9]), these algorithms are mainly designed for signal aquisition or tracking purposes, where coarse estimates for the channel time delays and amplitudes are sufficient for online signal decoding. Using the same algorithms for wireless location applications is not adequate for the following reasons: 1) Channel fading is mainly considered constant during the relatively short estimation period of these algorithms, thus totally ignored. This assumption cannot be made for wireless location applications where the estimation period could be much longer. 2) The low precision of the coarse estimates provided by these algorithms generally does not satisfy the precision levels needed in wireless location applications, especially the FCC requirements [5].

In this paper we first develop an efficient adaptive (sub-optimal) maximum likelihood procedure for estimating unknown parameters of a measured transmitted signal over a time-variant channel in the presence of additive white Gaussian noise. The properties of the proposed estimator are also studied. The proposed scheme is then used to estimate the time and amplitude of arrival of a known IS-95 CDMA sequence transmitted over a single path fading channel.

2 Problem Formulation

Consider the problem of estimating an unknown constant discrete time delay τ^o of a known real-valued sequence $\{s(n,\tau^o)\}$ transmitted over a single path time varying channel, from a measured sequence $\{r(n)\}_{n=1}^{K}$ that arises from the model[1]

$$r(n) = A\, x^o(n)\; s(n,\tau^o) + v(n) \quad , \tag{1}$$

where A is a constant *unknown* channel amplitude, $v(n)$ is additive white Gaussian noise, and $x^o(n)$ is a complex ergodic random process of known autocorrelation function $R_x(i)$ defined as

$$R_x(i) = \mathrm{E}\, x^o(n) x^o(n-i) \ .$$

The sequence $\{x^o(n)\}$ accounts for the time-varying nature of the *fading* channel gain over which the sequence $\{s(n,\tau^o)\}$ is transmitted, while A represents the gain of the *static* channel if fading

[1] By a known sequence $\{s(n,\tau^o)\}$ we mean one for which the dependency on n and τ^o is known. The actual sequence itself is not known.

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were not present. Without loss of generality, we will assume that the sequence $\{x^o(n)\}$ has unit power, i.e., $\frac{1}{K}\sum_{n=1}^{K}|x^o(n)|^2 = 1$. The maximum likelihood (ML) estimates of $\{\tau^o, \hat{x}(n)\}$ are given by [10]

$$\{\hat{\tau}, \hat{x}(n)\} = \arg \max_{\{\tau, x(n)\}} \left[P\left(r(1), \ldots, r(K)|\{\tau, x(n)\}\right)\right] ,$$

where the likelihood function $P(r|\{\tau, x(n)\})$ is equal to

$$C_1 \exp\left(-C_2 \frac{1}{K} \sum_{n=1}^{K} \left(r(n) - A\, x(n)\, s(n,\tau)\right)^2\right) ,$$

and C_1 and C_2 are positive constants that are independent of $\{\tau, x(n)\}$. Thus, the ML estimates of $\{\tau^o, x^o(n)\}$ are given by

$$\{\hat{\tau}, \hat{x}(n)\} = \arg \max_{\{\tau, x(n)\}} \left[J_{ML}(\tau, x(n))\right] ,$$

where the cost function J_{ML} is defined by

$$J_{ML}(\tau, x(n)) = \frac{2A}{K} \sum_{n=1}^{K} s(n,\tau) \operatorname{Re}\left(r(n) x^*(n)\right) - \frac{A^2}{K} \sum_{n=1}^{K} |x(n)|^2 s^2(n,\tau) . \quad (2)$$

This construction requires an infinite dimensional search, and is not feasible in practice even when τ and $x(n)$ are evaluated over a dense grid, which is generally the approach adopted in ML estimation over single path static channels (see, e.g., [7]).

3 Proposed Estimator

We now develop an efficient sub-optimal ML estimator that requires a finite number of search bins. To arrive at this estimator, we assume the case of slow channel variation, namely we assume that

A.1 $x^o(n)$ is piecewise constant over intervals of N samples

where N is a parameter that depends on the environment conditions. An optimal choice for N that is based on the available knowledge of the channel autocorrelation function $R_x(i)$ is discussed in the next section.

