A Computationally-Efficient FIR MMSE-DFE for Multi-User Communications

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Abstract

A new theoretical framework is introduced for analyzing the performance of a finite-length minimum-mean-square-error decision feedback equalizer (MMSE-DFE) in a multi-input multi-output (MIMO) environment. The framework includes transmit and receive diversity systems as special cases and quantifies the diversity performance improvement as a function of the number of transmit/receive antennas and equalizer taps. Closed-form expressions for computing the finite-length MIMO MMSE-DFE are presented for two common multi-user detection scenarios.

1 Introduction

In multi-user communication over linear, dispersive, and noisy channels, the received signal is composed of the sum of several transmitted signals corrupted by inter-symbol interference (ISI), inter-user interference (IUI), and noise. Examples include TDMA digital cellular systems with multiple transmit/receive antennas [1], wide-band asynchronous CDMA systems [2], where IUI is also known as multiple access interference (MAI), wide-band transmission over digital subscriber lines (DSL) [3], where IUI takes the form of near-end and far-end crosstalk between adjacent twisted pairs, and in high-density digital magnetic recording where IUI is due to interference from adjacent tracks [4].

Multi-user detection techniques for MIMO systems have been shown to offer significant performance advantages over single user detection techniques that treat IUI as additive colored noise and lumps its effects with thermal noise. Recently, it has been shown that the presence of ISI in these MIMO systems could enhance overall system capacity provided that effective multi-user detection techniques are employed [5, 6].

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The optimum maximum likelihood sequence estimation (MLSE) receiver for MIMO channels was developed in [7]; however, its exponential complexity increase with the number of users and channel memory makes its implementation costly for multi-user detection on severe-ISI channels. Two alternative transceiver structures which are widely used in practice for single-input single-output (SISO) dispersive channels, namely, Discrete Multitone (DMT) and minimum-mean-square-error decision feedback equalizer (MMSE-DFE) have been recently proposed for MIMO dispersive channels as well [6, 8, 2, 9, 10].

In this paper, we present a new analytical framework for analyzing the MIMO MMSE-DFE that is distinct from the work in [8, 2, 9, 10] in three key aspects. First, the MIMO MMSE-DFE feedforward and feedback matrix filters are restricted to be finite impulse response (FIR) for practical implementation and the decision delay is optimized, thus establishing finite-length analogs of the results in [8, 2, 9, 10]. Second, the assumption of an equal number of channel inputs and outputs made in [8, 2] is relaxed. Third, the special structure of the problem can be exploited to derive fast and parallelizable MIMO MMSE-DFE computation algorithms suitable for real-time implementation. These algorithms will be described elsewhere. As shown in [2], computing the MIMO MMSE-DFE for the infinite-length case requires computationally-intense spectral factorizations of matrix rational spectra.

2 The FIR MIMO MMSE-DFE

2.1 Input-Output Model

We consider the general case of a linear, dispersive, and noisy digital communication system with n_i inputs and n_o outputs. We use the standard complex-valued baseband equivalent signal

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A second approach for computing $\tilde{\mathbf{B}}_{opt}$ and $\mathbf{R}_{ee,min}$ utilizes the *block* Cholesky factorization

$$\mathbf{R} = \mathbf{R}_{xx}^{-1} + \mathbf{H}^*\mathbf{R}_{nn}^{-1}\mathbf{H}$$

$$= \begin{bmatrix} \mathbf{L}_1 & \mathbf{0} \\ \mathbf{L}_2 & \mathbf{L}_3 \end{bmatrix} \begin{bmatrix} \mathbf{D}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_2 \end{bmatrix} \begin{bmatrix} \mathbf{L}_1^* & \mathbf{L}_2^* \\ \mathbf{0} & \mathbf{L}_3^* \end{bmatrix} \stackrel{def}{=} \mathbf{LDL}^*$$

where $\mathbf{L}_1$ is of size $n_i(\Delta+1) \times n_i(\Delta+1)$. Using the result in (9) and (10) we get