Now introduce a sequence $\{x^u(m)\}$ that is an N undersampled version of $\{x^o(n)\}$, i.e.,

$$x^u(m) = x^o\left((m-1)N + 1\right) ,$$

for $m = 1, 2, \ldots, M$, where $M = K/N$ is assumed to be an integer. Using A.1 and (1), expression (2) becomes

$$J_{ML} = \frac{1}{K} \sum_{m=1}^{M} A^2 \left[\left(x^{u*}(m)x(m) + x^u(m)x^*(m)\right) \left(\sum_{n=n_o}^{mN} s(n,\tau^o)s(n,\tau)\right) - |x(m)|^2 \left(\sum_{n=n_o}^{mN} s^2(n,\tau)\right)\right] ,$$

where $n_o = (m-1)N + 1$. Consider further the choice of $x(m)$ that corresponds to the solution of

$$\frac{\partial J_{ML}}{\partial x(m)} = 0 ,$$

which is found to be $x(m) = x^u(m)$, and leads to the following cost function:

$$J_x(\tau) = \frac{1}{K} \sum_{m=1}^{M} \left[A^2 |x^u(m)|^2 \sum_{n=n_o}^{mN} s(n,\tau^o)s(n,\tau)\right] .$$

Although we still require a one dimensional search of $J_x(\tau)$ over τ, $J_x(\tau)$ cannot be maximized since no direct measurement can be obtained for $x^u(m)$. However, we can see that $J_x(\tau)$ reaches its maximum at $\tau = \tau^o$, which gives a maximum value of the time limited autocorrelation function $\sum_{n=n_o}^{mN} s(n,\tau^o)s(n,\tau)$. We also observe that this cost function is a sum of partial correlations that are weighted by the power of the received signal in each partial correlation interval. An alternate cost function that has the same maximum value at τ_o as $J_x(\tau)$ is given by

$$J_1(\tau) = \frac{1}{K} \sum_{m=1}^{M} \left[A^2 |x^u(m)|^2 \left|\sum_{n=n_o}^{mN} s(n,\tau^o)s(n,\tau)\right|^2\right] .$$

In this case, we are maximizing the square of the autocorrelation function. Note that this is implied by the fact that any autocorrelation function attains its maximum magnitude at $\tau = \tau^o$. Using A.1 and (1) leads to

$$J_1(\tau) = \frac{1}{K} \sum_{m=1}^{M} \left|\sum_{n=n_o}^{mN} \left(r(n)s(n,\tau) - v(n)s(n,\tau)\right)\right|^2 .$$

To arrive at a practically feasible algorithm, we further assume that:

A.2 The partial cross-correlation of $v(n)$ and $s(n,\tau)$ over N samples is considerably smaller than the partial autocorrelation of $s(n,\tau)$ over the same interval.

This assumption is actually practical since $v(n)$ and $s(n,\tau)$ are independent. In fact, it becomes true as N and $K \to \infty$.[2] In this case, our maximization problem is equivalent to maximizing

$$J(\tau) = \frac{1}{M} \sum_{m=1}^{M} \left|\frac{1}{N} \sum_{n=n_o}^{mN} r(n)s(n,\tau)\right|^2 .$$

Thus, the optimal ML estimate of τ^o, when A.1 and A.2 hold, and for sufficiently large K, becomes

$$\hat{\tau} = \arg \max_{\tau} \frac{1}{M} \sum_{m=1}^{M} \left|\frac{1}{N} \sum_{n=n_o}^{mN} r(n)s(n,\tau)\right|^2 . \quad (3)$$

Figure 1 shows a practical scheme for implementing (3). We can see that this scheme only requires a one dimensional search,

[2] However, if N is small, this cross correlation term will introduce an additive bias in the cost function. This bias term will be studied in the next section.

which is dramatically simpler than the optimal ML estimator given in (2). We will term the partial correlation operation $\frac{1}{N}\sum_{n=n_o}^{mN} r(n)s(n,\tau)$ *coherent integration or coherent averaging* since the phase of the samples of the sequence $\{r(n)s(n,\tau)\}$ is kept during this averaging process. Furthermore, we will term the averaging operation over the M partial correlations *non-coherent integration or non-coherent averaging* since the phase of each of the partial correlation samples is removed before performing the averaging operation.

Here, we may add that in order for A.1 and A.2 to hold simultaneously, a careful choice of N should be made. In the next section, we derive an optimal value for N that would maximize the estimation accuracy of our algorithm.