$$\tilde{\mathbf{B}}_{opt} = \begin{bmatrix} \mathbf{I}_{n_i(\Delta+1)} \\ \mathbf{L}_2\mathbf{L}_1^{-1} \end{bmatrix} \mathbf{C} = \begin{bmatrix} \mathbf{I}_{n_i} \\ \mathbf{L}_2\mathbf{L}_1^{-1}\mathbf{C} \end{bmatrix}$$

$$= \mathbf{L} \begin{bmatrix} \mathbf{e}_{n_i\Delta_{opt}} & \cdots & \mathbf{e}_{n_i(\Delta_{opt}+1)-1} \end{bmatrix} \qquad (13)$$

$$\mathbf{R}_{ee,min} = \mathbf{C}^*\mathbf{D}_1^{-1}\mathbf{C}$$

$$= diag(d_{n_i\Delta_{opt}}^{-1}, \cdots, d_{n_i(\Delta_{opt}+1)-1}^{-1})$$

where the index Δ_{opt} $(0 \leq \Delta_{opt} \leq N_f + \nu - 1)$ is chosen to minimize the trace and determinant of $\mathbf{R}_{ee,min}$. Using Equations (4) and (5), the feedforward matrix taps of (7) can be expressed as follows

$$\mathbf{W}_{opt}^* = \tilde{\mathbf{B}}_{opt}^*\mathbf{R}_{xx}\mathbf{H}^*(\mathbf{H}\mathbf{R}_{xx}\mathbf{H}^* + \mathbf{R}_{nn})^{-1}$$

$$= \tilde{\mathbf{B}}_{opt}^*(\mathbf{R}_{xx}^{-1} + \mathbf{H}^*\mathbf{R}_{nn}^{-1}\mathbf{H})^{-1}\mathbf{H}^*\mathbf{R}_{nn}^{-1} \qquad (14)$$

Case 2

Assume that users are ordered so that lower-indexed ones are detected first, more specifically, that *current decisions from lower-indexed users are used by higher-indexed users in making their decisions* , i.e., $\mathbf{B}_0$ is a lower-triangular matrix. In this case, a *standard* Cholesky factorization of the matrix $\mathbf{R} = \mathbf{R}_{xx}^{-1} + \mathbf{H}^*\mathbf{R}_{nn}^{-1}\mathbf{H}$ is performed and the matrix feedback filter is given by the n_i *adjacent* columns of $\mathbf{L}$ that correspond to a diagonal matrix with the smallest trace or determinant. Therefore, Equations (13) and (14) are used to compute MIMO MMSE-DFE filter settings *with the understanding that* $\mathbf{L}$ *is now a lower-triangular matrix not a block lower-triangular matrix.* This result is shown next.

Starting from Equation (8), we have

$$trace(\mathbf{R}_{ee}) = \sum_{k=0}^{n_i-1} \mathbf{e}_k^*\tilde{\mathbf{B}}^*\mathbf{L}^{-*}\mathbf{D}^{-1}\mathbf{L}^{-1}\tilde{\mathbf{B}}\mathbf{e}_k$$

$$\stackrel{def}{=} \sum_{k=0}^{n_i-1} \tilde{\mathbf{b}}_k^*\mathbf{L}^{-*}\mathbf{D}^{-1}\mathbf{L}^{-1}\tilde{\mathbf{b}}_k ,$$

where $\tilde{\mathbf{b}}_k^* \stackrel{def}{=} \tilde{\mathbf{B}}\mathbf{e}_k = \begin{bmatrix} \mathbf{0}_{1\times(n_i\Delta+k)} & 1 & \mathbf{b}_k^* \end{bmatrix}$. Therefore,

$$trace(\mathbf{R}_{ee}) = \sum_{k=0}^{n_i-1} \sum_{i=0}^{n_i(N_f+\nu)-1} d_i^{-1}\tilde{\mathbf{b}}_k^*\mathbf{L}^{-*}\mathbf{e}_i\mathbf{e}_i^*\mathbf{L}^{-1}\tilde{\mathbf{b}}_k$$

$$= \sum_{k=0}^{n_i-1} \sum_{i=0}^{n_i(N_f+\nu)-1} d_i^{-1}|\mathbf{e}_i^*\mathbf{L}^{-1}\tilde{\mathbf{b}}_k|^2$$

$$\stackrel{def}{=} \sum_{k=0}^{n_i-1} MSE_k .$$

To minimize this sum, we need to choose $\tilde{\mathbf{b}}_k$ to minimize each term MSE_k. It can be readily checked that MSE_0 is minimized by searching for the maximum diagonal elements of $\mathbf{D}$, call it $d_{n_i\Delta_{opt}}$ where $(0 \leq \Delta_{opt} \leq N_f + \nu - 1)$, and setting $\tilde{\mathbf{b}}_0 = \mathbf{L}\mathbf{e}_{n_i\Delta_{opt}}$ which results in MSE_0 equal to its minimum value of $d_{n,\Delta_{opt}}^{-1}$. Since we are restricting all n_i users to have the same decision delay, it is clear that $trace(\mathbf{R}_{ee})$ is minimized by setting $\tilde{\mathbf{b}}_k = \mathbf{L}\mathbf{e}_{(n_i\Delta_{opt})+k}$ (where $0 \leq k \leq n_i - 1$), which is equivalent to (13) and results in

$$trace(\mathbf{R}_{ee,min}) = \sum_{k=0}^{n_i-1} \sum_{m=0}^{n_i(N_f+\nu)-1} d_{n_im}^{-1}|\mathbf{e}_m^*\mathbf{e}_{\Delta_{opt}+k}|^2$$

$$= \sum_{k=0}^{n_i-1} d_{(n_i\Delta_{opt})+k}^{-1} ,$$

The performance measure adopted is the decision-point arithmetic SNR defined by

$$ASNR \stackrel{def}{=} \frac{\frac{1}{n_i(N_f+\nu)}trace(\mathbf{R}_{xx})}{\frac{1}{n_i}trace(\mathbf{R}_{ee,min})}$$

$$= \frac{\frac{1}{n_i(N_f+\nu)}trace(\mathbf{R}_{xx})}{\frac{1}{n_i}\sum_{k=0}^{n_i-1} d_{(n_i\Delta_{opt})+k}^{-1}} .$$

3 Computer Simulations

The channel impulse responses used in our computer simulations are unit-energy 4-tap FIR filters. The 4 taps are randomly-generated complex zero-mean uncorrelated Gaussian random variables. The input and noise processes are assumed to be uncorrelated. The performance results are calculated by averaging over 100 channel realizations.

In Figure 1, we plot the MIMO MMSE-DFE ASNR difference between the cases of $\mathbf{B}_0$ constrained to being a lower-triangular matrix or equal to the identity matrix. We assume $n_i = n_o = 2$, $N_f = 3$, and set the SNR of the second user (*the higher-indexed user*) at 10 dB while increasing the SNR of the first user (*the lower-indexed user*) from −10 dB to 30 dB. It can be seen from the figure that constraining $\mathbf{B}_0$ to be lower triangular always results in better performance than the case $\mathbf{B}_0 = \mathbf{I}$. As expected, this performance improvement increases as the SNR of the lower-indexed user (whose current decisions are also fed back and used in detecting the higher-indexed user) is increased.

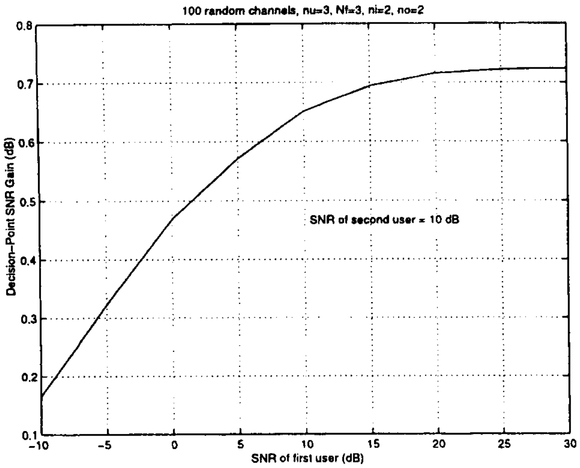

Figure 1: Variation of ASNR difference of the MIMO MMSE-DFE under the two constraints $\mathbf{B}_0 = \mathbf{I}$ and $\mathbf{B}_0$ is lower triangular versus SNR of first user assuming second user at 10 dB SNR and $N_f = \nu = 3$