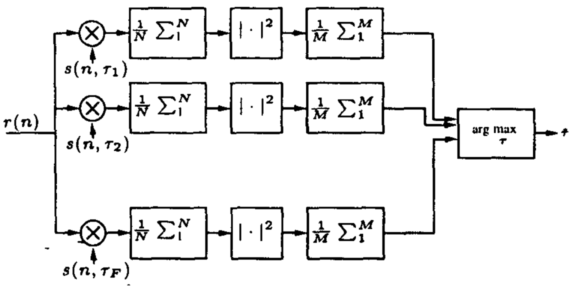

Figure 1. *Sub-optimal estimator for single path fading channels.*

4 Parameter Optimization and Amplitude Estimation

We now finalize the proposed algorithm by providing a design equation for the parameter N. We also show how to estimate A by a simple peak picking operation. In the following analysis, *assumptions* A.1 *and* A.2 *are not used.* Although these assumptions were used in deriving the estimation scheme, ignoring them in the following analysis will help us achieve the following goals: 1) Performance evaluation of the estimation scheme when assumptions A.1 and A.2 do not hold. 2) Arriving at an optimal value of the design parameter N. 3) Deriving an accurate estimation scheme for estimating the received signal amplitude A.

We will consider the case of an infinite received sequence length ($M \to \infty$). Thus, $J(\tau)$ becomes, by the law of large numbers,

$$J(\tau) = E\left|\frac{1}{N}\sum_{n=1}^{N} r(n)s(n,\tau)\right|^2 ,$$

in terms of the expectation operator. Using (1), one obtains

$$J(\tau) = E\left|\frac{1}{N}\sum_{n=1}^{N}\left(Ax^o(n)s(n,\tau^o)s(n,\tau) + v'(n,\tau)\right)\right|^2$$

where $v'(n,\tau) = v(n)s(n,\tau)$.

For mathematical tractability of the analysis, we impose the following assumption:

A.3 The sequence $\{s(n,\tau)\}$ is identically statistically independent (i.i.d), and is independent of the channel fading gain sequence $\{x^o(n)\}$.

Then, it is straightforward to show that at $\tau = \tau^o$, the cost function $J(\tau^o)$ is equal to

$$A^2\left(\frac{R_x(0)}{N} + \sum_{i=1}^{N-1}\frac{2(N-i)R_x(i)}{N^2}\right) + \frac{\sigma_v^2}{N} , \tag{4}$$

where σ_v^2 is the variance of the noise term $v(n)$. Equation (4) shows that as $M \to \infty$, the value of the cost function at $\tau = \tau^o$ is composed of two terms. The first term is proportional to A^2, while the second term is proportional to σ_v^2. Thus, a performance index that we might maximize is the signal-to-noise ratio (SNR), defined as the ratio of the signal and the noise terms at $\tau = \tau^o$. This SNR (S) is given, from (4), by

$$S = \frac{A^2}{\sigma_v^2}\left(R_x(0) + \sum_{i=1}^{N-1}\frac{2(N-i)R_x(i)}{N}\right) .$$

The SNR at the input of our scheme is $\frac{A^2}{\sigma_v^2}$. Thus, the SNR gain introduced by the algorithm (S_G) is given by

$$\begin{aligned} S_G &= \frac{S}{A^2/\sigma_v^2} \\ &= R_x(0) + \sum_{i=1}^{N-1}\frac{2(N-i)R_x(i)}{N} . \end{aligned} \tag{5}$$

The optimal value of the coherent averaging period (N_{opt}) is obtained by maximizing the SNR gain given in (5) with respect to N. Thus, N_{opt} is computed by equating

$$\frac{dS_G}{dN} = \sum_{i=1}^{N_{opt}-1}\frac{2(N_{opt}-i)R_x(i) - 2N_{opt}R_x(i)}{N_{opt}^2}$$

to zero, which directly leads to

$$\sum_{i=1}^{N_{opt}-1} iR_x(i) = 0 . \tag{6}$$

This shows that the coherent integration interval N *should be adapted* based on the available knowledge of the channel according to (6).