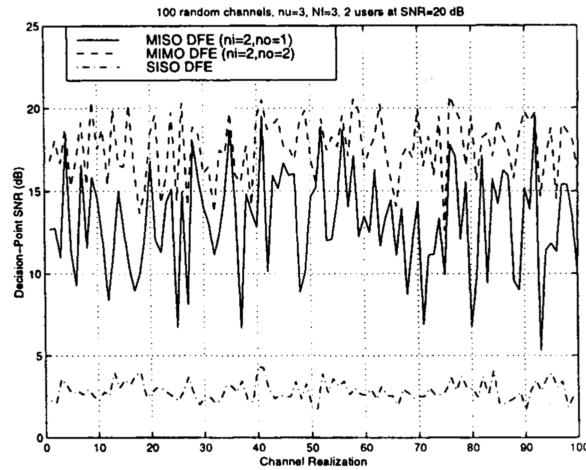

Figure 3: Variation of ASNR of the MIMO, MISO, and SISO MMSE-DFE in presence of 2 users at 20 dB SNR for different channel realizations and $N_f = \nu = 3$

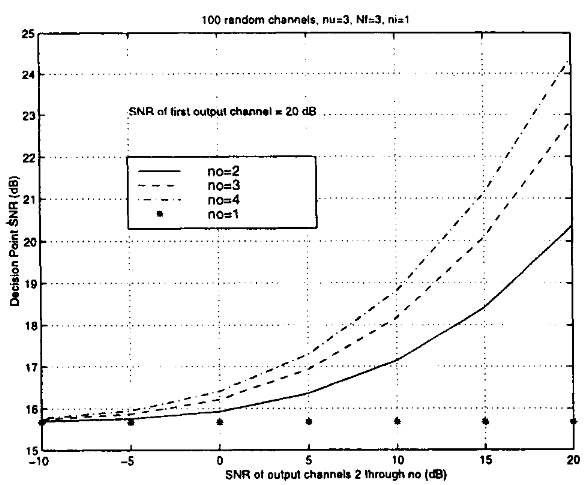

Figure 2: Variation of ASNR of the SIMO MMSE-DFE with n_0 and SNR of output channels 2 through n_o for $N_f = \nu = 3$ assuming SNR of first output channel equals 20 dB

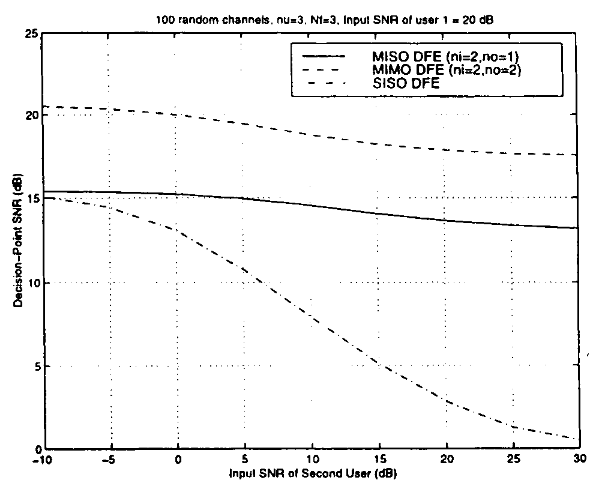

Figure 4: Variation of ASNR of the MIMO, MISO, and SISO MMSE-DFE in presence of 2 users versus SNR of second user assuming first user at 20 dB SNR and $N_f = \nu = 3$