We will now use this analysis to obtain an accurate estimate of the amplitude A. Equation (4) shows that the amplitude of the output of our proposed estimation algorithm suffers from two biases. The first bias is an additive noise bias that is caused by non-coherent integration and that increases with the noise variance. This noise bias is given by

$$B_n = \frac{\sigma_v^2}{N} , \tag{7}$$

and it vanishes as $N \rightarrow \infty$. The second bias is a multiplicative fading bias that arises from coherent averaging and increases as the channel rapidly changes. It is given from (4) by

$$B_f = \frac{R_x(0)}{N} + \sum_{i=1}^{N-1} \frac{2(N-i)R_x(i)}{N^2} \quad . \tag{8}$$

It is clear that this multiplicative fading bias is less than or equal to unity. This bias is due to phase misalignment of the complex channel coefficients at different time instants, which leads to a degradation in the coherent integration output. It also vanishes for the case of static channels since B_f is equal to unity in this case, i.e., when A.1 becomes true. This explains why previous conventional designs ignored this bias as fading was not considered in these designs [7]. Clearly, both biases degrade the precision of the estimation of A significantly, especially at low signal-to-noise ratios and fast channel variations.

Two correction factors are therefore needed to correct for the noise and fading biases. Both of the biases should be estimated and used to correct the maximum of the cost function $J(\tau)$. Then, the amplitude estimate $\hat{A}$ can be taken as

$$\hat{A} = \sqrt{C_f\left(J(\tau^o) - C_n\right)} \quad , \tag{9}$$

where C_n and C_f are the two needed correction factors that are given by

$$C_n = \hat{B}_n \quad , \quad C_f = \frac{1}{\hat{B}_f} \quad , \tag{10}$$

where $\hat{B}_n$ and $\hat{B}_f$ are estimates for B_n and B_f, respectively. Obtaining accurate estimates for B_n and B_f is actually feasible in practical applications. A successful application for this correction technique is given in the next section in the context of CDMA location finding (see [11]). We may also add that neither of these correction factors is needed for the estimation of τ^o as the precision of this estimate depends only on the SNR gain over A^2/σ_v^2 given in (5).

5 CDMA IS-95 Example

Figure 2 shows the scheme that is used to implement our estimation procedure in the case of an IS-95 CDMA system. The scheme evaluates the cost function, picks the argument τ that maximizes it, assigns this argument to the time of arrival estimate. The scheme also uses an estimate of the maximum Doppler frequency to calculate the fading bias. It then equalizes the maximum of the cost function for two fading and noise biases and uses the equalized value for amplitude estimation. The estimation algorithm used in this specific case is given in Table 1. Further motivation and explanation of this structure is given in [11], along with a study of its sensitivity to errors in the fading channel Doppler estimate.

Figures 3 and 4 show the estimation mean absolute TOA error and the AOA mean square error versus the received signal chip energy-to-noise ratio over a Rayleigh fading channel for M=128 and various values of the maximum Doppler frequency. The figure also shows that the proposed algorithm outperforms conventional estimation algorithms significantly (see, e.g., [9]). Several other simulation results are given in [11] along with the results of a recent field trial that was performed using our estimation scheme and resulted in a root mean square location error of 57 meters [12], which meets the current required FCC requirements in the case of single path channels.[3]

Table 1: The proposed estimation algorithm.

Given a received sequence $\{r(n)\}_{n=1}^{K}$ that arises from the model:

$$r(n) = A\, x^o(n) s(n, \tau^o) + v(n) \quad ,$$

a (sub-optimal) maximum likelihood estimation algorithm for the time and amplitude of arrival (τ^o, A) that maximizes the signal-to-noise ratio gain at the output of the estimation scheme is given as follows:

$$\begin{aligned} \hat{\tau} &= \arg\max_{\tau} J(\tau) \quad , \\ \hat{A} &= \sqrt{C_f\left(\max_{\tau}(J(\tau)) - C_n\right)} \quad , \end{aligned}$$

where

$$J(\tau) = \frac{1}{M}\sum_{m=1}^{M}\left|\frac{1}{N_{opt}}\sum_{n_o}^{mN_{opt}} (r(n)s(n,\tau))\right|^2 \quad ,$$

N_{opt} is the solution of $\quad \sum_{i=1}^{N_{opt}-1} iR_x(i) = 0$,

$$\begin{aligned} n_o &= (m-1)N_{opt} + 1 \quad , \\ M &= \frac{K}{N_{opt}} \quad , \\ C_n &= \frac{1}{K_n N_{opt}} \sum_{i=1}^{K_n} |r(i)|^2 \quad , \\ C_f &= \left[\frac{R_x(0)}{N_{opt}} + \sum_{i=1}^{N_{opt}-1} \frac{2(N_{opt}-i)R_x(i)}{N_{opt}^2}\right]^{-1} \quad , \end{aligned}$$

$R_x(i)$ is the autocorrelation function of the sequence $\{x^o(n)\}$, and $K_n \leq N_{opt} M$.

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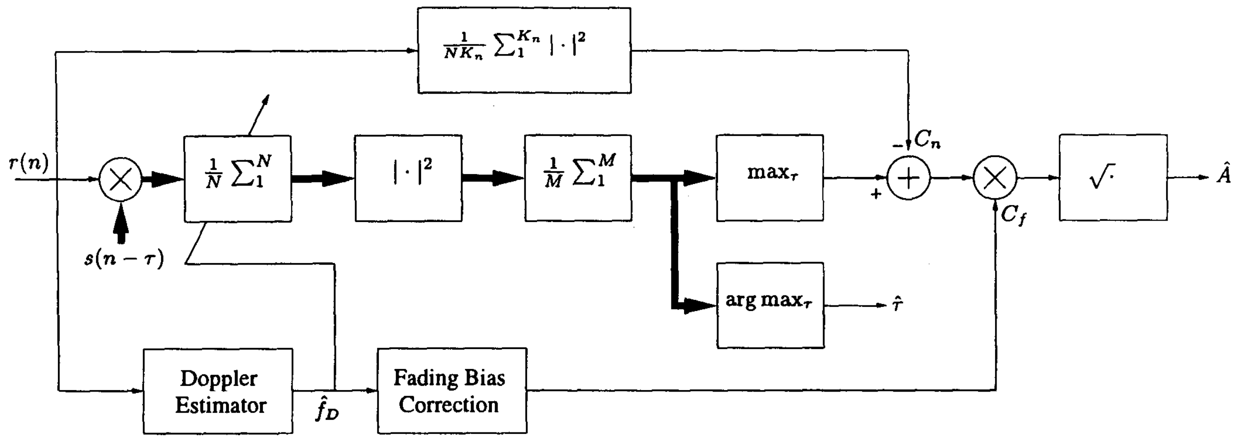

Figure 2. *CDMA estimation scheme.*

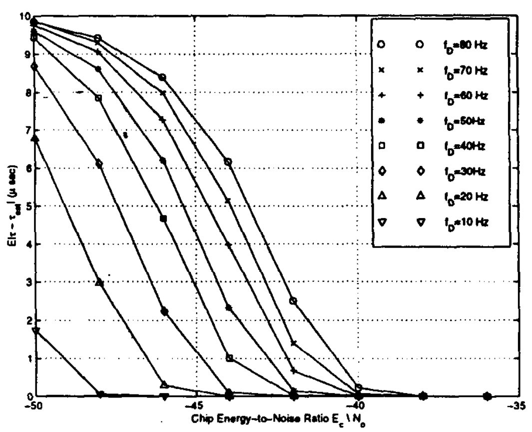

Figure 3. *Mean absolute TOA error vs. E_c/N_o.*

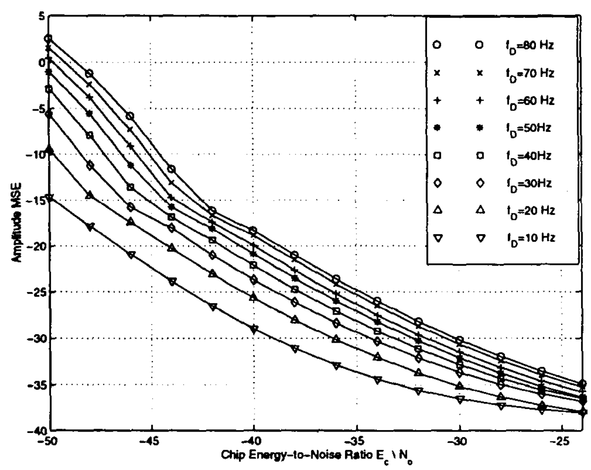

Figure 4. *AOA MSE error vs. E_c/N_o.*

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